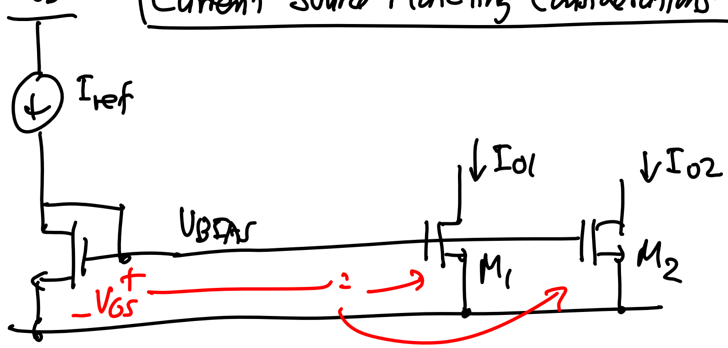
Lecture 12: Op Amps & Emitter Coupled Pair

- **Announcements**:
  - Pre-Lecture materials online
  - Change Th OH to 2-3 p.m. (right after class)
- **Lecture Topics**:
  - Op Amp Review
  - Emitter Coupled Pair (ECP)
  - Half Circuits
  - Source Coupled Pair w/ Current Mirror Load
- ------------------------------------
- **Last Time**:

Current Source Matching Considerations



$$\therefore \boxed{\frac{\Delta I_D}{I_D} = \frac{\Delta(W/L)}{(W/L)} - \frac{\Delta V_t}{(V_{ov}/2)}}$$

this could be (−), so this doesn't necessarily help!

Fractional Current Mismatch

Geometry Based Component ← helps

Independent of Bias Pt.

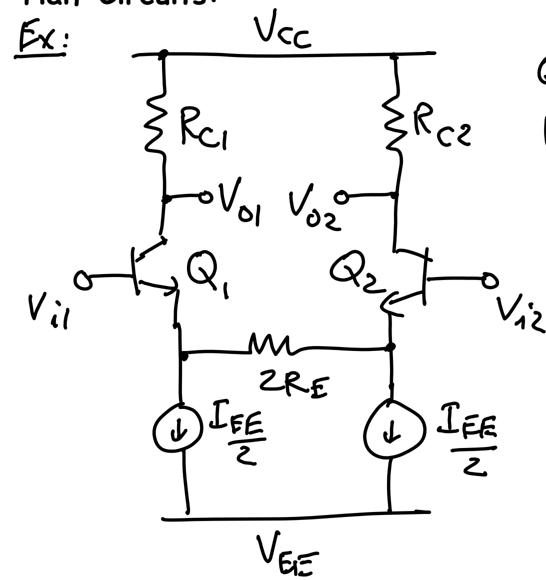
Increases as $V_{ov}\downarrow$

Today: $V_{DD}\downarrow \rightarrow V_{ov}\downarrow$

For each new Xsistor generation, must make $(W/L)\uparrow$

- Now, start on Op Amps & ECP using the Pre-Lecture handout
- Half Circuits:

Ex:

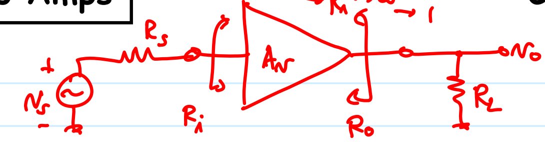


$Q_1 = Q_2$

$R_{C1} = R_{C2} = R_C$

EE 140 | Ideal Op Amps | CTN | 1

$$\text{Gain}: \frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \cdot A_v \cdot \frac{R_L}{R_L + R_o}$$
$R_i \to \infty \to 1$, $R_o \to 0 \to 1$



Ideal Voltage Amplifier

→ ideal when $\frac{v_o}{v_s} = A_v$; i.e., when source and load R's do not influence the gain of the amplifier.

For this to occur, the voltage division at the input & output must be eliminated. This happens when:

$R_i = \infty$, $R_o = 0$ } These resistance values define an ideal voltage amplifier.

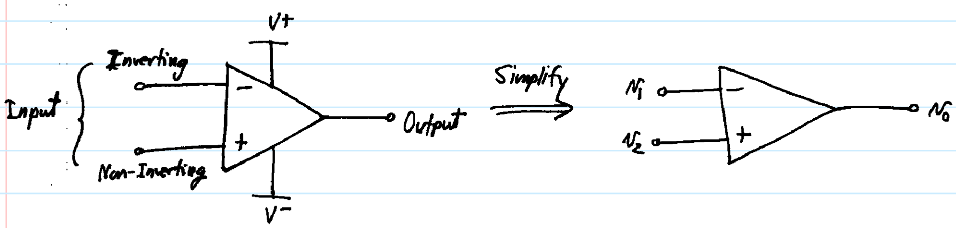
We'll look at other amplifier types later.

This, then, naturally leads us to:

Ideal Operational Amplifiers (Op Amps)

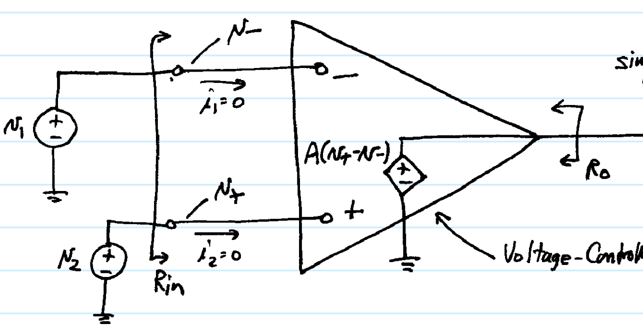
↳ The work horse of analog electronics → combinations of op amps w/ feedback components allow the implementation of analog computers, sampled-data systems, analog filters, A/D Converters, DAC's, instrumentation amplifiers

In general, have a minimum of 5 terminals:



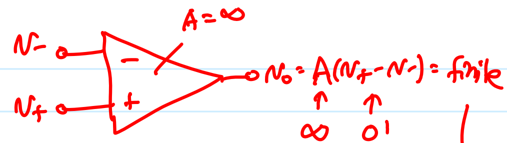
Perhaps the best way to define an op amp is thru its equivalent ckt:

Equivalent Ckt. of an Ideal Op Amp:



$$v_o = A(v_+ - v_-) = A(v_2 - v_1)$$

EE 140 | Ideal Op Amps | CTN | 2



Properties of Ideal Op Amps:

① $R_{in} = \infty$ $\xrightarrow{\text{leads to}}$ ④ $i_+ = i_- = 0$

② $R_o = 0$

③ $A = \infty$ $\xrightarrow{\text{leads to}}$ ⑤ $v_+ = v_-$, assuming v_o = finite

↳ Why? Because for $\infty(v_+ - v_-) = v_o$ = finite

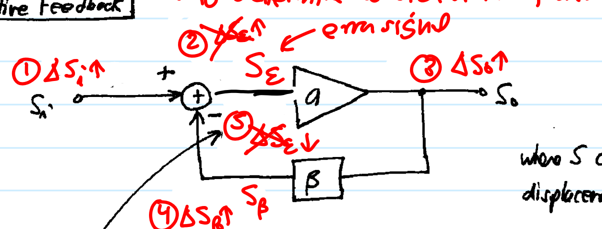
$\therefore \underbrace{v_+ - v_-}_{= \frac{v_o}{\infty}} = 0 \rightarrow v_+ = v_-$

⇒ virtual short ckt. (virtual ground)

Big assumption! (v_o = finite)

How can we assume this? ⟹ only when there is an appropriate negative feedback path!

$\boxed{\text{Negative Feedback}}$ → to determine whether or not there's (−) FB, do perturbation analysis:



where S could be a current, voltage, displacement, etc., ...

Negative feedback acts to oppose or subtract from input.

$$\left.\begin{aligned} S_o &= a S_\varepsilon \\ S_\varepsilon &= S_i - \beta S_o \end{aligned}\right\} \Longrightarrow \begin{aligned} &S_o = a(S_i - \beta S_o) \\ &S_o(1 + a\beta) = a S_i \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1 + a\beta}} \end{aligned}$$

overall transfer function

"+" → neg. FB ✓

$$[a \rightarrow \infty] \Longrightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$$

$$\therefore S_o = \frac{1}{\beta} S_i = \text{finite} ✓$$

(when there is neg. FB around the amplifier)

In Summary:

① Neg. FB can insure S_o = finite even with $a = \infty$.

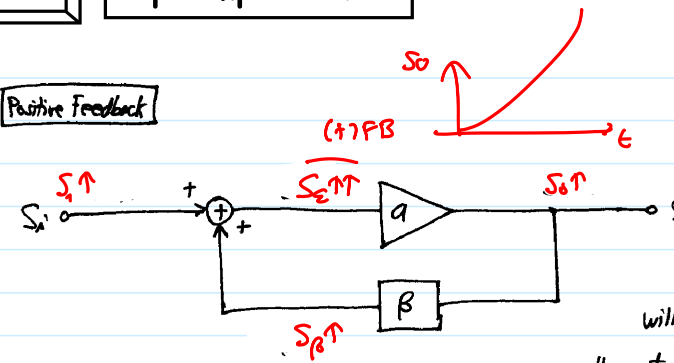
② Overall Gain dependent (or overall T.F.) dependent only on external components. (e.g., β)

③ Overall (closed-loop) gain $\frac{S_o}{S_i}$ is independent of amplifier gain a.

↖ very important! ⇒ as you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain.
i.e., if you're shooting for $a = 50{,}000$, you might get 47,000 or 60,000 instead.

EE 140 | Op Amp Circuits | CTN | 3

Contrast w/ Positive Feedback



$\Rightarrow$ output blow up! (for $a\beta > 1$)

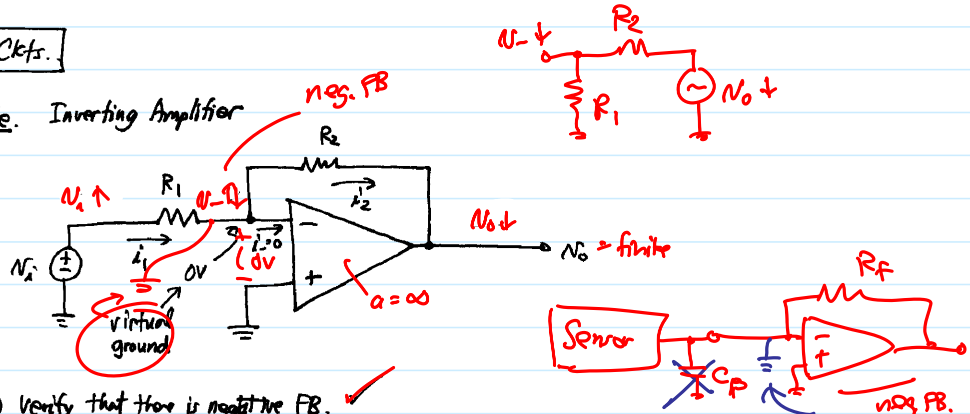
will be the case for $a = \infty$

$\Rightarrow$ usually get oscillation!

Thus, for a bounded, controllable function, need negative FB around an op amp.

Op Amp Ckts.

Example. Inverting Amplifier



① Verify that there is negative FB. ✓

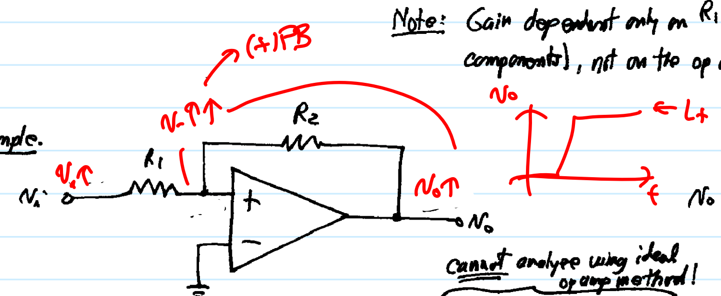
② $\therefore v_o$ = finite $\rightarrow v_+ = v_-$ $\rightarrow$ node attached to (−) terminal is virtual ground

③ $i_- = 0 \;\therefore\; i_1 = i_2$

$$\left.\begin{aligned} i_1 &= \frac{v_i - 0}{R_1} = \frac{v_i}{R_1} = i_2 \\ v_o &= 0 - i_2 R_2 = -i_2 R_2 \end{aligned}\right\} \Rightarrow v_o = -\left(\frac{v_i}{R_1}\right)R_2 = -\frac{R_2}{R_1}v_i \quad \therefore \boxed{\frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$

Note: Gain dependent only on R_1 & R_2 (external components), not on the op amp gain.

Example.



$v_o = L^+$ or L^- depending on initial conds.

$v_+ = (+) \longrightarrow L^+$

$v_+ = (-) \longrightarrow L^-$.

Cannot analyze using ideal op amp method!

① Verify that there is neg. FB ✗

$\therefore v_o \neq$ finite, $v_+ \neq v_-$

$\Rightarrow$ this ckt. will "rail out"

 CTN 2/23/12

EE 140 | Basic Op Amp Design | CTN | 4

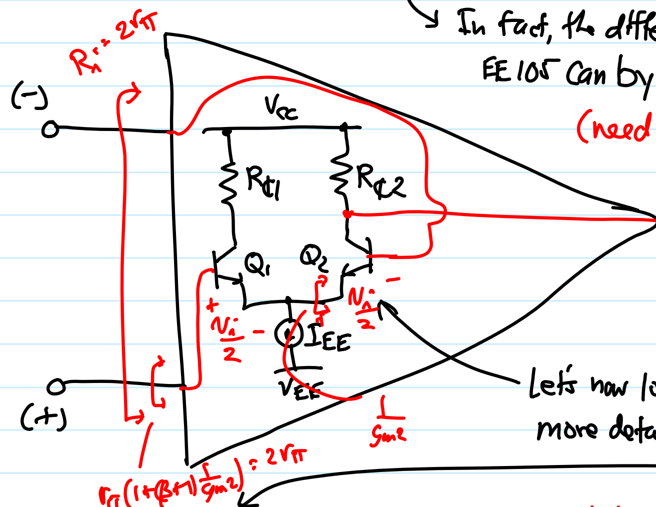
How does one make an op amp? (It turns out, you already know!)

⇒ Basic Needed Attributes:

1. Gain (voltage gain).
2. Two inputs, (+) and (−).
3. One output equal to the difference of the inputs multiplied by some gain.

↳ In fact, the differential pair you studied in EE105 can by itself serve as an op amp!

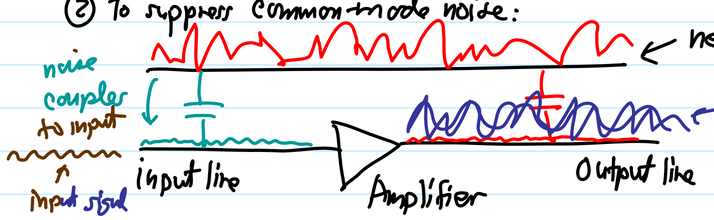
(need only decide which of the inputs is the (+) and (−) terminals)



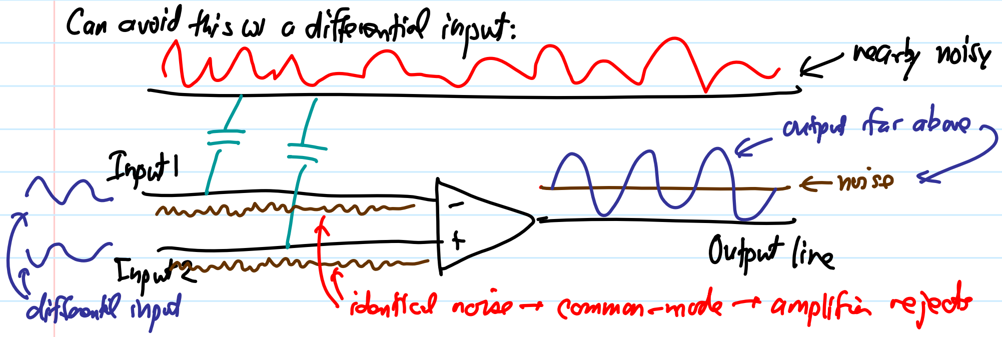
Let's now look at the diff. pair in more detail! ... but first

Why have 2 inputs?

1. To get a virtual short between v_+ and v_- for op amp ckts. ←
2. To suppress common-mode noise:

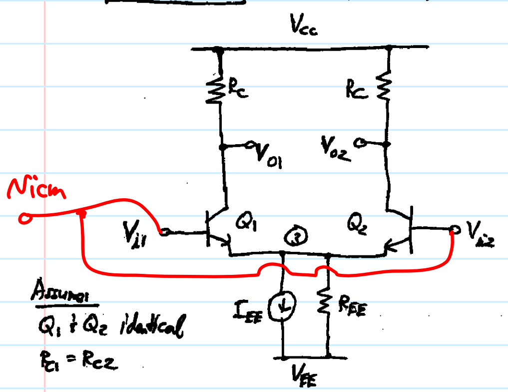


Can avoid this w/ a differential input:



EE 140 | Differential Pair (Bipolar) | CTN | 5

Differential Pair (Emitter Coupled Pair)



Assume:
Q_1 & Q_2 identical
$R_{C1} = R_{C2}$

Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

Definition: $V_{id} = V_{i1} - V_{i2}$ (differential input)

$V_{icm} = \frac{V_{i1} + V_{i2}}{2}$ (common-mode input)

$$\Rightarrow \begin{cases} V_{i1} = V_{icm} + \frac{V_{id}}{2} \\ V_{i2} = V_{icm} - \frac{V_{id}}{2} \end{cases}$$

Differential Gain $= A_d = \frac{V_{o1} - V_{o2}}{V_{id}} = \frac{V_{od}}{V_{id}}$ (want this to be large for this differential amplifier)

Common-Mode Gain $= A_{cm} = \frac{V_{o1}}{V_{cm}}$ or $\frac{V_{o2}}{V_{cm}}$ (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio $= CMRR = \frac{A_{dm}}{A_{cm}}$ (should be very high to favor the differential-mode and reject the common-mode)

⇒ we also want a high Common-Mode Input Range to reject DC input offsets

⇒ Note: No need for bypass capacitors (large) to the inputs or outputs → can just use direct coupling!

Biasing & Large Signal Common-Mode Behavior

Case: $R_{EE} = \infty$ → ideal current source biasing → $I_{E1} = I_{E2} = \frac{I_{EE}}{2}$ → $V_{o1} = V_{o2} \Rightarrow V_{od} = 0$

If $V_{cm} \uparrow$ → $V_{③} \uparrow$, but current draw from I_{EE} stays constant ∴ I_{C1} & I_{C2} stay constant → bias pt. doesn't change

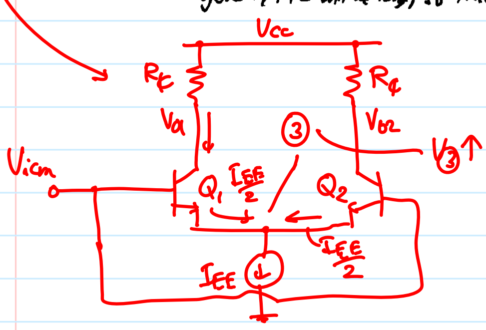
$g_m = \frac{1}{2}\frac{I_{EE}}{V_T}$

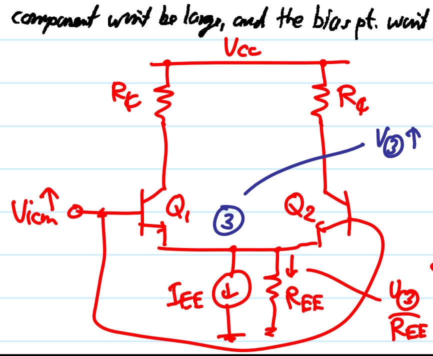
Case: R_{EE} = finite

$V_{③} = V_{i1} - V_{BE(on)}$

If $V_{icm} \uparrow$ → $V_{③} \uparrow$ → $I_{E1} = I_{E2} \uparrow$ (current draw $= I_{EE} + \frac{V_{③}}{R_{EE}}$)

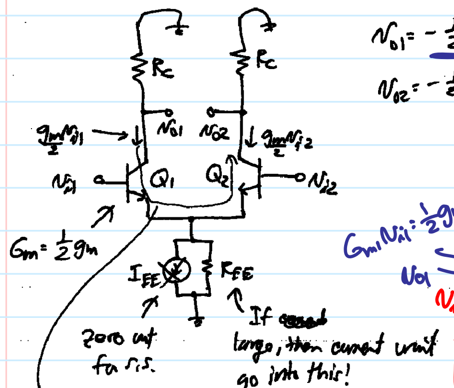
⇒ in general, R_{EE} will be large, so this component won't be large, and the bias pt. won't Δ much





EE 140 | Differential Mode Analysis | CTN | 6

Small-Signal Analysis of Diff. Pair



$$v_{o1} = -\frac{1}{2} g_m v_{i1} R_c + \frac{1}{2} g_m v_{i2} R_c$$

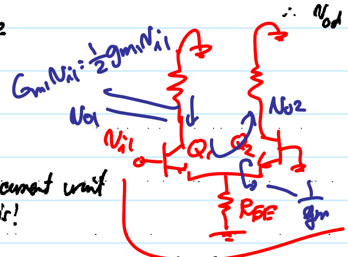
$$v_{o2} = -\frac{1}{2} g_m v_{i2} R_c + \frac{1}{2} g_m v_{i1} R_c$$

$$v_{o1} = -\frac{1}{2} g_m R_c (v_{i1} - v_{i2})$$

$$v_{o2} = +\frac{1}{2} g_m R_c (v_{i1} - v_{i2})$$

$$\therefore v_{od} = v_{o1} - v_{o2} = -g_m R_c (v_{i1} - v_{i2})$$

$$\therefore \boxed{\frac{v_{od}}{v_{id}} = A_{dm} = -g_m R_c}$$

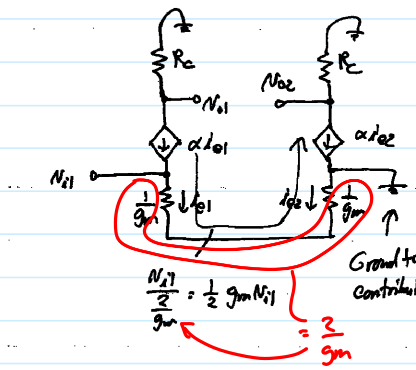


$$G_m = \frac{g_m}{1 + g_m\left(\frac{1}{g_m}\right)} = \frac{1}{2} g_m$$

one voltage source @ a time; then use superposition.

⇒ Easiest to see this happening using the T-model: (for those who must see the model ckt.)

This time also get the $\frac{v_{o1} - v_{o2}}{v_{i1}}$ gain!



$$v_{o1} = -\frac{1}{2} g_m R_c v_{i1}$$

$$v_{o2} = +\frac{1}{2} g_m R_c v_{i1}$$

$$v_{o1} - v_{o2} = -g_m R_c v_{i1}$$

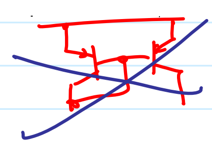
$$\therefore \boxed{\frac{v_{od}}{v_{i1}} = -g_m R_c}$$

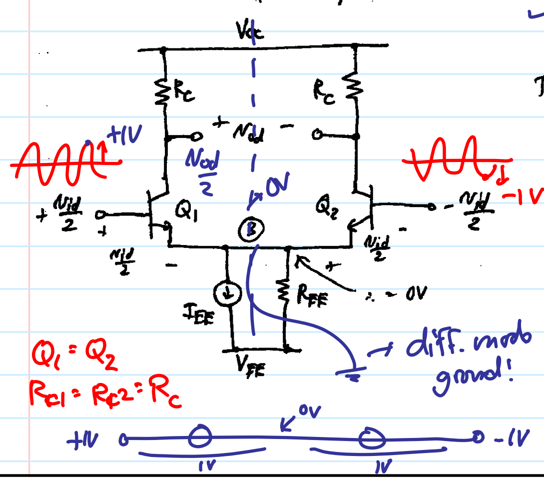
$$\boxed{\frac{v_{o2}}{v_{i1}} = \frac{1}{2} g_m R_c}$$

$$\boxed{\frac{v_{o1}}{v_{i1}} = -\frac{1}{2} g_m R_c}$$

Diff. Mode Analysis

Assume a ckt. w/ only diff. input:





Total current thru I_{EE} = const.

⟶ V_E = const. as input changes

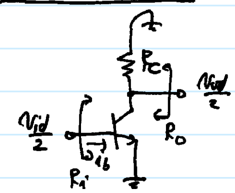
⟶ ③ acts as an incremental ground! ⟶ $V_③ = 0V$ (always!)

∴ we can ground ③, and then, have a **Differential Half Ckt.**

Note: Can really only make this for a purely symmetrical ckt.!

EE 140 | Common-Mode Analysis | CTN | 7

Differential Half Ckt.



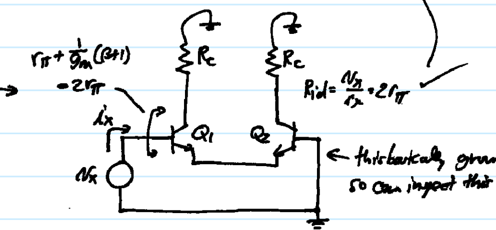
By inspection: $\dfrac{v_{od}/2}{v_{id}/2} = \dfrac{v_{od}}{v_{id}} = \boxed{A_{dm} = -g_m R_C}$

$\dfrac{v_{id}/2}{i_b} = r_\pi \longrightarrow R_{id} = \dfrac{v_{id}}{i_b} = \boxed{2r_\pi = R_{id}}$

$\dfrac{v_{od}/2}{i_o} = r_o \| R_C \longrightarrow R_{od} = \dfrac{v_{od}}{i_o} = 2(r_o\|R_C) \approx \boxed{2R_C = R_{od}}$

S.S. params. determined w/ $I_C = \frac{I_{EE}}{2}$

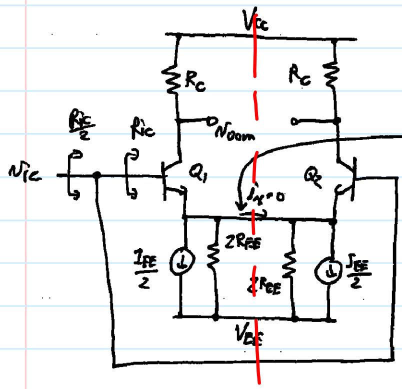
First define



← this basically ground, so can ignore this

Common-Mode Analysis

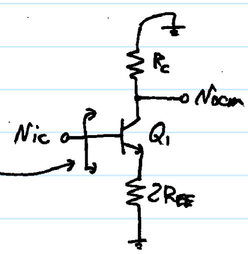
Assume a pure CM input → tie inputs together



By symmetry, $i_x = 0$. ⇒ thus, really have the equivalent of an open ckt. here

∴ ⇒ can split the ckt. into CM half-ckts.!

S.S. CM Half-Ckt.



$R_{ic} = r_\pi + (\beta+1)(2R_{EE})$
@ each input

$$\boxed{A_{cm} = \frac{v_{ocm}}{v_{ic}} = -\frac{g_m R_C}{1+g_m(2R_{EE})} \approx -\frac{R_C}{2R_{EE}}}$$

Want small for large CMRR ∴ want R_{EE} = large!

Common-Mode Rejection Ratio = $\text{CMRR} = \dfrac{A_{dm}}{A_{cm}} = \dfrac{-g_m R_C}{-\dfrac{g_m R_C}{1+g_m(2R_{EE})}} \Rightarrow \boxed{\text{CMRR} = 1 + 2g_m R_{EE}}$

! Want as ideal a current source as possible!

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the ECP.

