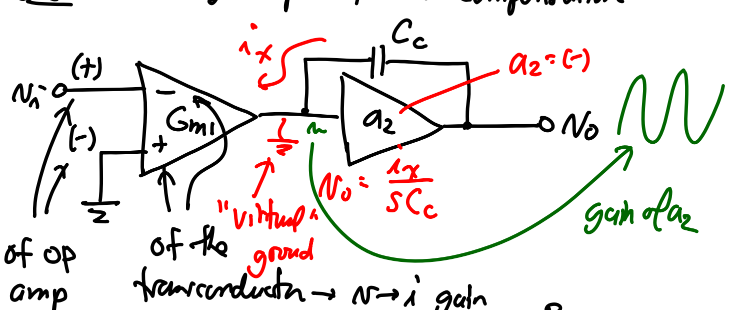
- **Announcements**:
  - Your project cannot use ideal current sources; all you have is Vdd and Vss
  - You need to design your own current sources if you need them
  - You should also try to minimize area, as usual (e.g., a design using an enormous resistor will not be as good as one using a much smaller one, or no resistor at all)
- **Today**:
  - CMOS Op Amp Compensation

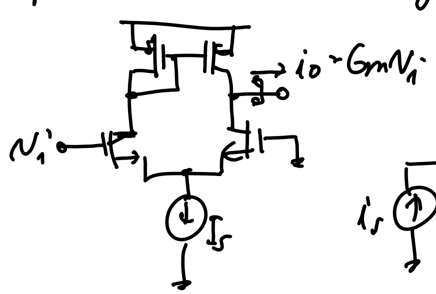
Last Time –

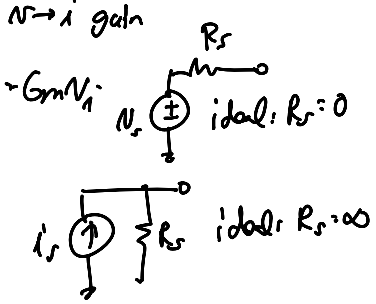
$\boxed{\text{Choosing } C_c}$ (assume no RHP zeros & $|p_3| >> |p_2|$)

→ assume $\frac{1}{sC_c}$ << (surrounding resistance) @ high freq.

① Case: Two-stage Amplifier, Miller compensation

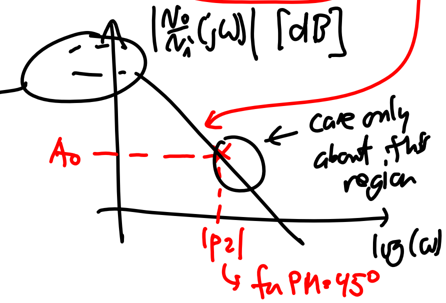






$v_o = \frac{i_x}{sC_c}$, $i_x = G_{m1} v_i$ } $v_o = \frac{G_{m1}}{sC_c} v_i$ → $\frac{v_o}{v_i}(s) = \frac{G_{m1}}{sC_c}$

This doesn't work for



$$\left|\frac{v_o}{v_i}(j\omega)\right| = \frac{G_{m1}}{\omega C_c}$$ ⇒ this should equal A_o @ the freq. ω corresponding to the target phase margin

For PM = 45°: $\omega_{ulg} = \omega @ |T(j\omega)| = 1$ "ulg" = "unity loop gain"

For PM = 45° → $\omega_{ulg} = \omega_2$ ← freq. of the 2nd pole in the $a(j\omega)$ transfer function

$$\left|\frac{v_o}{v_i}(j\omega_2)\right| = A_o = \frac{G_{m1}}{\omega_2 C_c}$$

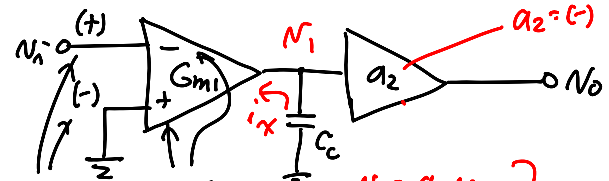
$$\boxed{C_c = \frac{G_{m1}}{\omega_2 A_o}}$$ ← For PM = 45° (provided high-order poles are far away, i.e., $|p_3| >> |p_2|$)

For PM = 60°:

$$\omega_{ulg} = \frac{\omega_2}{1.73} \rightarrow \left|\frac{v_o}{v_i}\left(j\frac{\omega_2}{1.73}\right)\right| = A_o = \frac{G_{m1}}{\left(\frac{\omega_2}{1.73}\right)C_c}$$

$$\boxed{C_c = \frac{1.73\, G_{m1}}{\omega_2 A_o}}$$ ← For PM = 60°

② Case: Two-Stage Amplifier, Shunt C_c Compensation



$$\left.\begin{aligned} v_o &= a_2 v_1 \\ v_1 &= -\frac{G_{m1} v_i}{sC_c} \end{aligned}\right\} v_o = -\frac{G_{m1}a_2}{sC_c} v_i$$

$$\therefore \frac{v_o}{v_i}(s) = -\frac{G_{m1}a_2}{sC_c}$$ Closed loop gain A_o must again intersect this curve @ the right ω_{ulg} for a given PM

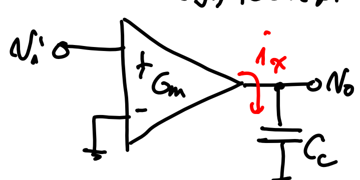
For PM = 45°:

$$\left|\frac{v_o}{v_i}(j\omega_{ulg})\right| = A_o = \frac{G_{m1}a_2}{\omega_{ulg}C_c}$$

For PM = 45°: $\omega_{ulg} = \omega_2 \rightarrow$ $\boxed{C_c = \frac{G_{m1}a_2}{\omega_2 A_o}}$ ← For PM = 45°

For PM = 60°: $\boxed{C_c = \frac{1.73 G_{m1}a_2}{\omega_2 A_o}}$

Case ③: Single-Stage Amplifier, Shunt C_c Compensation
e.g., telescopic op amp



$$\left.\begin{aligned} v_o &= \frac{i_x}{sC_c} \\ i_x &= G_{m1} v_i \end{aligned}\right\} \frac{v_o}{v_i}(s) = \frac{G_{m1}}{sC_c}$$

same as Case ①!

$\boxed{C_c = \frac{G_{m1}}{\omega_2 A_o}}$ ← PM = 45° $\boxed{C_c = \frac{1.73 G_{m1}}{\omega_2 A_o}}$ ← PM = 60°

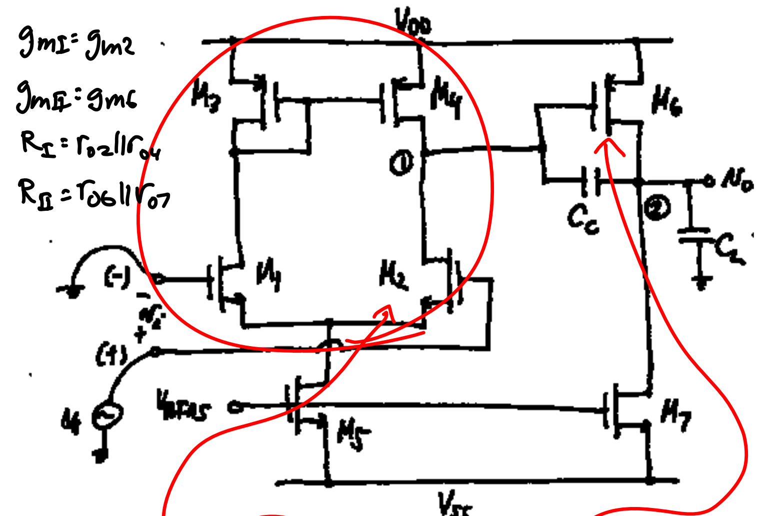
CMOS 2-Stage Op Amp Compensation

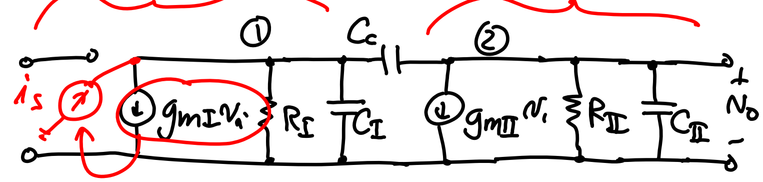
$g_{mI} = g_{m2}$

$g_{mII} = g_{m6}$

$R_I = r_{o2} \| r_{o4}$

$R_{II} = r_{o6} \| r_{o7}$





KCL①: $i_s = \frac{v_1}{R_I} + sC_I v_1 + (v_1 - v_o)sC_c$

KCL②: $g_{mII} v_1 + \frac{v_o}{R_{II}} + sC_{II} v_o + (v_o - v_1)sC_c = 0$

$$\frac{v_o}{i_s} = \frac{(g_{mII} - sC_c)R_I R_{II}}{\left\{1 + s\left[(C_{II}+C_c)R_{II} + (C_I + C_c)R_I + g_{mII}R_I R_{II} C_c\right] + s^2 R_I R_{II}(C_I C_{II} + C_c C_I + C_c C_{II})\right\}}$$

$$= \frac{N(s)}{D(s)}$$

This Xfer fcn has 2 poles & a zero!

The zero: $N(s) = 0 \rightarrow \boxed{z = \frac{g_{mII}}{C_c}}$ ← (+) and real

$z = s$

The poles:

$$D(s) = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right) = 1 - s\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{p_1 p_2}$$

$$[p_2 \gg p_1] \quad \approx 1 - \frac{s}{p_1} + \frac{s^2}{p_1 p_2}$$

↑ i.e., p_1 is a dominant pole

Thus:

$$p_1 = -\frac{1}{(C_{II}+C_c)R_{II} + (C_I+C_c)R_I + g_{mII}R_I R_{II} C_c}$$

As $C_c \uparrow \rightarrow p_1 \downarrow \rightarrow \approx -\frac{1}{g_{mII}R_I R_{II} C_c}$

For the 2nd pole:

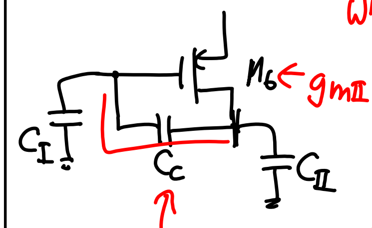
$$p_1 p_2 = \frac{1}{R_I R_{II}(C_I C_{II} + C_c C_I + C_c C_{II})}$$

$$\boxed{p_2 \simeq -\frac{g_{mII} C_c}{C_I C_{II} + C_c C_I + C_c C_{II}}}$$

As $C_c \uparrow \rightarrow \boxed{p_2 \simeq -\frac{g_{mII}}{C_I + C_{II}}}$ ← This is higher than before!

$C_c \uparrow \rightarrow p_2 \uparrow$

When $C_c \uparrow$, this becomes a short!



g_{m6} becomes "diode-connected"!

$$\omega_2 = \frac{1}{\left(\frac{1}{g_{mII}}\right)(C_I + C_{II})} = \frac{g_{mII}}{C_I + C_{II}}$$

Remarks.

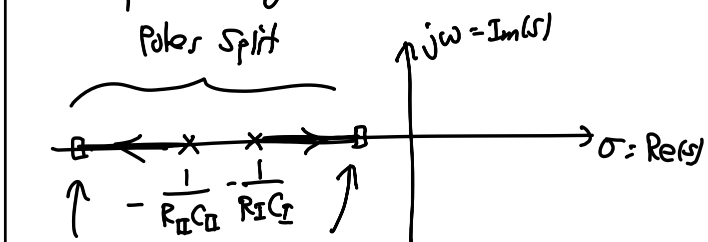
① Note that as $C_c \uparrow \rightarrow p_1 \downarrow \rightarrow 0$

② As $C_c \uparrow \rightarrow p_2 \uparrow \rightarrow p_2 = \frac{g_{mII}}{C_I + C_{II}}$

③ With $C_c = 0$ (i.e., before compensation)

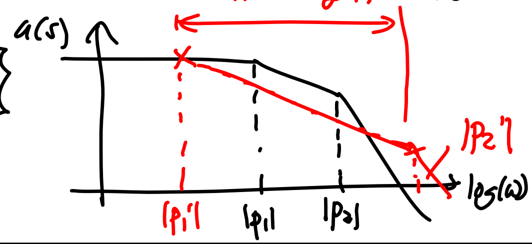
$$p_1 = -\frac{1}{R_I C_I}, \quad p_2 = -\frac{1}{R_{II} C_{II}}$$

④ On a pole zero diagram:



much larger! ∴ very good!

But what about the RHP zero?



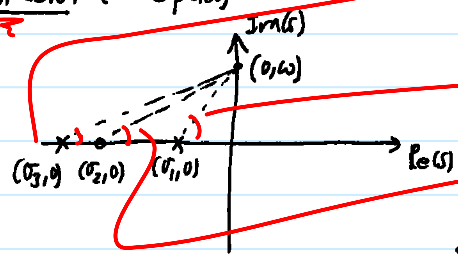
RHP Zero

Again, we're mainly concerned here w/ phase margin; i.e., stability.

How does a RHP zero affect the PM?

⇒ compare a LHP zero w/ a RHP zero:

① LHP zero: (and 2 poles)



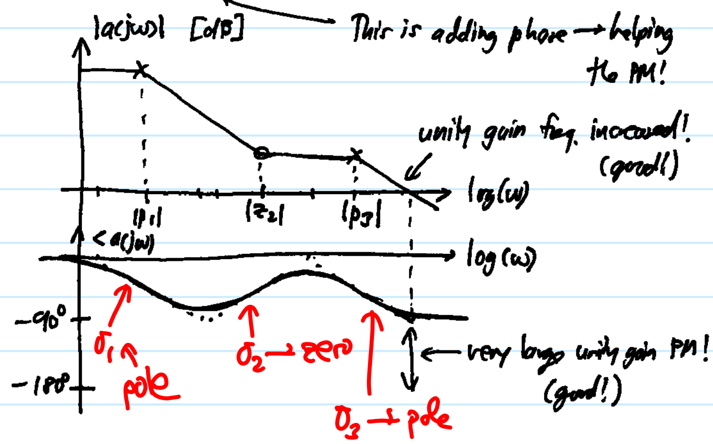
$$|H(j\omega)| = \frac{\sqrt{\omega^2+\sigma_2^2}}{\sqrt{\omega^2+\sigma_1^2}\sqrt{\omega^2+\sigma_3^2}} \qquad \frac{\omega-0}{0-\sigma_2}$$

$$\measuredangle H(j\omega) = +\tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)$$

$$= \tan^{-1}\left(\frac{\omega}{|\sigma_2|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_1|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_3|}\right)$$

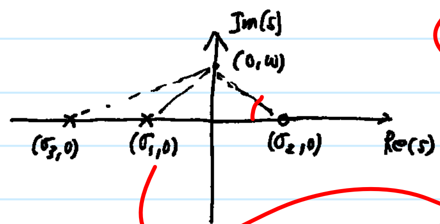
This is adding phase → helping the PM!

Thus:



A LHP zero can really improve the performance & stability of an op amp FB ckt.

② RHP zero: (and 2 poles)



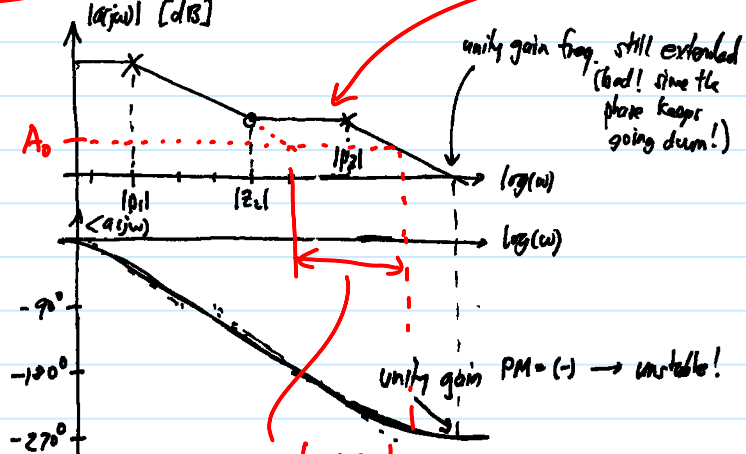
$$|H(j\omega)| = \frac{\sqrt{\omega^2+\sigma_2^2}}{\sqrt{\omega^2+\sigma_1^2}\sqrt{\omega^2+\sigma_3^2}}$$ → same as above!

now $\sigma_2 = (+)$

$$\measuredangle H(j\omega) = +\tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)$$

$$= +\tan^{-1}\left(-\frac{\omega}{|\sigma_2|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_1|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_3|}\right)$$

Thus:



A RHP zero is detrimental because:
① extends the ω_u
② while continuing to drop the phase
⇓
instability!

Problem!

⇒ to solve, must first understand where the zero comes from!

much more phase shift occurs before $|T(j\omega)| = 1$!
→ more unstable! ✗

$s + \sigma_z = s - p_1$