

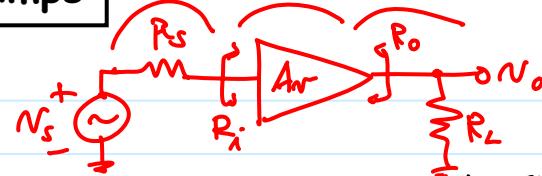
Lecture 13: OpAmps & ECPAnnouncements:

- Dusan's Office Hours:
  - ↳ So far, not many takers on Thursdays
  - ↳ Thus, move to Tuesdays, 2:30-4:30 p.m.
- No lecture Thursday, next week, 3/12
  - ↳ Make up lecture on next Tuesday evening, at 7 p.m., in 247 Cory
- Homeworks now due at 8 p.m. on Tuesdays

Today:

- Start Op Amps
  - ↳ Ideal op amps
  - ↳ Diff pairs
  - ↳ Offset Voltage,  $V_{OS}$
  - ↳ Finite Gain
  - ↳ 2 stage op amps
  - ↳ Finite BW
  - ↳ Stability
  - ↳ Compensation of Op Amps
  - ↳ Slew Rate
  - ↳ Power Supply Rejection
  - ↳ Settling time
  - ↳ Single-stage cascade op amps for better bandwidth performance
- Some of the above is review
  - ↳ We will do the review material using pre-made lecture notes
  - ↳ Slow down for things are new to you

$$\frac{R_o}{R_s+R_o} \cdot A_{vI} \cdot \frac{R_L}{R_o+R_L} = \text{Gain}$$

Ideal Voltage Amplifier

→ ideal when  $\frac{N_o}{N_S} = A_{vI}$ ; i.e., when source and load R's do not influence the gain of the amplifier.

For this to occur, the voltage division at the input & output must be eliminated.  
This happens when:

$R_I = \infty$  } These resistance values define an  
 $R_o = 0$  } ideal voltage amplifier.

We'll look at other amplifier types later.

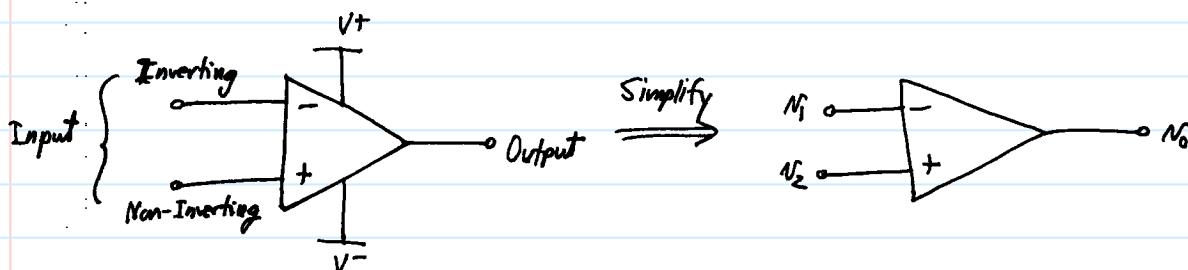
This, then, naturally leads us to:

Ideal Operational Amplifiers (Op Amps)

The work horse of analog electronics → combinations of op amps w/ feedback components allow the implementation of analog computers, sampled-data systems, analog filters, A/D Converters, DAC's, instrumentation amplifiers

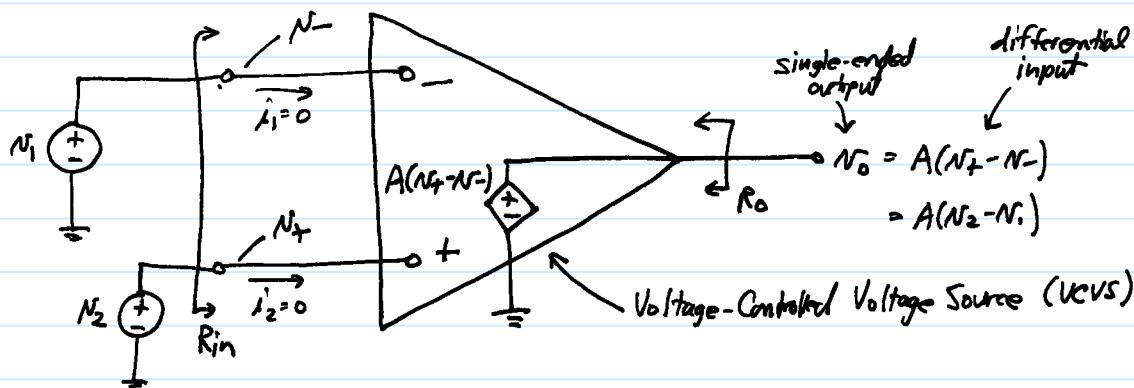
In general,

have a minimum of 5 terminals:



Perhaps the best way to define an op amp is thru its equivalent ckt:

Equivalent Ckt. of an Ideal Op Amp:



Properties of Ideal Op Amps:

$$\textcircled{1} \quad R_{in} = \infty \quad \xrightarrow{\text{leads to}} \quad \textcircled{4} \quad i_+ = i_- = 0$$

$$\textcircled{2} \quad R_o = 0$$

$$\textcircled{3} \quad A = \infty \quad \xrightarrow{\text{leads to}} \quad \textcircled{5} \quad V_+ = V_- \text{, assuming } N_o = \text{finite}$$

↳ Why? Because for  $\infty(V_+ - V_-) = V_o = \text{finite}$

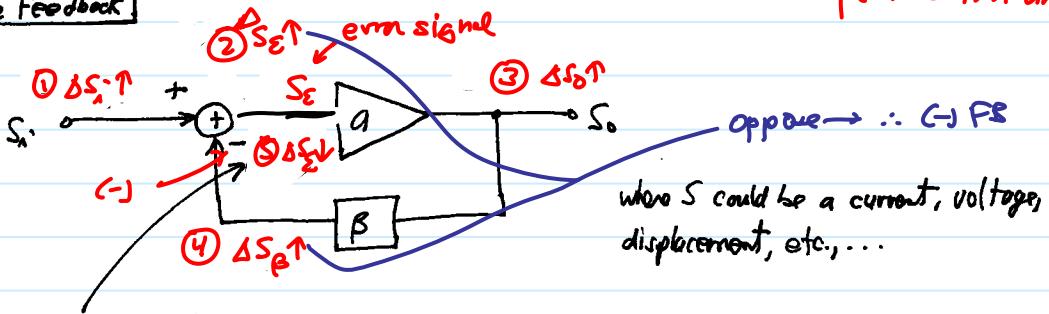
$$\therefore \underbrace{V_+ - V_-}_{\frac{N_o}{\infty}} = 0 \rightarrow V_+ = V_- \Rightarrow \text{virtual short abt. (virtual ground)}$$

Big assumption! ( $N_o = \text{finite}$ )

How can we assume this?  $\Rightarrow$  only when there is an appropriate negative feedback path!

### Negative Feedback

→ to determine whether or not there's  $\text{C} \rightarrow \text{FB}$ , do a perturbation analysis:



Negative feedback acts to oppose or subtract from input.

$$\left. \begin{array}{l} S_o = aS_e \\ S_e = S_i - \beta S_o \end{array} \right\} \Rightarrow S_o = a(S_i - \beta S_o)$$

$$S_o(1 + a\beta) = aS_i \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1 + a\beta}}$$

overall transfer function  
 $a\beta = \text{loop gain}$   
 $\beta = \text{feedback factor}$

$$[a \rightarrow \infty] \Rightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$$

$$\therefore S_o = \frac{1}{\beta} S_i = \text{finite} \checkmark$$

(when there is neg. FB around the amplifier)

### In Summary:

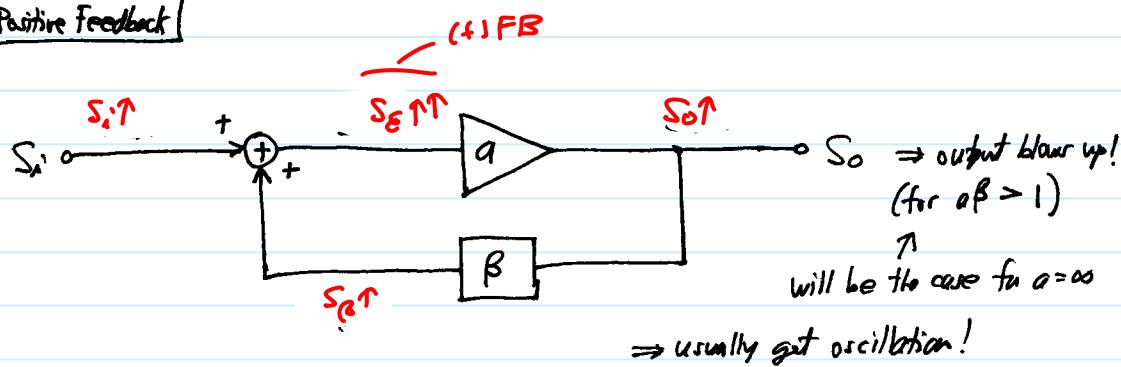
① Neg. FB can insure  $S_o = \text{finite}$  even with  $a = \infty$ .  
Overall

② Gain dependent (or overall T.F.) dependent only on external components. (e.g.,  $\beta$ )

③ Overall (closed-loop) gain  $\frac{S_o}{S_i}$  is independent of amplifier gain  $a$ .

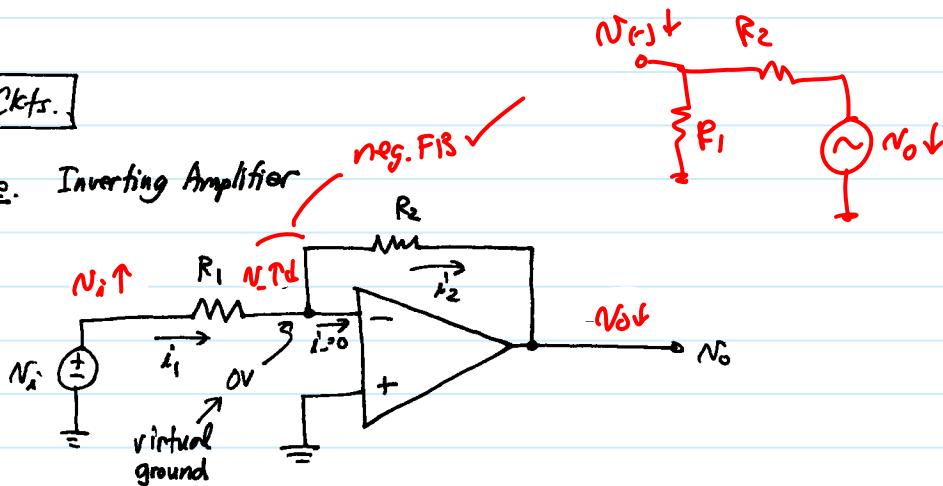
↖ very important!  $\Rightarrow$  as you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain.  
i.e., if you're shooting for  $a = 50,000$ , you might get 47,000 or 60,000 instead.

Contract w/ Positive Feedback



Thus, for a bounded, controllable function, need negative FB around an op amp.

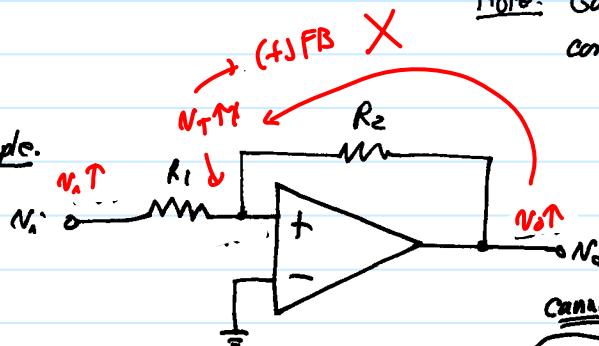
## Op Amp Ckt's.

Example. Inverting Amplifier

① Verify that there is negative FB. ✓

②  $\therefore N_o \neq \text{finite} \rightarrow N_+ = N_- \rightarrow$  node attached to (-) terminal is virtual ground③  $i_- = 0 \therefore i_1 = i_2$ 

$$\left. \begin{aligned} i_1 &= \frac{N_i - 0}{R_1} = \frac{N_i}{R_1} = i_2 \\ N_o &= 0 - i_2 R_2 = -i_2 R_2 \end{aligned} \right\} \Rightarrow N_o = -\left(\frac{N_i}{R_1}\right) R_2 = -\frac{R_2}{R_1} N_i \therefore \frac{N_o}{N_i} = -\frac{R_2}{R_1}$$

Note: Gain dependent only on  $R_1$  &  $R_2$  (external components), not on the op amp gain.Example.

① Verify that there is neg. FB X

$N_o = L^+$  or  $L^-$  depending on initial condns.

$(+)\text{rail}$

$(-) \text{ rail}$

cannot analyze using ideal op amp method!

$\therefore N_o \neq \text{finite}, N_+ \neq N_-$

$\Rightarrow$  this ckt. will "rail out"

$$\begin{aligned} N_+ = (+) &\rightarrow L^+ \\ N_+ = (-) &\rightarrow L^- \end{aligned}$$

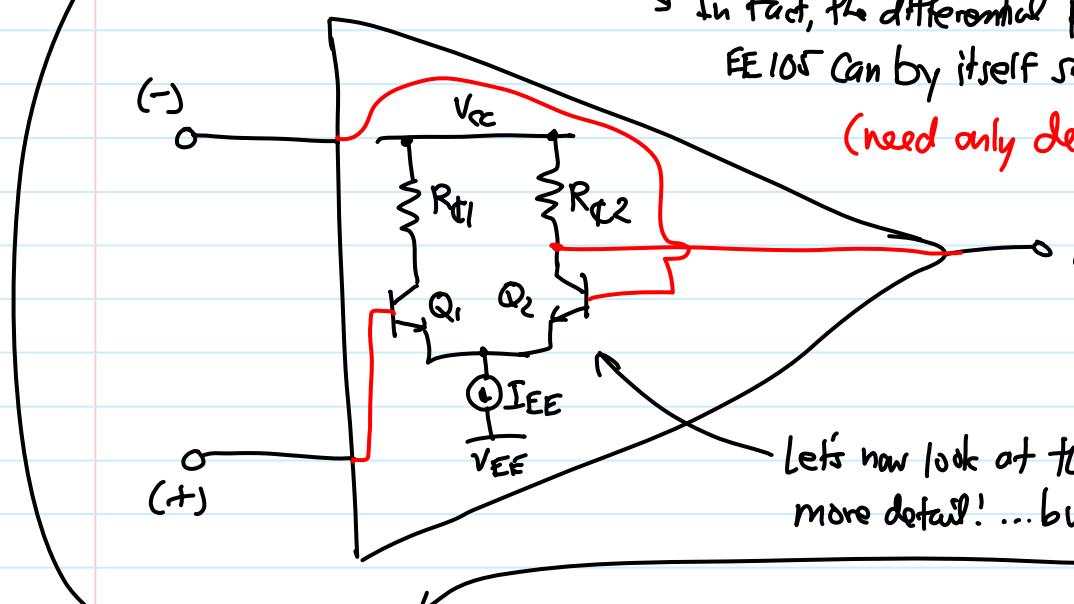
How does one make an op amp? (It turns out, you already know!)

⇒ Basic Needed Attributes:

- ① Gain (voltage gain).
- ② Two inputs, (+) and (-).
- ③ One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE 105 can by itself serve as an op amp!

(need only decide which of the inputs is the (+) and (-) terminals)

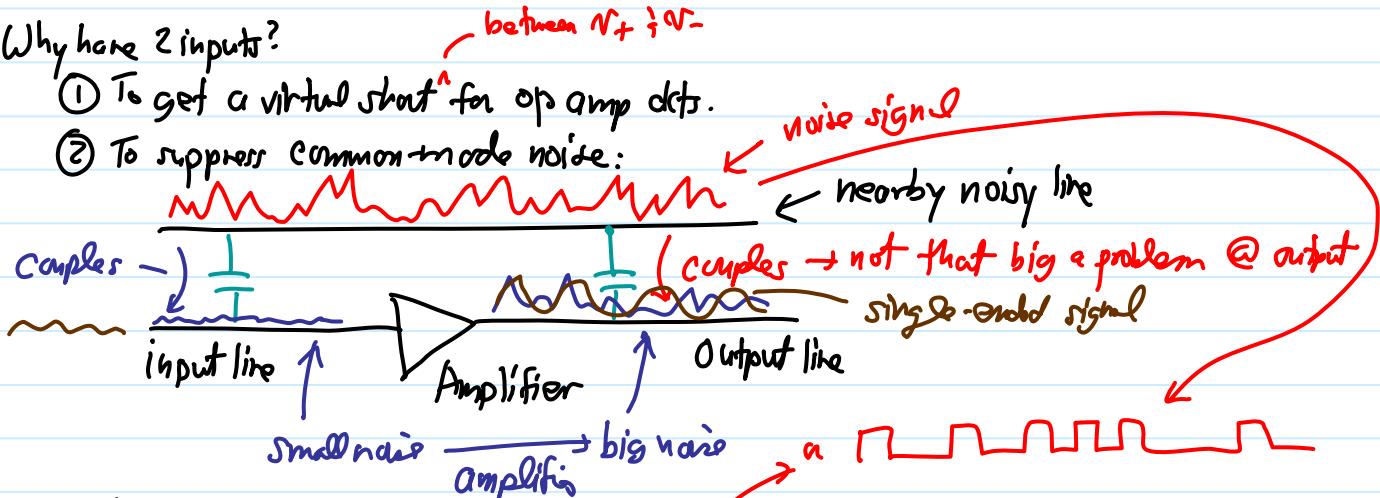


Let's now look at the diff. pair in more detail! ...but first

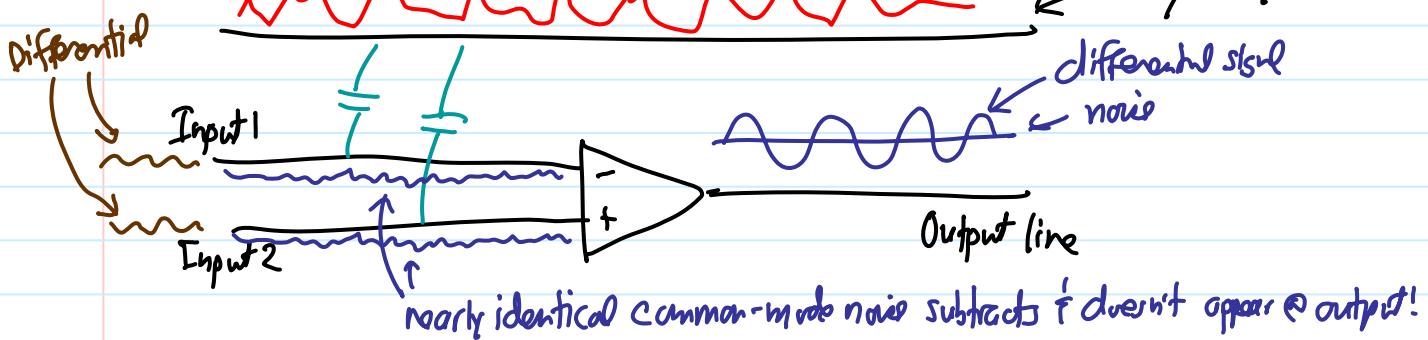
Why have 2 inputs?

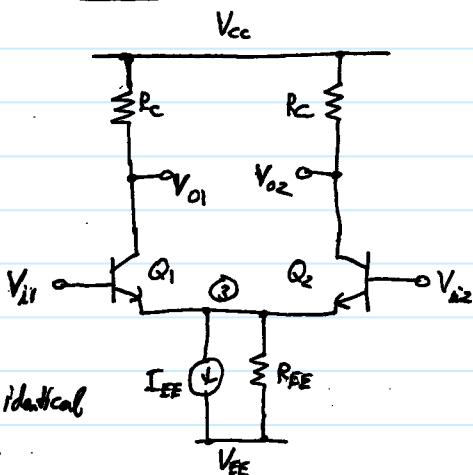
① To get a virtual short for op amp dc's.

② To suppress common-mode noise:



Can avoid this w/ a differential input:



**Differential Pair (Emitter-Coupled Pair)**

Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

Definition:  $V_{id} = V_{i1} - V_{i2}$  (differential input)  
 $V_{icm} = \frac{V_{i1} + V_{i2}}{2}$  (common-mode input)

$$\Rightarrow \begin{cases} V_{i1} = V_{icm} + \frac{V_{id}}{2} \\ V_{i2} = V_{icm} - \frac{V_{id}}{2} \end{cases}$$

Differential Gain =  $A_{id} = \frac{V_{o1} - V_{o2}}{V_{id}} = \frac{V_{od}}{V_{id}}$  (want this to be large for this differential amplifier)

Common-Mode Gain =  $A_{icm} = \frac{V_{o1}}{V_{icm}} \approx \frac{V_{o2}}{V_{icm}}$  (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio = CMRR =  $\frac{A_{id}}{A_{icm}}$  (should be very high to favor the differential mode and reject the common-mode)

⇒ we also want a high Common-Mode Input Range to reject DC input offsets

⇒ Note: No need for bypass capacitors (large) to the inputs or outputs → can just use direct coupling!

**Biasing & Large Signal Common-Mode Behavior**

Case:  $R_{EE} = \infty$  → ideal current source biasing →  $I_{E1} = I_{E2} = \frac{I_{EE}}{2} \rightarrow V_{o1} = V_{o2} \Rightarrow V_{od} = 0$

If  $V_{icm} \uparrow \rightarrow V_{EE} \uparrow$ , but current draw from I<sub>EE</sub> stays constant ∵  $I_{C1} \neq I_{C2}$  stay constant → bias pt.

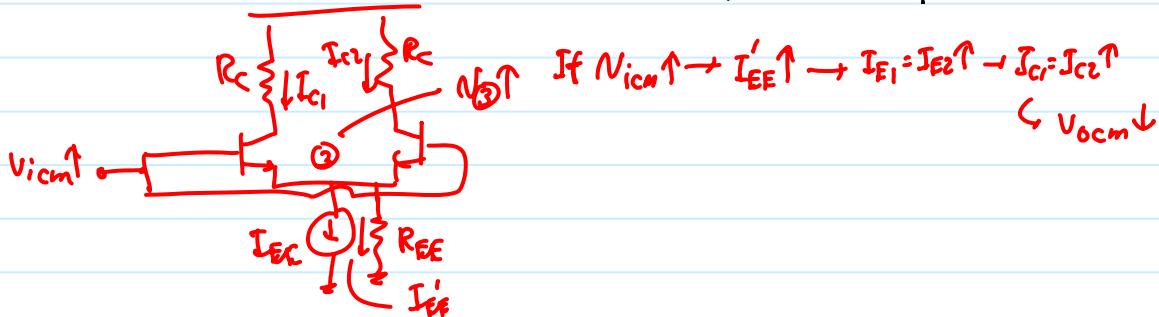
$$g_m = \frac{1}{2} \frac{I_{EE}}{V_T}$$

doesn't change  
 $V_{icm}$  stayed const.

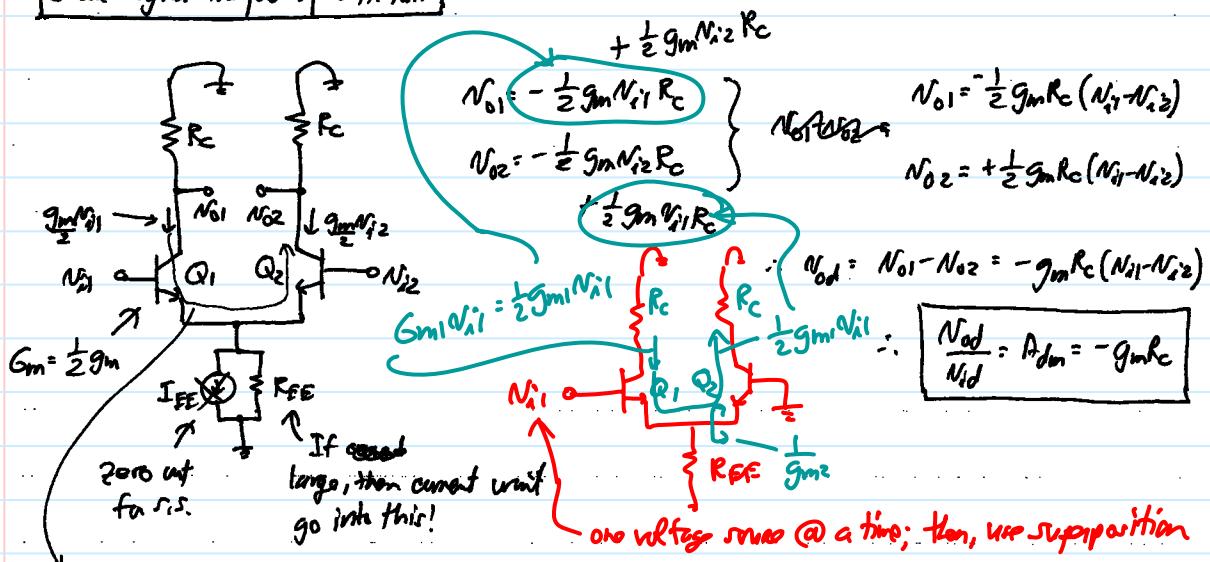
Case:  $R_{EE}$  finite  $\rightarrow V_{EE} = V_{i1} - V_{BE(on)}$

If  $V_{icm} \uparrow \rightarrow V_{EE} \uparrow \rightarrow I_{E1} > I_{E2} \uparrow$  (current draw =  $I_{EE} + \frac{V_{od}}{R_{EE}}$ )

⇒ in general,  $R_{EE}$  will be large, so this component won't be large, and the bias pt. won't change much

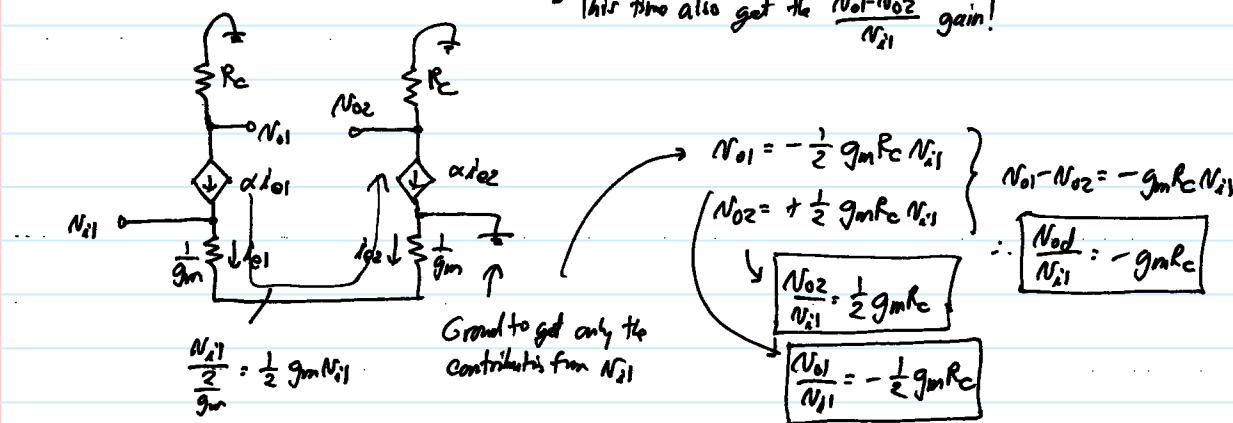


## Small-Signal Analysis of Diff. Pair



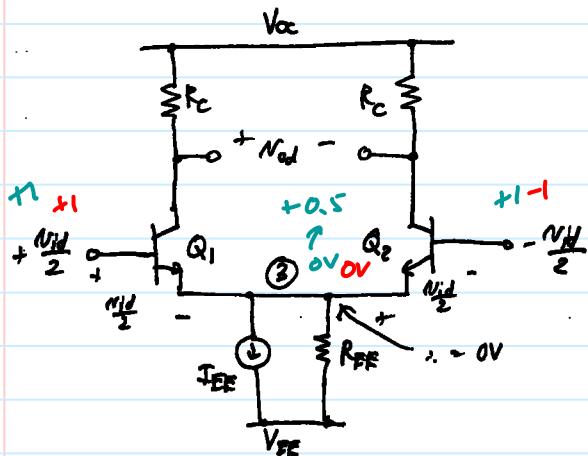
→ Easiest to see this happening using the T-model: (for those who must see the model click.)

→ This time also get the  $\frac{N_1 - N_2}{N_1}$  gain!



## Diff. Mode Analysis

Assume a def. w/ only diff. input:



Total current thru  $I_{EE}$  = const.

→  $V_E = \text{const.}$  as input changes

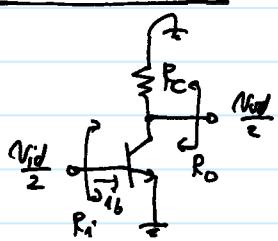
→ ③ acts as an inertial ground! →  $V_0 = 0V$  (always!)

5. we can ground ③, and thus, have

a Differential Half. Ct.

Note: Can really only make this for a purely symmetrical obj.!

Differential Half Ckt.



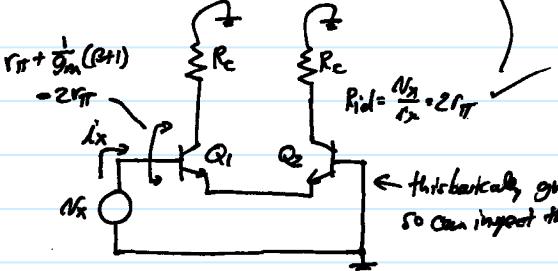
$$\text{By Inspection: } \frac{N_{id}/2}{N_{id}/2} = \frac{N_{id}}{N_{id}} = A_{dm} = -g_m R_c$$

$$\frac{N_{id}/2}{i_b} = r_{\pi} \rightarrow R_{id} \frac{N_{id}}{i_b} = 2r_{\pi} > R_{id}$$

$$\frac{N_{id}/2}{i_b} = r_o || R_c \rightarrow R_{id} \frac{N_{id}}{i_b} = 2(r_o || R_c) \approx 2R_c = R_{id}$$

S.S. params. determined  
w/  $I_c = \frac{I_{FE}}{2}$

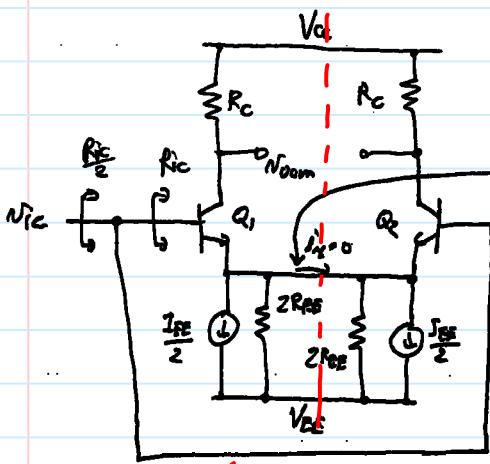
First define



← this is actually grounded,  
so can inject there

Common-Mode Analysis

Assume a pure CM input → tie inputs together



$$(2R_{EE})(2R_{EE}) \cdot R_{FE}$$

By symmetry,  $i_X = 0 \Rightarrow$  thus, really have the equivalent of an open ckt. has

∴ can split the ckt. into CM half-ckt's!

S.S. CM Half-Ckt.

$$R_{ic} = r_{\pi} + (\beta + 1)(2R_{EE}) \quad @ \text{each input}$$

$$A_{cm} = \frac{N_{icm}}{N_{ic}} = -\frac{g_m R_c}{1 + g_m (2R_{EE})} \approx -\frac{R_c}{2R_{EE}}$$

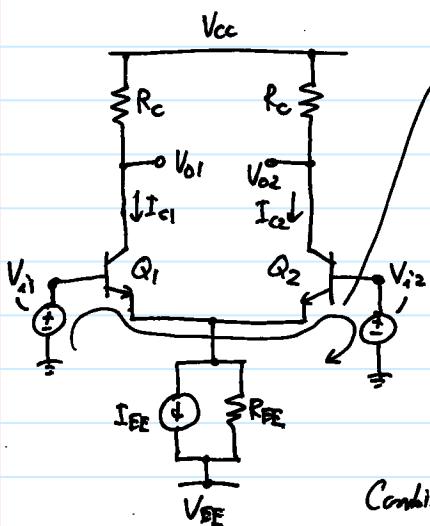
want small for large CMRR ∴ want

$R_{EE}$  large!

$$\text{Common-Mode Rejection Ratio} = \text{CMRR} = \frac{A_{dm}}{A_{cm}} = \frac{-g_m R_c}{-\frac{g_m R_c}{1 + g_m (2R_{EE})}} = \boxed{\text{CMRR} = 1 + 2g_m R_{EE}}$$

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the EOP.

## Large Signal ECP Performance

Find  $I_{c1} \neq I_{c2}$ :

$$\text{EVL: } V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$$

$$I_{c1} = I_{s1} \exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{c1}}{I_{s1}}\right), V_{be2} = V_T \ln\left(\frac{I_{c2}}{I_{s2}}\right)$$

$$V_{i1} = V_T \ln\left(\frac{I_{c1}}{I_{c2}} \frac{I_{c2}}{I_{s1}}\right) - V_{i2} = 0 \rightarrow \ln\left(\frac{I_{c1}}{I_{c2}}\right) = \frac{V_{i1} - V_{i2}}{V_T} = \frac{V_{id}}{V_T}$$

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{id}}{V_T}\right) \quad (1)$$

$$I_{EE} = I_{c1} + I_{c2} = \alpha (I_{c1} + I_{c2}) \quad (2)$$

Combine (1) &amp; (2) to get:

$$I_{c1} = \frac{\alpha I_{EE}}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)}, \quad I_{c2} = \frac{\alpha I_{EE}}{1 + \exp\left(\frac{V_{id}}{V_T}\right)} \quad (3)$$

Find  $V_{od}$ :

$$V_{o1} = V_{cc} - I_{c1} R_C$$

$$V_{o2} = V_{cc} - I_{c2} R_C$$

$$V_{od} = V_{o1} - V_{o2} = (I_{c2} - I_{c1}) R_C$$

$$= \alpha_F I_{EE} R_C \left\{ \frac{1}{1 + \exp\left(\frac{V_{id}}{V_T}\right)} - \frac{1}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)} \right\}$$

$$\times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)}$$

$$\times \frac{\exp\left(+\frac{V_{id}}{2V_T}\right)}{\exp\left(+\frac{V_{id}}{2V_T}\right)}$$

$$= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right) + \exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

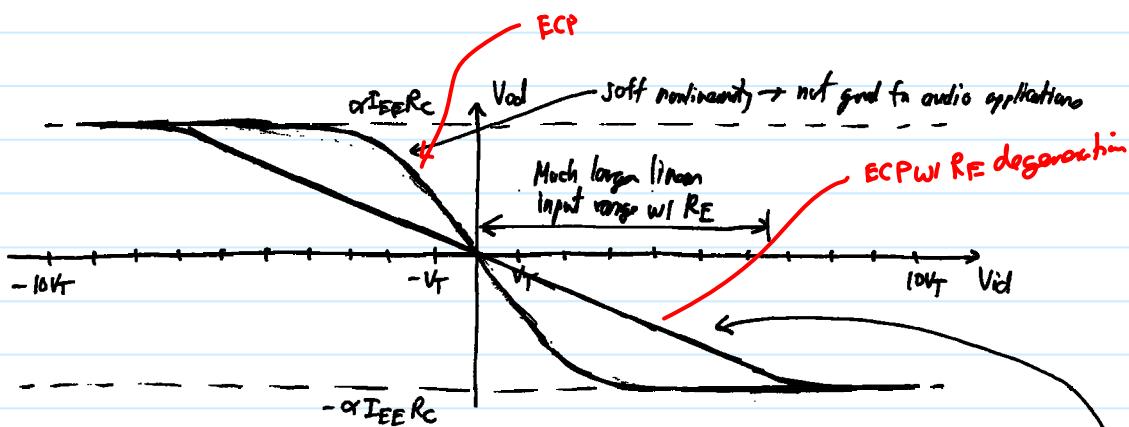
$$= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_C \frac{\sinh\left(\frac{V_{id}}{2V_T}\right)}{\cosh\left(\frac{V_{id}}{2V_T}\right)}$$

$$\begin{cases} \sinh u = \frac{1}{2}(e^u - e^{-u}) \\ \cosh u = \frac{1}{2}(e^u + e^{-u}) \end{cases} \quad u = -\frac{V_{id}}{2V_T}$$

$$\therefore V_{od} = \alpha_F I_{EE} R_C \tanh\left(-\frac{V_{id}}{2V_T}\right)$$

From our knowledge of the Taylor series for  
 $\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \dots$

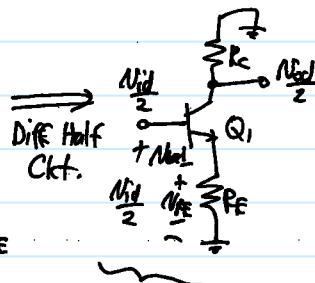
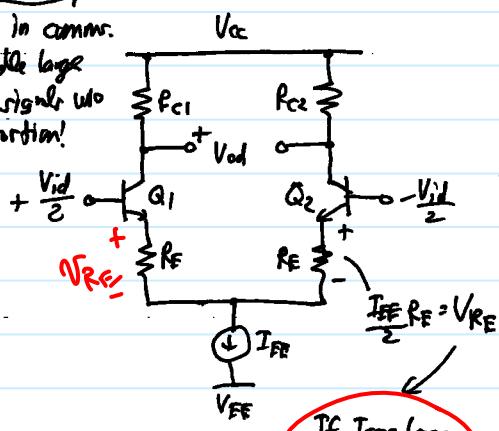
this is fairly linear for small  $V_{id}$ , but gets nonlinear  
 abruptly when  $V_{id}$  approaches a threshold value!



In the above curve, the  $\frac{V_{id}}{V_{id}}$  Xfer function is really only linear for  $Vid \ll V_T \rightarrow$  beyond  $V_T$ , start to enter the nonlinear realm of curve  $\rightarrow$  causes signal distortion: e.g., phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xfer in amplifiers)

Needed in common:  
to handle large  
input signals w/o  
distortion!



$$A_{differential} = -\frac{g_m R_C}{1 + g_m R_E}$$

$\Rightarrow$  s.r. gain reduced, but the linear range is increased

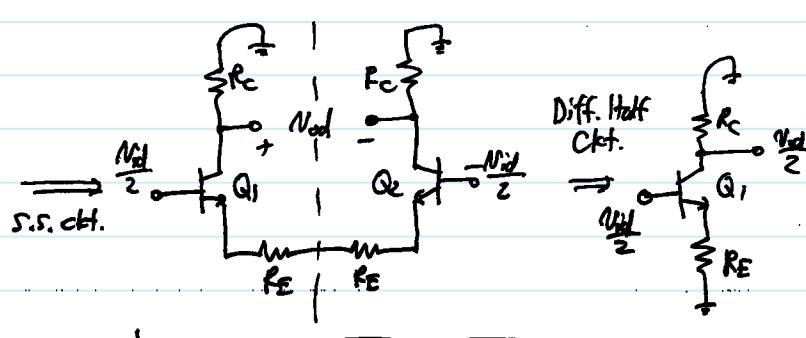
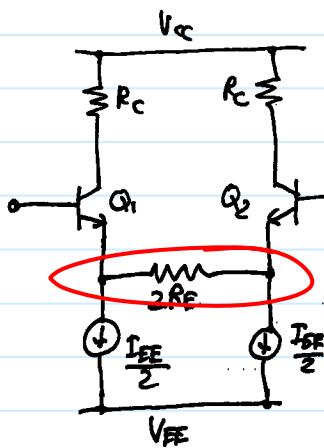
If  $I_{FE}$  is large,  
then this can  
force large  
supply voltage

$$\frac{V_{id}}{2} = V_{be1} + V_{RE}$$

This can utilize

$V_{be1} \ll V_T$  if this absorbs some of the input voltage!

Alternative biasing Technique If Need Larger DC Current:-



Same S.S. performance w/o the need to drop  
a DC voltage across  $R_E \rightarrow$  ~~good/better~~

Can use lower  $V_{cc}$  &  $V_{ee}$ .