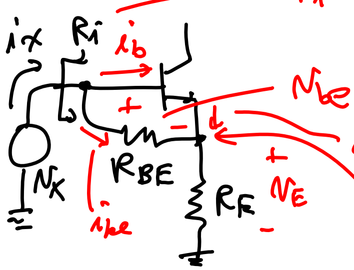
Logistics:
- I will miss Thursday, Feb. 12
- Make up lecture time:
  - Tonight, Tuesday, 7 p.m., 247 Cory

Today:
- High Frequency Cut-Off Examples
- Short-Circuit Time Constant Analysis for Low Frequency Cut-Off

Problem 1(c)

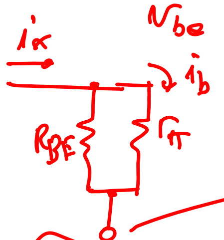
$R_i = \frac{v_x}{i_x}$



$i_e = (\beta+1) i_b$

$i_x = i_b + i_{be}$

$$v_x = \underbrace{i_x (r_\pi \| R_{BE})}_{v_{be}} + \underbrace{\overbrace{[(\beta+1) i_b + i_x - i_b]}^{(\beta i_b + i_x) R_E} R_E}_{v_E}$$



Current Divider:

$$\left[i_b = i_x \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right) \right]$$

$$v_x = i_x \left[(r_\pi \| R_{BE}) + \left[\beta \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right) + 1 \right] R_E \right]$$

$$R_i = \frac{v_x}{i_x} = (r_\pi \| R_{BE}) + (\beta' + 1) R_E$$

$$\beta' = \beta \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right)$$

Insp. Analysis Formula:

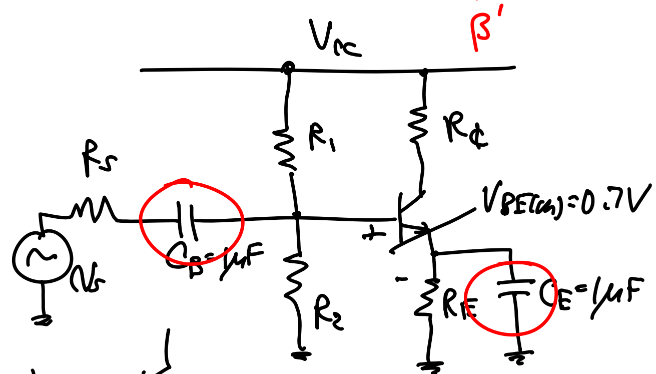
$$R_i = r_\pi + (\beta+1) R_E \cong r_\pi (1 + g_m R_E)$$

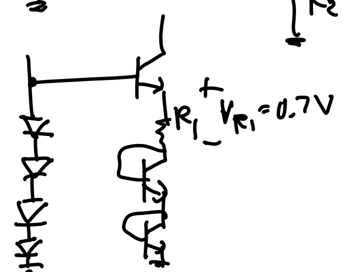
$$R_i = (r_\pi \| R_{BE})(1 + g_m R_E)$$

$$= (r_\pi \| R_{BE}) + \underbrace{g_m \left(\frac{r_\pi R_{BE}}{r_\pi + R_{BE}} \right)}_{\beta} R_E$$

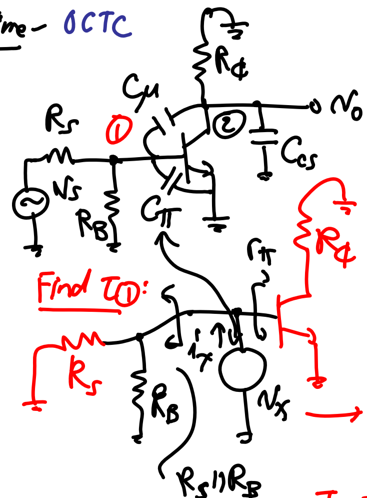
$$= (r_\pi \| R_{BE}) + \underbrace{\beta \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right)}_{\beta'} R_E$$

= ✓





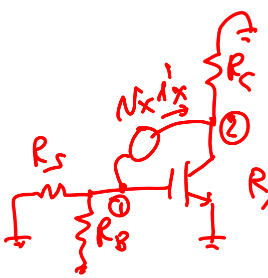
Last Time – OCTC



$R = \frac{v_x}{i_x} = r_\pi || R_s || R_B$

$\tau_1 = C_\pi (r_\pi || R_s || R_B)$

Find $\tau_{\mu o}$: replace C_μ w/ v_x



use inspection formula

"O" means "in-shunt"

$R_{\mu o} = \frac{v_x}{i_x} = R_{①} + R_{②} + g_m R_{①} R_{②}$

$R_{\mu o} = R_s || R_B || r_\pi + r_o || R_c$
$\quad + g_m (R_s || R_B || r_\pi)(r_o || R_c)$

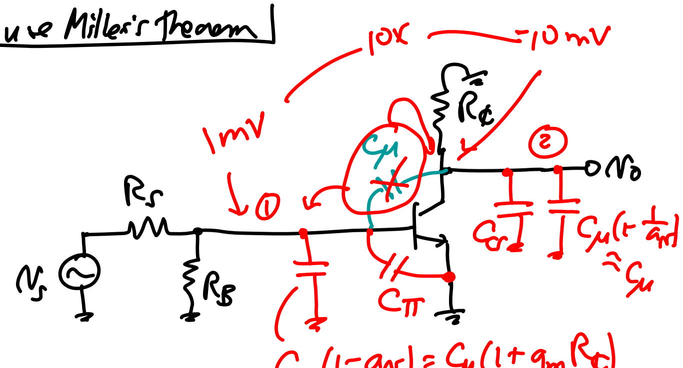
$\tau_{\mu o} = R_{\mu o} C_\mu$

Find $\tau_②$:

$\tau_② = C_{cs} (r_o || R_c) \approx C_{cs} R_c$

$$\boxed{\omega_H = \frac{1}{\tau_① + \tau_{\mu o} + \tau_②}}$$

Now, use Miller's Theorem



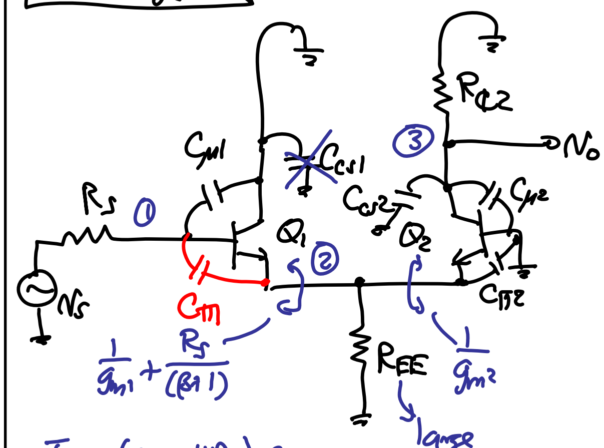
$C_\mu (1 - a_v) = C_\mu (1 + g_m R_c)$

$\tau_① = (r_\pi || R_s || R_B)(C_\pi + C_\mu (1 + g_m R_c))$

$\tau_② = R_c (C_\mu + C_{cs})$

$\omega_H = \frac{1}{\tau_① + \tau_②}$

Multi-Stage Ex.



$\frac{1}{g_{m1}} + \frac{R_s}{(\beta + 1)}$ $\quad \frac{1}{g_{m2}}$

$\tau_① = (2 r_\pi || R_s) C_{\mu 1}$

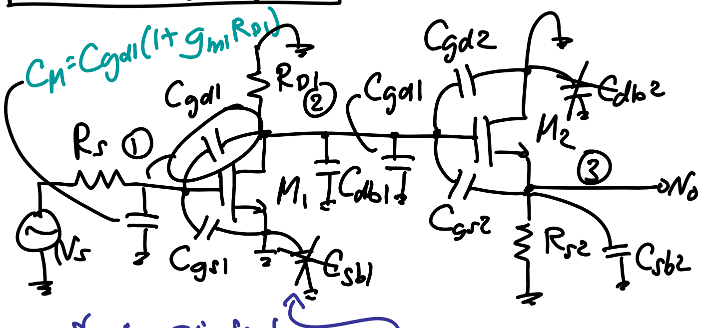
$\tau_{\pi 1} = C_{\pi 1} \left(r_{\pi 1} || \frac{R_s + 1/g_{m2}}{1 + g_{m1}\left(\frac{1}{g_{m2}}\right)} \right)$

$\tau_② = C_{\pi 2} \left[\left(\frac{1}{g_{m1}} + \frac{R_s}{\beta_{\pi 1}} \right) || \frac{1}{g_{m2}} || \cancel{R_{EE}} \right]$

$$\tau_3 = (C_{\mu 2} + C_{cs2}) R_{t2}$$

$$\boxed{\omega_H = \frac{1}{\tau_1 + \tau_{\pi 1} + \tau_2 + \tau_3}}$$

MOS Two-Stage Amplifier



Step 1: Eliminate caps.

Step 2: Determine the constants.

$$\tau_1 = [C_{gs1} + C_{gd1}(1+g_{m1}R_{D1})]R_S$$

$$\tau_2 = [C_{gd1} + C_{db1} + C_{gd2}](r_o \| R_{D1}) \quad \sim R_{D1}$$

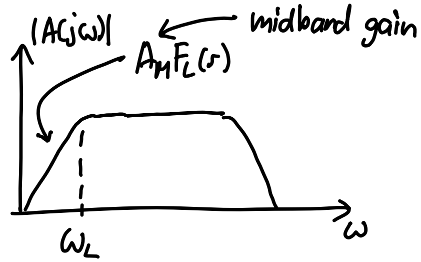
$$\tau_3 = C_{sb2}\left(\frac{1}{g_{m2}+g_{mb2}} \| R_{s2}\right)$$

$$\tau_{gs2} = C_{gs2}\left(\frac{R_{D1} + R_{s2}}{1+(g_{m2}+g_{mb2})R_{s2}}\right)$$

$$\boxed{\omega_H = \frac{1}{\tau_1 + \tau_2 + \tau_3 + \tau_{gs2}}}$$

Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall:



In general, for the low freq. response:

$$F_L(s) = \frac{s^{n_L} + d_1 s^{(n_L-1)} + \cdots}{s^{n_L} + e_1 s^{(n_L-1)} + \cdots}, \quad n_L = \#\text{poles} = \#\text{zeros}$$

We can express the coefficient e_1 by:

$$e_1 = \omega_{p1} + \omega_{p2} + \cdots + \omega_{pn_L}$$

Similar analysis to that used for OCTC...

For the case of a dominant pole:
↳ i.e., the highest freq. pole

$$F_L(s) \cong \frac{s}{s+\omega_L} = \frac{s}{s+e_1} \rightarrow e_1 \cong \omega_{p1} = \omega_L$$

$$\therefore \boxed{\omega_L \cong e_1 = \sum_j \omega_{pj} = \sum_j \frac{1}{C_j R_{js}} = \sum_j \frac{1}{\tau_{js}}}$$

where $C_j \triangleq$ various large (>10nF) capacitors in the ckt. (e.g., the bypass caps.)

$R_{js} \triangleq$ driving point resistance seen between the terminals of C_j determined with:

① all large capacitors short-circuited, except C_j, which is replaced by the test voltage source for R determination

For reading, can go to Sedra & Smith

(2) all independent sources eliminated (i.e., short voltage sources, open current sources)

(3) open all H.F. capacitors (i.e., small caps in the pF range, or < 1nF)

Again, for the case where there are no dominant poles, a reasonable approximation is:

$$\omega_L \cong \sqrt{\omega_{P1}^2 + \omega_{P2}^2 - 2\omega_{Z1}^2 - 2\omega_{Z2}^2}$$