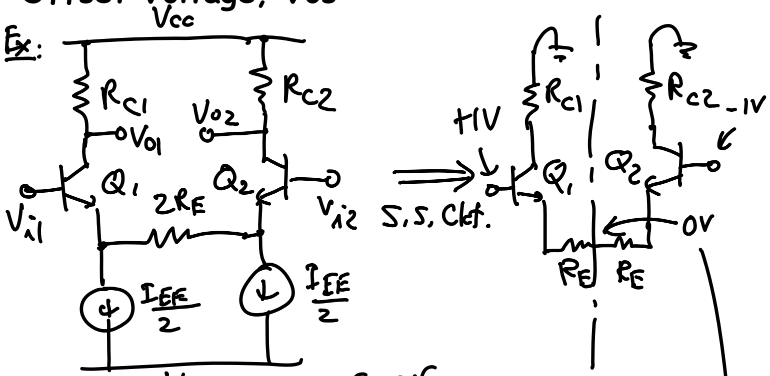
Announcements:

- No lecture Thursday, next week, 3/12
  - Make up lecture on next Tuesday evening, at 7 p.m., in 247 Cory
- Homeworks now due at 8 p.m. on Tuesdays

Today:

- Diff pairs
- Offset Voltage, Vos

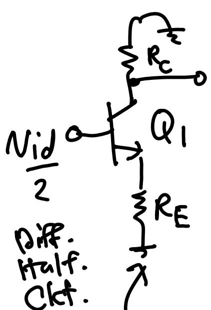
Ex:



$Q_1 = Q_2 \rightarrow r_{o1} = r_{o2} = r_o$, $g_{m1} = g_{m2} = g_m$

$R_{C1} = R_{C2} = R_C$

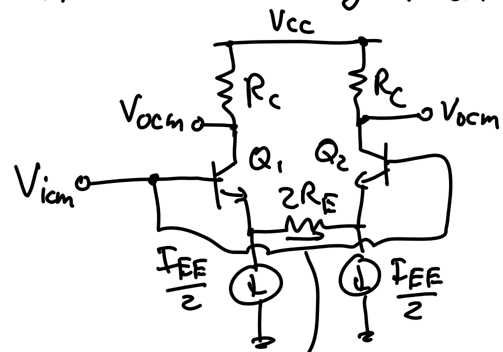
(i) Find the differential-mode gain: (A_{dm})



$$\frac{(v_{od}/2)}{(v_{id}/2)} = -\frac{g_m(r_o \| R_C)}{1 + g_m R_E}$$

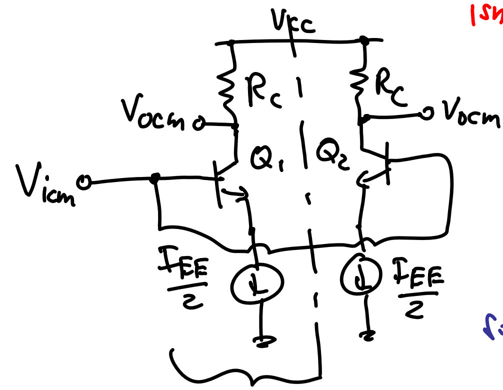
$$A_{dm} = \frac{v_{o1} - v_{o2}}{v_{i1} - v_{i2}} = \frac{v_{od}}{v_{id}} =$$

(ii) Find the common-mode gain, A_{cm}:

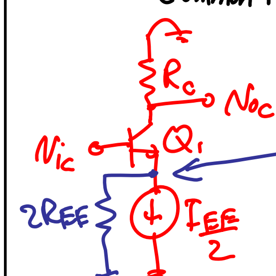


How much current goes through this resistor?

$i = 0$ ∴ the resistor effectively isn't there!



Common-Mode Half Ckt



r_o ( ) → R_C

$$A_{cm} = \frac{v_{oc}}{v_{ic}} = -\frac{g_m(r_o \| R_C)}{1 + g_m(\infty)} = 0$$

for the ideal case!

In reality:

$$A_{cm} = \frac{v_{oc}}{v_{ic}} = -\frac{g_m(r_o \| R_C)}{1 + g_m(2R_{EE})}$$

If there is a mismatch in the ckt., e.g., $R_{C1} \neq R_{C1}$, then we can also define:

$A_{cm-dm} \triangleq$ common-mode input to differential-mode output gain

$$= \frac{v_{od}}{v_{ic}} = \frac{v_{o1}-v_{o2}}{v_{i1}} = \frac{v_{o1}-v_{o2}}{v_{i2}} \quad (\text{w}/\, v_{i1}=v_{i2})$$

$A_{dm-cm} \triangleq$ differential-mode input to common-mode output gain

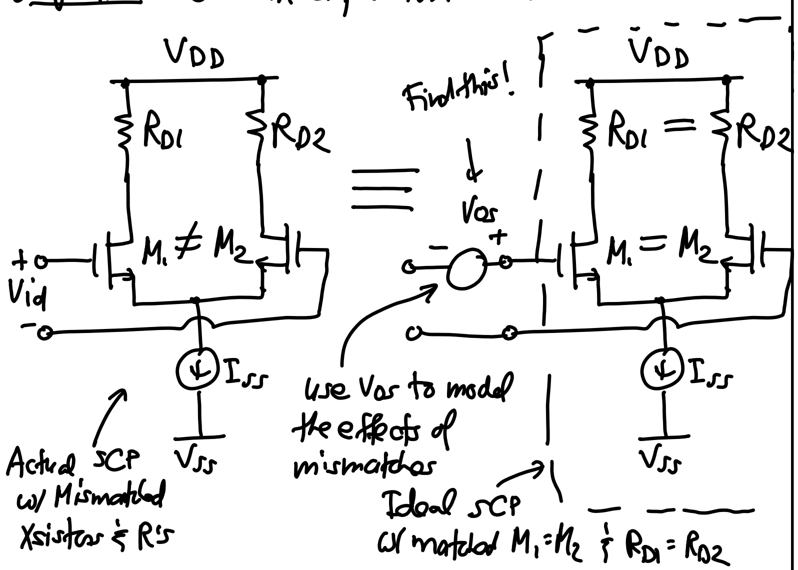
$$= \frac{v_{oc}}{v_{id}} = \frac{v_{oc}}{v_{i1}-v_{i2}} \quad \left[\text{w}/\, v_{oc} = \tfrac{1}{2}(v_{o1}+v_{o2})\right]$$

You will be experiencing these in your HW.

V_{OS} of a Mismatched SCP

First, go to prepared material on offset voltage & current.

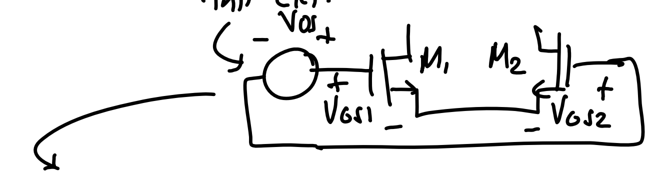
Objective: Derive an expression for V_{OS}.



V_{OS} arises due to variations in:

1. Xsistors M_1 & M_2 → $\frac{W}{L}$ & V_t vary
2. $R_{D1} \neq R_{D2}$ → cause variations in gain

Definition. $V_{OS} = V_{id}$ needed to get $V_{od} = 0$V in this ckt.



KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS1} - V_{GS2}$$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D}}{\mu_n C_{ox}(W/L)_2}} \quad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$ | $\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$

$I_D = \frac{I_{D1}+I_{D2}}{2}$ | $\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$

$\Delta V_t = V_{t1} - V_{t2}$ | $\Delta R_D = R_{D1} - R_{D2}$

$V_t = \frac{1}{2}(V_{t1}+V_{t2})$ | $R_D = \frac{1}{2}(R_{D1}+R_{D2})$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1): $2I_D\left(1+\frac{\Delta I_D}{2I_D}\right)$

$$V_{os} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}$$

$\frac{W}{L}\left[1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{os} \rightarrow V_{od} = 0 \quad \therefore I_{D1}R_{D1} = I_{D2}R_{D2}$

→ mismatch in I_D must be opposite that of R_D

$$\therefore \frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Mismatch

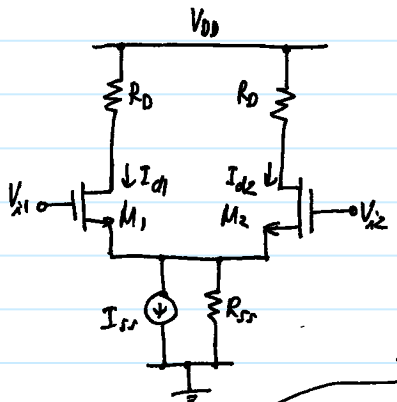
Bias Independent

Geometric Variations (i.e., layout)

Scale w/ overdrive

⇒ Now, go to Bipolar Mismatch prepared material.

MOSFET Source-Coupled Pair



Assume: M_1 & M_2 are identical.

Find $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$.

$\Longrightarrow$ approach: get $V_{id} = f(\Delta I_d) \longrightarrow$ then invert to get $\Delta I_d = f(V_{id})$

$$I_{d1} = \underbrace{\tfrac{1}{2}\mu_n C_{ox}\left(\tfrac{W}{L}\right)_1}_{k}(V_{gs1}-V_t)^2 \Longrightarrow V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{k}}$$

$$\therefore\ V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}}$$

Define. $\left.\begin{aligned} \Delta I_d &= I_{d1} - I_{d2} \\ I_d &= \frac{I_{d1}+I_{d2}}{2} \end{aligned}\right\}\quad \begin{aligned} I_{d1} &= I_d + \frac{\Delta I_d}{2} \\ I_{d2} &= I_d - \frac{\Delta I_d}{2} \end{aligned}$

$$V_{id} = \sqrt{\frac{2\left(I_d + \frac{\Delta I_d}{2}\right)}{k}} - \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{k}} \Longrightarrow \frac{k}{2}V_{id}^2 = I_d + \cancel{\frac{\Delta I_d}{2}} - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2} + I_d - \cancel{\frac{\Delta I_d}{2}}$$

$$\frac{k}{2}V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2}$$

$\Longrightarrow$ now rearrange to get ΔI_d (algebra)

Solve for ΔI_d: $\quad \Delta I_d = \frac{k}{2}V_{id}\left(\frac{2I_{SS}}{k/2} - V_{id}^2\right)^{1/2} = \boxed{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{id}\sqrt{\left(\frac{2I_{SS}}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\right) - V_{id}^2} = \Delta I_d}$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

V_{GS} for $I_D = \frac{I_{SS}}{2}$

$$|V_{id}| \le \sqrt{\frac{2I_{SS}}{k}} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{2}\,(V_{GS} - V_t)$$

if true then input devices are both saturated

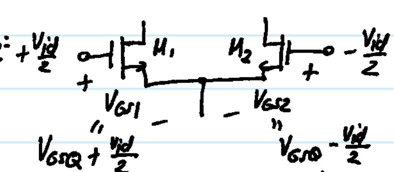
Thus, to extend the linear input range:

① $I_{SS}\uparrow \longrightarrow (V_{GS}-V_t)\uparrow$

② $W\downarrow$

③ $L\uparrow$

To derive this:



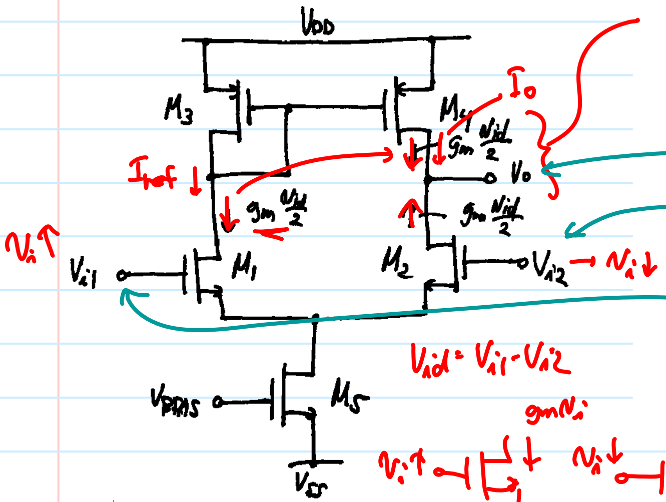
When $\frac{V_{id}}{2} \ge V_{GSQ} - V_t = \Delta V$ then M_2 will cut-off

$\therefore\ V_{id} \le 2(V_{GSQ} - V_t) \longrightarrow$ to maintain saturation

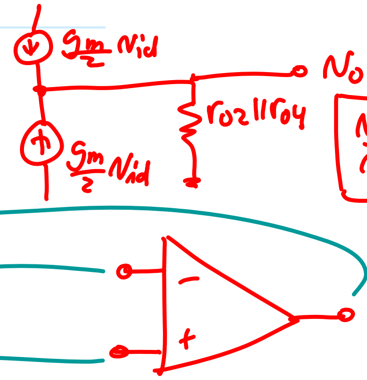
$$V_{GSQ} - V_t = \sqrt{\frac{2I_{d2}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \frac{V_{id}}{2}$$

then plug in ΔI_d & solve for V_{id}

MOS Differential Stage w/ Current Mirror Load

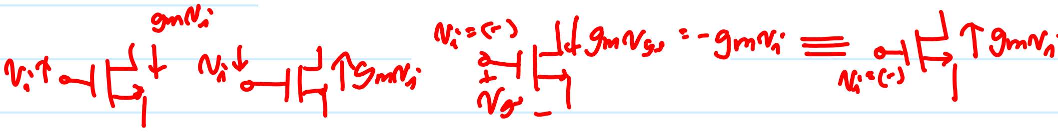


$V_{id} = V_{i1} - V_{i2}$

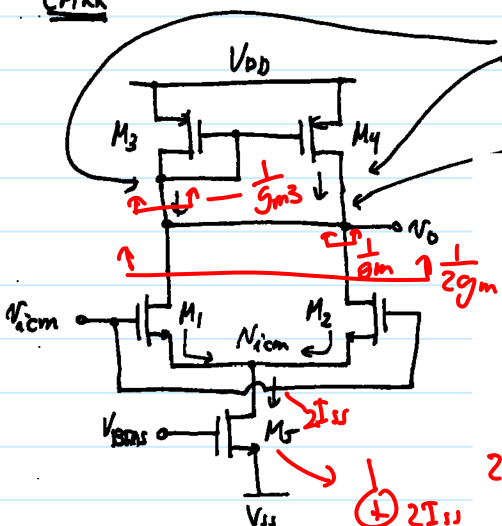


$v_o = (g_m v_{id})(r_{o2} \| r_{o4})$

$$\boxed{\frac{v_o}{v_{id}} = g_m (r_{o2} \| r_{o4})}$$



CMRR –

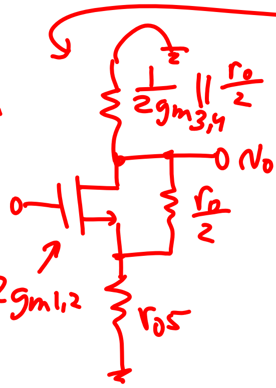


$V_{DS} = V_{D4}$ (they track each other) → provided the ckt. is completely symmetrical.

⇒ Thus, can short the drains of M_3 & M_4

↳ truly symmetric → we can draw a half ckt.

↳ or just combine two ckts together!



$$\boxed{A_{cm} \cong - \frac{2g_{m1,2}\left(\frac{1}{2g_{m3,4}}\right)}{1 + 2g_{m1,2} r_{o5}}}$$

Thus:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \cancel{g_{m1,2}}(r_{o1,2} \| r_{o3,4})(1 + 2g_{m1,2} r_{o5})\left(\frac{g_{m3,4}}{\cancel{g_{m1,2}}}\right)$$

$$\boxed{CMRR = (1 + 2g_{m1,2} r_{o5})\, g_{m3,4}(r_{o1,2} \| r_{o3,4})}$$

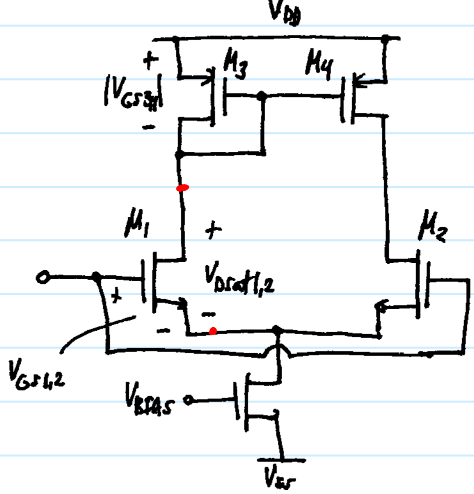
Common-Mode Input Range – Range of input voltages for which all devices remain in saturation.

Low End – must keep M_5 saturated

$$V_{icm(min)} = CMR- = V_{SS} + V_{OV5} + V_{GS1,2}$$

$$\boxed{CMR- = V_{SS} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}(W/L)_5}} + V_{t1,2} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox}(W/L)_{1-2}}}}$$

High End - keep M_1, M_2 saturated



$$V_{icm(max)} = CMR+ = V_{DD} - |V_{GS\,3,4}| - V_{ov1,2} + V_{GS1,2}$$

$$V_{icm(max)} = CMR+ = V_{DD} - \sqrt{\frac{I_{SS}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4}}} - |V_{t3,4}| + V_{t1,2}$$

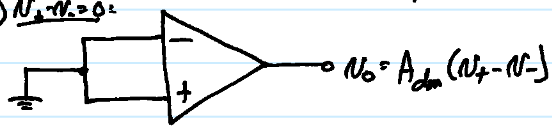
Device Mismatch Effects in Diff. Amplifiers

⇒ up to this point, we've assumed that Q_1 & Q_2 are perfectly matched
⇒ in actual ckt., get device mismatches due to processing variations

The Result:

→ Output not zero when input is zero → $v_{od} \neq 0$ when $v_{id} = 0$!

① $v_+ - v_- = 0$:



Ideal Case: $v_o = 0$

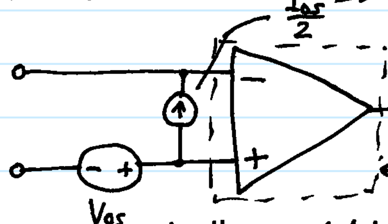
Reality: $v_o \neq 0$, even w/ $(v_+ - v_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if Q_1 & Q_2 not matched. (for BJT & JFET only.)

To model those effects, introduce:

① Input Offset Voltage, V_{OS} → Typ. 1-5 mV for BJT

② Input Offset Current, I_{OS} → Typ. $I_{OS} = 10$ nA for BJT



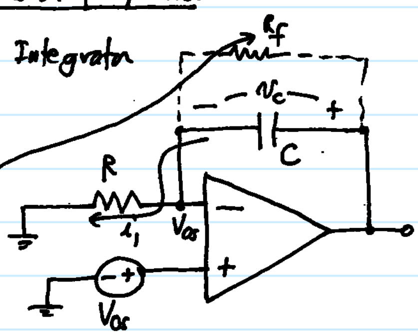
$\frac{I_{OS}}{2}$ → $I_{OS} \triangleq I_{B1} - I_{B2}$ (difference between input currents)

← Ideal Op Amp → well, it could have finite A_v, R_i, & R_o!

V_{OS} → voltage needed to set $v_o = 0$ in this ckt. (negating the effect of mismatches)

Effect of V_{OS} on Op Amp Ckts. –

Example. Integrator



$$v_o = V_{OS} + \frac{1}{C}\int_0^t i_1 dt$$
$$= V_{OS} + \frac{1}{C}\int_0^t \frac{V_{OS}}{R} dt$$
$$= V_{OS}\left(1 + \frac{t}{RC}\right) + v_C\big|_{t=0}$$

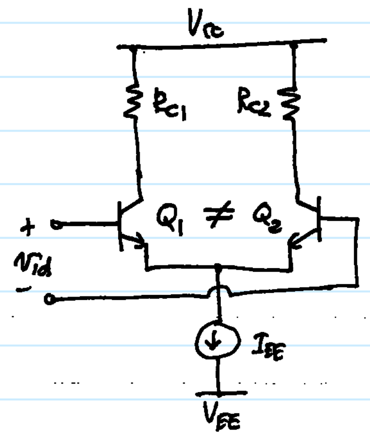
→ will continue to increase until op amp hits the voltage rails

Fix: Place an R_f in shunt w/ the C
→ then $v_o = V_{OS}\left(1 + \frac{R_f}{R}\right)$, and railing doesn't happen
⇒ but, usually R_f = large to allow the C to dominate the integrator Xfer Function ∴ $v_o = V_{OS}\left(1 + \frac{R_f}{R}\right)$ can be quite large → still want V_{OS} = small

V_{OS} is even more important in setting the resolution of A/D converters and other precision ckts.

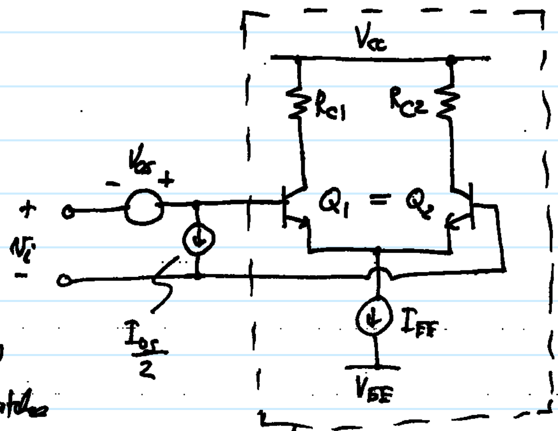
V_{OS} in a Mismatched ECP

<u>Objective:</u> Derive an expression for V_{OS}.



$\equiv$

Equate to an ideal ECP + use V_{OS} + I_{OS} to model the effect of mismatches



Actual ECP w/ Mismatched Xsistors + R's

Ideal ECP w/ Matched Q_1 + Q_2 and $R_{C1} = R_{C2}$

<u>Input Offset Voltage V_{OS} arises due to variations in:</u>

① Xsistors, Q_1 + Q_2 → I_S + β vary: $I_S = \dfrac{q n_i^2 \bar{D}_n A}{N_A W_B(V_{CB})}$ ← "function of"

∴ $I_{S1} \neq I_{S2}$ can be caused by:

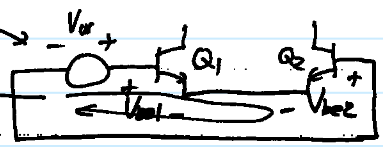
(i) $A_1 \neq A_2$ (etching tolerance limits)

(ii) $N_{A1} \neq N_{A2}$ (doping variations of base)

(iii) $W_B = f(V_{CB})$ (width variations exacerbated by V_{CB} diff.)

② $R_{C1} \neq R_{C2}$ → cause gain variation

<u>Definition.</u> $V_{OS} = V_{id}$ to get $V_{od} = 0$, ~~which occurs when~~ in this ckt.:



<u>KVL:</u> $V_{OS} - V_{be1} + V_{be2} = 0$

$$V_{OS} = V_{be1} - V_{be2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} = V_T \ln\left(\frac{I_{C1}}{I_{C2}} \frac{I_{S2}}{I_{S1}}\right)$$

<u>Find $\frac{I_{C1}}{I_{C2}}$ in terms of design elements:</u>

[When $V_{id} = V_{OS}$ → $V_{od} = 0V$] → $V_{od} = (\cancel{V_{CC}} - I_{C1}R_{C1}) - (\cancel{V_{CC}} - I_{C2}R_{C2}) = 0$

$$I_{C1}R_{C1} = I_{C2}R_{C2} \longrightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}}$$

$$\boxed{V_{OS} = V_T \ln\left(\frac{R_{C2}}{R_{C1}} \frac{I_{S2}}{I_{S1}}\right)}$$

This is an exact equation for V_{OS}. It's often more useful + intuitive to express this in terms of percent variations (and eventually standard deviations).

V_{OS} of a Mismatched ECP

Convert to Percent Variation Form –

Define. $R_c = \frac{R_{c1}+R_{c2}}{2}$, $\Delta R_c = R_{c1}-R_{c2}$; $I_s = \frac{I_{s1}+I_{s2}}{2}$, $\Delta I_s = I_{s1}-I_{s2}$

Objective: Express V_{os} in terms of percent variations $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$.

In general: $\Delta X = X_1 - X_2$, $X = \frac{X_1+X_2}{2}$ $\Rightarrow$ $X_1 = X + \frac{\Delta X}{2}$, $X_2 = X - \frac{\Delta X}{2}$ $\Rightarrow$ Thus: $R_{c1} = R_c + \frac{\Delta R_c}{2}$, $R_{c2} = R_c - \frac{\Delta R_c}{2}$; $I_{s1} = I_s + \frac{\Delta I_s}{2}$, $I_{s2} = I_s - \frac{\Delta I_s}{2}$

With these formulations:

$$V_{OS} = V_T \ln\left[\frac{R_{c2}}{R_{c1}}\frac{I_{s2}}{I_{s1}}\right] = V_T \ln\left\{\frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}}\;\frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}}\right\} = V_T \ln\left\{\frac{1 - \frac{\Delta R_c}{2R_c}}{1 + \frac{\Delta R_c}{2R_c}}\;\frac{1 - \frac{\Delta I_s}{2I_s}}{1 + \frac{\Delta I_s}{2I_s}}\right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \dots\right] \Rightarrow V_{os} \cong V_T\left\{-\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s}\right\}$$

taking the first term assuming $\Delta R \ll R_c$ & $\Delta I_s \ll I_s$

$$\boxed{V_{OS} = V_T\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}$$

Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically varying parameters for a given process run & layout, one usually expresses terms in the form of variances when specifying V_{os}:

→ since $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like powers:

$$\sigma_{V_{os}} = V_T\sqrt{\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta I_s/I_s}}$$

Ex: Typ. $\sigma_{\Delta R_c/R_c} \sim 0.01$, $\sigma_{\Delta I_s/I_s} \sim 0.05$

$\therefore\ \sigma_{V_{os}} = (26m)\sqrt{(0.01)^2 + (0.05)^2} = \underline{1.3mV}$ Typ. V_{os} for BJT ~ $\underline{1\text{-}5mV}$

V_{os} Drift w/ Temperature

$$\frac{dV_{os}}{dT} = \underbrace{\frac{kT}{q}\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}_{\text{indep. of } T}\frac{1}{T} = \frac{V_{os}}{T}$$

[in kelvin]

Ex. $\frac{dV_{os}}{dT} = \frac{1.3m}{300K} = 4.3\,\mu V/°C$ drift around $T = 300K$.

I_{os} in a Mismatched ECP

By Definition: $I_{os} = I_{B1} - I_{B2} = \boxed{\dfrac{I_{c1}}{\beta_1} - \dfrac{I_{c2}}{\beta_2} = I_{os}}$

To express in percent variations:

$$\begin{cases} I_{c1} = I_c + \dfrac{\Delta I_c}{2} \\ I_{c2} = I_c - \dfrac{\Delta I_c}{2} \end{cases} \qquad \begin{cases} \beta_1 = \beta + \dfrac{\Delta\beta}{2} \\ \beta_2 = \beta - \dfrac{\Delta\beta}{2} \end{cases}$$

$$\therefore I_{os} = \frac{I_c + \frac{\Delta I_c}{2}}{\beta + \frac{\Delta\beta}{2}} - \frac{I_c - \frac{\Delta I_c}{2}}{\beta - \frac{\Delta\beta}{2}} = \frac{I_c}{\beta}\left\{\frac{1+\frac{\Delta I_c}{2I_c}}{1+\frac{\Delta\beta}{2\beta}} - \frac{1-\frac{\Delta I_c}{2I_c}}{1-\frac{\Delta\beta}{2\beta}}\right\}$$

$$\left[\frac{1}{1+x} \approx 1 - x + x^2 - \cdots\right] \rightarrow \;= \frac{I_c}{\beta}\left\{\left(1+\frac{\Delta I_c}{2I_c}\right)\left(1-\frac{\Delta\beta}{2\beta}\right) - \left(1-\frac{\Delta I_c}{2I_c}\right)\left(1+\frac{\Delta\beta}{2\beta}\right)\right\}$$

$$= \frac{I_c}{\beta}\left\{\cancel{1} + \frac{\Delta I_c}{2I_c} - \frac{\Delta\beta}{2\beta} - \cancel{\frac{\Delta I_c}{2I_c}\frac{\Delta\beta}{2\beta}} - \cancel{1} + \frac{\Delta I_c}{2I_c} - \frac{\Delta\beta}{2\beta} + \cancel{\frac{\Delta I_c}{2I_c}\frac{\Delta\beta}{2\beta}}\right\}$$

$$\boxed{I_{os} = \frac{I_c}{\beta}\left\{\frac{\Delta I_c}{I_c} - \frac{\Delta\beta}{\beta}\right\}}$$

But for $V_{od} = 0V \Rightarrow \dfrac{I_{c1}}{I_{c2}} = \dfrac{R_{c2}}{R_{c1}} \longrightarrow \dfrac{\Delta I_c}{I_c} = -\dfrac{\Delta R_c}{R_c}$

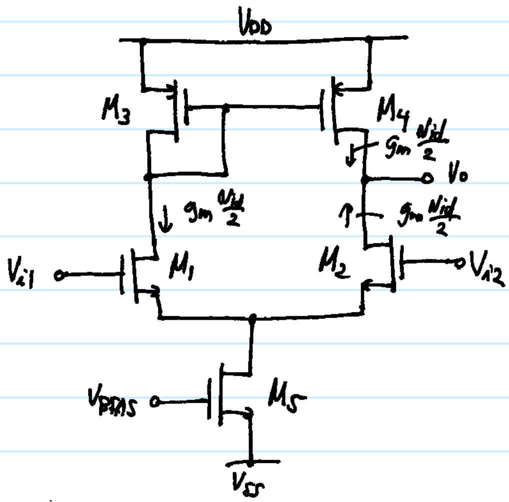
$$\therefore \boxed{I_{os} = -\frac{I_c}{\beta}\left(\frac{\Delta R_c}{R_c} + \frac{\Delta\beta}{\beta}\right)}$$

Ex. Typ: $\sigma_{\Delta\beta/\beta} = 0.1$, $\sigma_{\Delta R_c/R_c} = 0.01$

$$\longrightarrow I_{os} = -\frac{I_c}{\beta}\left[\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta\beta/\beta}\right]^{1/2} \cong -0.1\frac{I_c}{\beta} \approx -0.1 I_B = I_{os}$$

Diff. Pair w/ I-Mirror Load V_{OS}

MOS Differential Stage w/ Current Mirror Load



all the same FB effects, etc...

Small Signal Gain: (similar to BJT)

$$\frac{v_o}{v_{id}} = g_{m2}(r_{o2}\|r_{o4}) = \frac{g_{m2}}{g_{ds2}+g_{ds4}} = \frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}}{\lambda_2 I_{D2}+\lambda_4 I_{D4}}$$

$$= \frac{\sqrt{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{SS}}}{\frac{I_{SS}}{2}(\lambda_2+\lambda_4)} \Rightarrow \boxed{\frac{v_o}{v_{id}} = \frac{2}{\lambda_2+\lambda_4}\sqrt{\frac{\mu_n C_{ox}(W/L)_2}{I_{SS}}}}$$

Offset Voltage – $V_{os} = V_{gs1} - V_{gs2}$ when $V_{od} = 0V$

$$V_{os} = \Delta V_{t1,2} + \Delta V_{t3,4}\left(\frac{g_{m3,4}}{g_{m1,2}}\right) + \frac{(V_{GS}-V_t)_{1,2}}{2}\overbrace{\left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}}\right]}^{\left[\frac{\Delta(W/L)_{1,2}}{(W/L)_{1,2}} - \frac{\Delta(W/L)_{3,4}}{(W/L)_{3,4}}\right]}$$

Via similar derivation to what we just did

For small V_{os}: ① small $(V_{GS}-V_t)$

② $g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2}$ & $\left(\frac{W}{L}\right)_{3,4} < \left(\frac{W}{L}\right)_{1,2}$