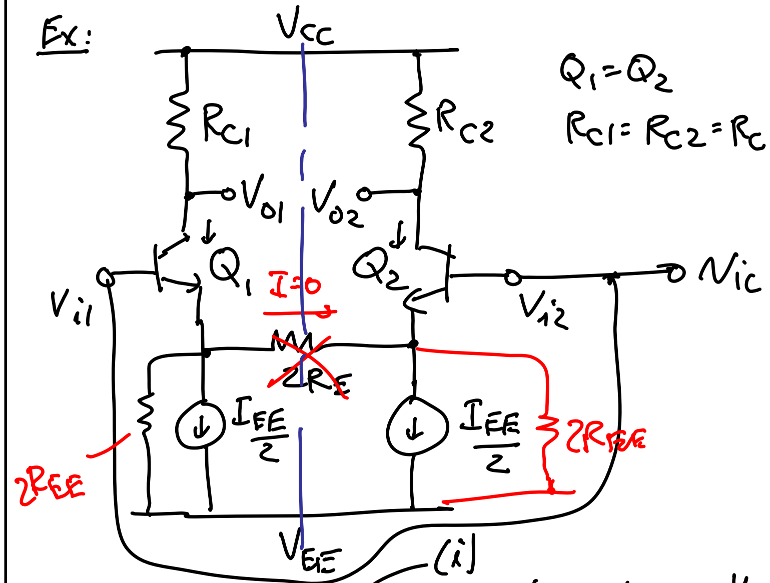
Lecture 13: SCP & Current Mirror Load

- **Announcements:**
  - Pre-Lecture materials online
  - Reminder: Changed Th OH to 2-3 p.m. (right after class)
  - HW#4 had some misplaced problems that should have been on a later HW
  - Problems 2 and 4 from HW#4 will be deferred to later HW's
  - I will need to miss next Thursday's lecture; once again, there will be a make-up lecture in a room and time TBD
- **Lecture Topics:**
  - Half Circuits Examples
  - Source Coupled Pair w/ Current Mirror Load
- ------------------------------------
- **Last Time:**
- Going though the handout on Op Amps and Differential Pairs
- Continue with this now …
- Problem with LCD projector
  - Wasted some time trying to fix it
  - Used the board for the first portion of lecture, until the right people came to fix the projector
  - On the board, went through differential and common-mode half-circuit analysis, including emitter degenerated differential pairs
  - Some of the notes at the end of this

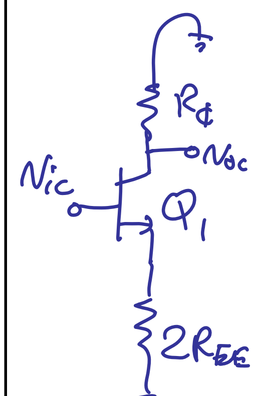
- Projector fixed … continue with half circuit example

Ex:



→ did differential part on the board… see the end of these notes…

(ii) Find the common-mode gain:



$$A_{cm} = \frac{v_{oc}}{v_{ic}} = \frac{-g_m R_C}{1+g_m(2R_{EE})}$$

If there is a mismatch in the ckt., e.g., $R_{C1} \neq R_{C2}$, then we can also define:

$A_{cm\text{-}dm} \triangleq$ common-mode input to differential-mode output gain

$$= \frac{v_{od}}{v_{ic}} = \frac{v_{o1}-v_{o2}}{v_{i1}} = \frac{v_{o1}-v_{o2}}{v_{i2}} \quad (\text{w/ } v_{i1}=v_{i2})$$

$A_{dm\text{-}cm} \triangleq$ differential-mode input to common-mode output gain

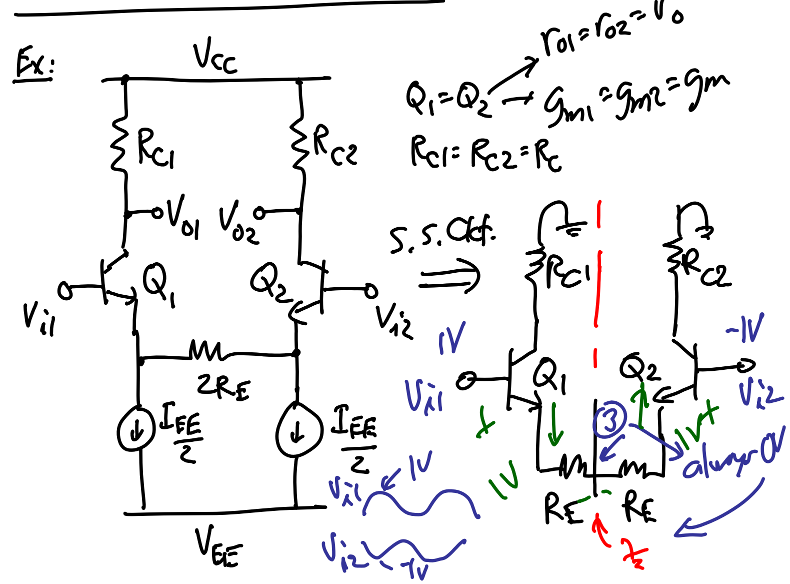
$$= \frac{v_{oc}}{v_{id}} = \frac{v_{oc}}{v_{i1}-v_{i2}} \quad \left[\text{w/ } v_{oc} = \tfrac{1}{2}(v_{o1}+v_{o2})\right]$$

You will be experiencing these in a future HW.

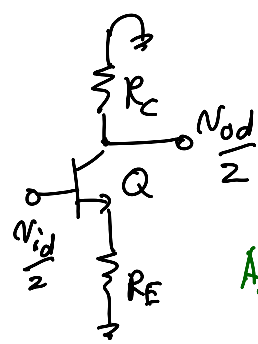
over for repeat of chalkboard material

Parts Done on the Chalkboard

Ex:



(i) Find the differential-mode gain: (A_{dm})



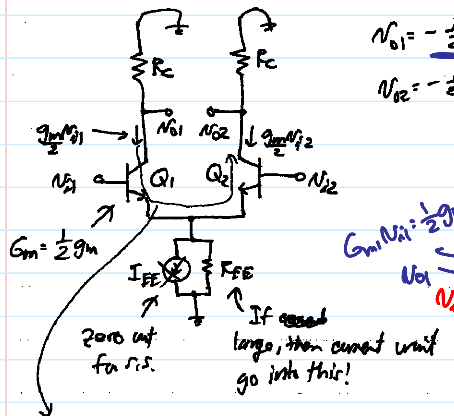
$$\frac{(v_{od}/2)}{(v_{id}/2)} = -\frac{g_m(r_o \| R_C)}{1+g_m R_E}$$

$$A_{dm} = \frac{v_{o1}-v_{o2}}{v_{i1}-v_{i2}} = \frac{v_{od}}{v_{id}} =$$

Diff. Half Ckt.

EE 140 | Differential Mode Analysis | CTN | 6

Small-Signal Analysis of Diff. Pair



$v_{o1} = -\frac{1}{2} g_m v_{i1} R_c \; + \frac{1}{2} g_m v_{i2} R_c$

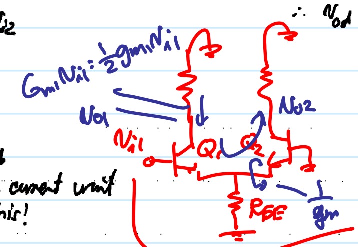
$v_{o2} = -\frac{1}{2} g_m v_{i2} R_c \; + \frac{1}{2} g_m v_{i1} R_c$

$v_{o1} = -\frac{1}{2} g_m R_c (v_{i1} - v_{i2})$

$v_{o2} = +\frac{1}{2} g_m R_c (v_{i1} - v_{i2})$

$\therefore v_{od} = v_{o1} - v_{o2} = -g_m R_c (v_{i1} - v_{i2})$

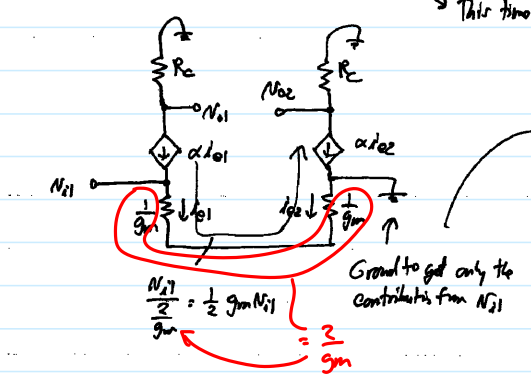
$\therefore \boxed{\frac{v_{od}}{v_{id}} = A_{dm} = -g_m R_c}$



$G_m = \frac{g_m}{1 + g_m(\frac{1}{g_m})} = \frac{1}{2} g_m$

one voltage source @ a time; then use superposition.

⇒ Easiest to see this happening using the T-model: (for those who must see the model ckt.)
↳ This time also get the $\frac{v_{o1} - v_{o2}}{v_{i1}}$ gain!



$v_{o1} = -\frac{1}{2} g_m R_c v_{i1}$

$v_{o2} = +\frac{1}{2} g_m R_c v_{i1}$

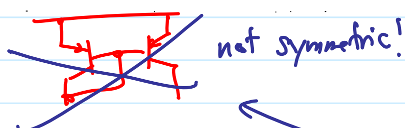
$v_{o1} - v_{o2} = -g_m R_c v_{i1}$

$\therefore \boxed{\frac{v_{od}}{v_{i1}} = -g_m R_c}$

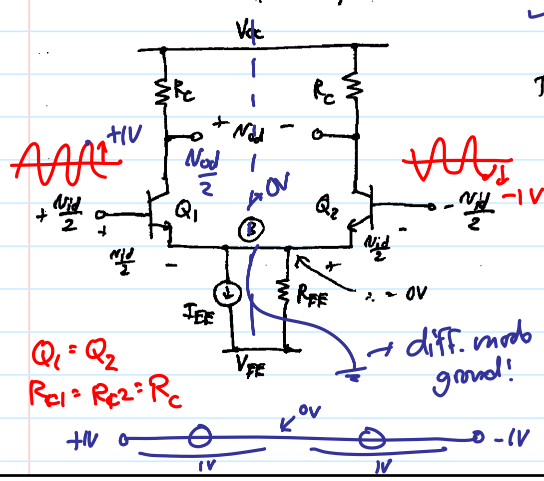
$\boxed{\frac{v_{o2}}{v_{i1}} = \frac{1}{2} g_m R_c}$

$\boxed{\frac{v_{o1}}{v_{i1}} = -\frac{1}{2} g_m R_c}$

Diff. Mode Analysis



Assume a ckt. w/ only diff. input:



Total current thru I_{EE} = const.

⟶ V_E = const. as input changes

⟶ ③ acts as an incremental ground! → $V_③ = 0V$ (always!)

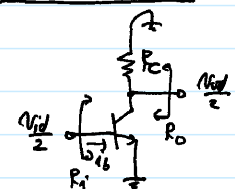
∴ we can ground ③, and thus, have a $\boxed{\text{Differential Half Ckt.}}$

Note: Can really only make this for a purely symmetrical ckt.!

EE 140 | Common-Mode Analysis

CTN 7

Differential Half Ckt.



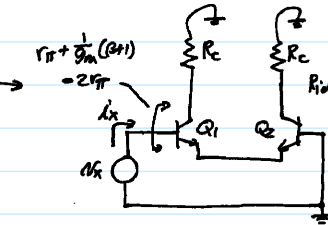
By inspection: $\dfrac{v_{od}/2}{v_{id}/2} = \dfrac{v_{od}}{v_{id}} = \boxed{A_{dm} = -g_m R_C}$

$\dfrac{v_{id}/2}{i_b} = r_\pi \longrightarrow R_{id} = \dfrac{v_{id}}{i_b} = \boxed{2r_\pi = R_{id}}$

$\dfrac{v_{od}/2}{i_o} = r_o \| R_C \longrightarrow R_{od} = \dfrac{v_{od}}{i_o} = 2(r_o \| R_C) \approx \boxed{2R_C = R_{od}}$

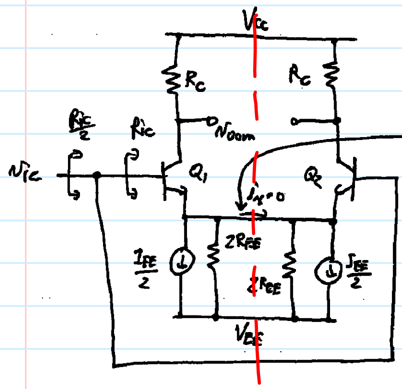
S.S. params. determined w/ $I_C = \frac{I_{EE}}{2}$

First define



Common-Mode Analysis

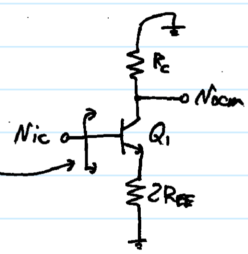
Assume a pure CM input → tie inputs together



By symmetry, $i_x = 0$. ⇒ thus, really have the equivalent of an open ckt. here

∴ ⇒ can split the ckt. into CM half-ckts.!

S.S. CM Half-Ckt.



$\boxed{A_{cm} = \dfrac{v_{ocm}}{v_{ic}} = -\dfrac{g_m R_C}{1 + g_m(2R_{EE})} \approx -\dfrac{R_C}{2R_{EE}}}$

Want small for large CMRR ∴ want R_{EE} = large!

$R_{ic} = r_\pi + (\beta+1)(2R_{EE})$ @ each input

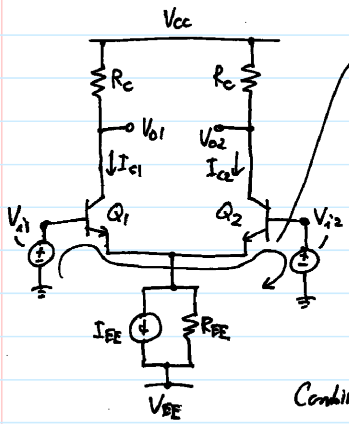
Common-Mode Rejection Ratio = $\text{CMRR} = \dfrac{A_{dm}}{A_{cm}} = \dfrac{-g_m R_C}{-\dfrac{g_m R_C}{1+g_m(2R_{EE})}} \Longrightarrow \boxed{\text{CMRR} = 1 + 2g_m R_{EE}}$

! Want as ideal a current source as possible!

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the ECP.

EE 140 | Large Signal ECP Performance | CTN | 8

Large Signal ECP performance



Find I_{c1} & I_{c2}:

KVL: $V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$

$I_{c1} = I_{s1}\exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{c1}}{I_{s1}}\right), \quad V_{be2} = V_T \ln\left(\frac{I_{c2}}{I_{s2}}\right)$

$V_{i1} - V_T \ln\left(\frac{I_{c1}}{I_{c2}}\frac{I_{s2}}{I_{s1}}\right) - V_{i2} = 0 \rightarrow \ln\frac{I_{c1}}{I_{c2}} = \frac{V_{i1}-V_{i2}}{V_T} = \frac{V_{id}}{V_T}$

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{id}}{V_T}\right) \qquad (1)$$

$$I_{EE} = I_{e1} + I_{e2} = \frac{1}{\alpha}(I_{c1} + I_{c2}) \qquad (2)$$

Combine (1) & (2) to get:

$$I_{c1} = \frac{\alpha I_{EE}}{1+\exp\left(-\frac{V_{id}}{V_T}\right)}, \qquad I_{c2} = \frac{\alpha I_{EE}}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \qquad (3)$$

Find V_{od}:

$V_{o1} = V_{cc} - I_{c1}R_c$
$V_{o2} = V_{cc} - I_{c2}R_c$

$V_{od} = V_{o1} - V_{o2} = (I_{c2} - I_{c1})R_c$ using (3)

$$= \alpha_F I_{EE} R_c \left\{ \frac{1}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)} - \frac{1}{1+\exp\left(-\frac{V_{id}}{V_T}\right)} \times \frac{\exp\left(+\frac{V_{id}}{2V_T}\right)}{\exp\left(+\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right) + \exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_c \frac{\sinh\left(-\frac{V_{id}}{2V_T}\right)}{\cosh\left(-\frac{V_{id}}{2V_T}\right)}$$

$$\left[\begin{array}{l} \sinh u = \frac{1}{2}(e^u - e^{-u}) \\ \cosh u = \frac{1}{2}(e^u + e^{-u}) \end{array} \right\} u = -\frac{V_{id}}{2V_T} \Bigg]$$

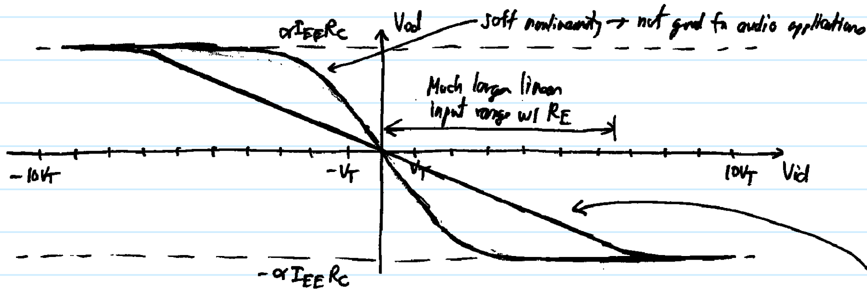
$$\therefore \boxed{V_{od} = \alpha_F I_{EE} R_c \tanh\left(-\frac{V_{id}}{2V_T}\right)}$$

From our knowledge of the Taylor series for

$\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \ldots$

this is fairly linear for small Vid, but gets nonlinear abruptly when Vid approaches a threshold value!

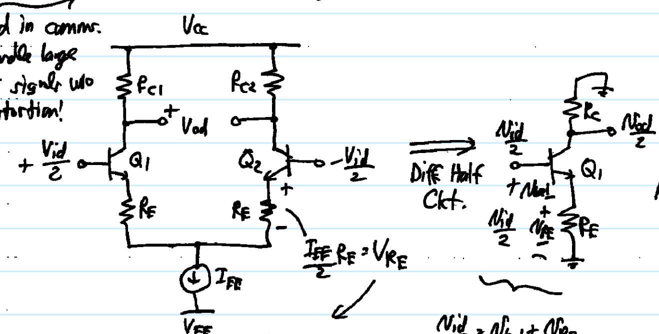
EE 140 | Large Signal ECP Performance | CTN | 9



In the above curve, the $\frac{V_{od}}{V_{id}}$ Xfer function is really only linear for $V_{id} \ll V_T$ → beyond V_T, start to enter the nonlinear realm of curve → causes signal distortion: e.g., phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xistor amplifiers)

Needed in comm. to handle large input signals w/o distortion!



$$A_{dm} = -\frac{g_m R_C}{1 + g_m R_E}$$

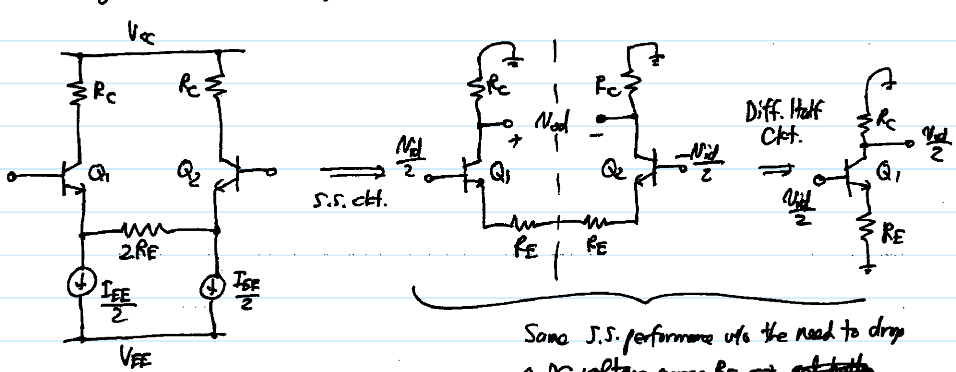
⇒ s.s. gain reduced, but the linear range is increased

If I_{EE} = large, then this can force large supply voltages

$\frac{v_{id}}{2} = v_{be1} + v_{R_E}$

This can still be $v_{be1} \ll V_T$ if this absorbs some of the input voltage!

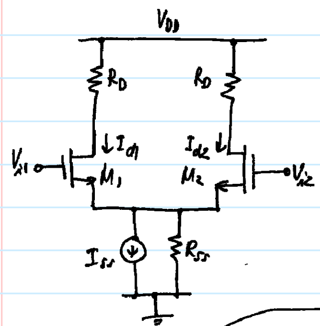
Alternative Biasing Technique If Need Larger DC Currents: –



Same s.s. performance w/o the need to drop a DC voltage across R_E → ~~get both~~ can use lower V_{CC} & V_{EE}.

EE 140 | **MOS Source-Coupled Pair** | CTN | 10

MOSFET Source-Coupled Pair



Assume: M_1 & M_2 are identical.

Find $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$.

$\Rightarrow$ approach: get $V_{id} = f(\Delta I_d)$ $\rightarrow$ then invert to get $\Delta I_d = f(V_{id})$

$$I_{d1} = \underbrace{\tfrac{1}{2}\mu_n C_{ox}\left(\tfrac{W}{L}\right)_1}_{k}(V_{gs1}-V_t)^2 \Rightarrow V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{k}}$$

$$\therefore V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}}$$

Define. $\Delta I_d = I_{d1} - I_{d2}$, $I_d = \dfrac{I_{d1}+I_{d2}}{2}$ $\Bigg\}$ $I_{d1} = I_d + \dfrac{\Delta I_d}{2}$, $I_{d2} = I_d - \dfrac{\Delta I_d}{2}$

$$V_{id} = \sqrt{\frac{2\left(I_d + \frac{\Delta I_d}{2}\right)}{k}} - \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{k}} \Rightarrow \frac{k}{2}V_{id}^2 = I_d + \cancel{\frac{\Delta I_d}{2}} - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2} + I_d - \cancel{\frac{\Delta I_d}{2}}$$

$$\frac{k}{2}V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2}$$

$\Rightarrow$ now rearrange to get ΔI_d (algebra)

Solve for ΔI_d:

$$\Delta I_d = \frac{k}{2}V_{id}\left(\frac{2I_{SS}}{k/2} - V_{id}^2\right)^{1/2} = \boxed{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{id}\sqrt{\left(\frac{2I_{SS}}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\right) - V_{id}^2} = \Delta I_d}$$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

V_{GS} for $I_D = \frac{I_{SS}}{2}$

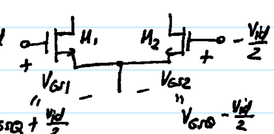
$$|V_{id}| \le \sqrt{\frac{2I_{SS}}{k}} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{2}\,(V_{GS}-V_t)$$

if true then input devices are both saturated

Thus, to extend the linear input range:

1. $I_{SS}\uparrow \longrightarrow (V_{GS}-V_t)\uparrow$
2. $W\downarrow$
3. $L\uparrow$

To derive this:



When $\frac{V_{id}}{2} \ge V_{GSQ} - V_t = \Delta V$ then M_2 will cut-off

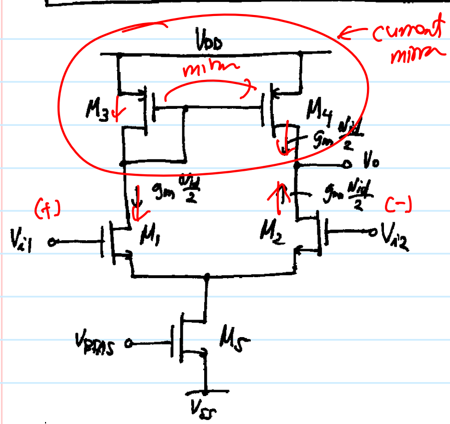
$\therefore V_{id} \le 2(V_{GSQ}-V_t) \rightarrow$ to maintain saturation

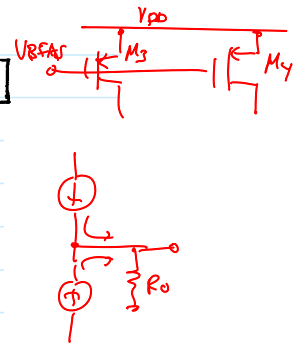
$$V_{GSQ} - V_t = \sqrt{\frac{2I_{d2}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \frac{V_{id}}{2}$$

then plug in ΔI_d & solve for V_{id}

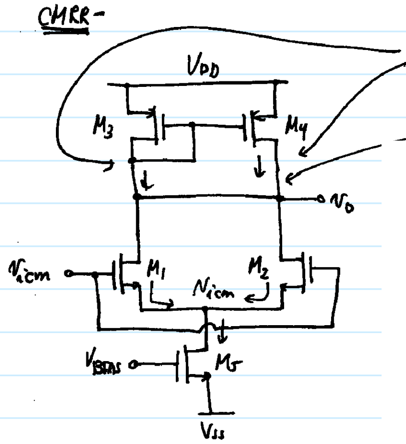
EE 140 | Diff. Pair w/ Current Mirror Load | CTN | 11

MOS Differential Stage w/ Current Mirror Load





CMRR -



$V_{D3} = V_{D4}$ (they track each other) → provided the ckt. is completely symmetrical.

⇒ Thus, can short the drains of M_3 & M_4

Common-Mode Input Range - Range of input voltages for which all devices remain in saturation.

Low End - must keep M_5 saturated

