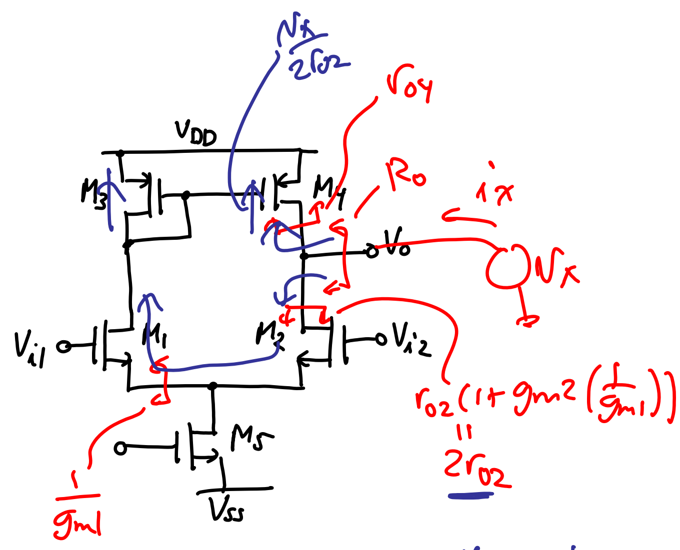
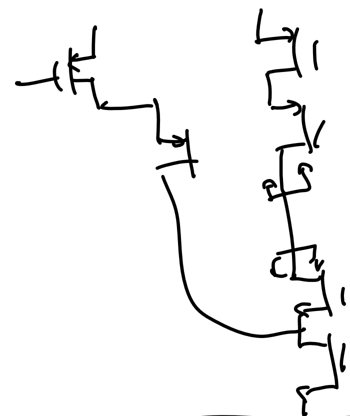
Lecture 14: SCP & Vos

- Announcements:
  - I will need to miss next Thursday's lecture
  - Makeup lecture Friday, 3/9, 3-4:30 p.m., in 277 Cory
- Lecture Topics:
  - Source Coupled Pair w/ Current Mirror Load
  - Input Offset Voltage (Vos)
  - Op Amp Non-Ideality
  - Op Amp Finite Gain-BW
- ----------------------------------
- Last Time:
- Going though the handout on Op Amps and Differential Pairs
- Continue with this now …



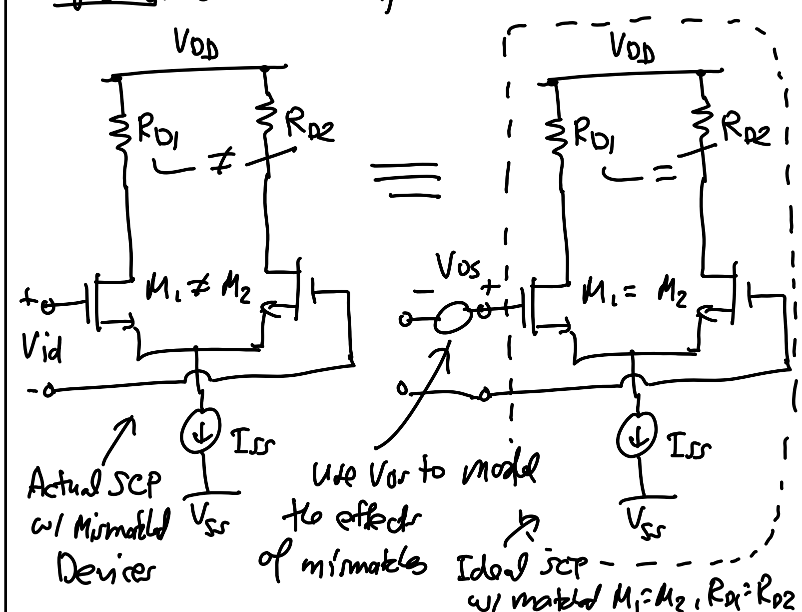
$$i_x = \frac{v_x}{r_{o4}} + \frac{v_x}{2r_{o2}} + \frac{v_x}{2r_{o2}} = \frac{v_x}{r_{o4}} + \frac{v_x}{r_{o2}}$$

$$R_o = \frac{v_x}{i_x} = r_{o2} \| r_{o4}$$



$\boxed{\text{Vos of a mismatched SCP}}$

Objective: Derive an expression for Vos.

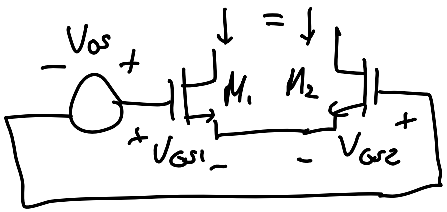


V_{os} arises due to variations in:

① Xsistors, M_1 & M_2 → $\frac{W}{L}$ & V_t vary

② $R_{D1} \neq R_{D2}$ → cause variations in gain

Define: V_{os} = V_{id} needed to get $V_{od} = 0V$ in this ckt.



KVL: $V_{os} - V_{GS1} + V_{GS2} = 0$

$\therefore V_{os} = V_{GS1} - V_{GS2}$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{os} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \qquad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$
$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$V_t = \frac{1}{2}(V_{t1} + V_{t2})$	$R_D = \frac{1}{2}(R_{D1} + R_{D2})$

Rearranging:

$I_{D1} = I_D + \frac{\Delta I_D}{2}$	$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2}$	$V_{t1} = V_t + \frac{\Delta V_t}{2}$
$I_{D2} = I_D - \frac{\Delta I_D}{2}$	$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2}$	$V_{t2} = V_t - \frac{\Delta V_t}{2}$

Substituting into (1): $2I_D\left(1 + \frac{\Delta I_D}{2I_D}\right)$

$$V_{os} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}$$

$\frac{W}{L}\left[1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{OS} \rightarrow V_{od} = 0$ $\therefore I_{D1} R_{D1} = I_{D2} R_{D2}$

mismatch in I_D must be opposite that of R_D

$$\therefore \frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R_D}{R_D} - \frac{\Delta (W/L)}{(W/L)}\right\}}$$

Threshold Voltage Mismatch

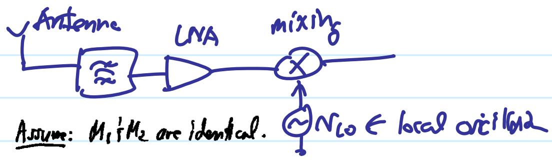
Bias Dependent

Geometric Variations (i.e., layout & litho)

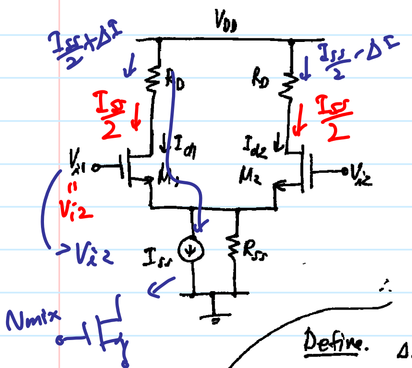
scale w/ overdrive

EE 140 | MOS Source-Coupled Pair

CTN (10)



MOSFET Source-Coupled Pair



Assume: M_1 & M_2 are identical.

Find $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$.

⟹ approach: get $V_{id} = f(\Delta I_d)$ → then invert to get $\Delta I_d = f(V_{id})$

$$I_{d1} = \underbrace{\tfrac{1}{2}\mu_n C_{ox}\left(\tfrac{W}{L}\right)_1}_{k}(V_{gs1}-V_t)^2 \Longrightarrow V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{k}}$$

$$\therefore\ V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}}$$

Define. $\Delta I_d = I_{d1} - I_{d2}$, $I_d = \frac{I_{d1}+I_{d2}}{2}$ $\Big\}$ $I_{d1} = I_d + \frac{\Delta I_d}{2}$, $I_{d2} = I_d - \frac{\Delta I_d}{2}$

$$V_{id} = \sqrt{\frac{2\left(I_d + \frac{\Delta I_d}{2}\right)}{k}} - \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{k}} \Rightarrow \frac{k}{2}V_{id}^2 = I_d + \cancel{\frac{\Delta I_d}{2}} - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2} + I_d - \cancel{\frac{\Delta I_d}{2}}$$

$$\frac{k}{2}V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2}$$

⟹ now rearrange to get ΔI_d (algebra)

Solve for ΔI_d:
$$\Delta I_d = \frac{k}{2}V_{id}\left(\frac{2I_{ss}}{k/2} - V_{id}^2\right)^{1/2} = \boxed{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{id}\sqrt{\left(\frac{2I_{ss}}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\right) - V_{id}^2} = \Delta I_d}$$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

$$|V_{id}| \le \sqrt{\frac{2I_{ss}}{k}} = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{2}\,(V_{GS}-V_t)$$

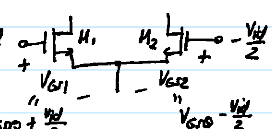
V_{GS} for $I_D = \frac{I_{ss}}{2}$

if true then input devices are both saturated

Thus, to extend the linear input range:

① $I_{ss}\uparrow \longrightarrow (V_{GS}-V_t)\uparrow$
② $W\downarrow$
③ $L\uparrow$

To derive this:



When $\frac{V_{id}}{2} \ge V_{GSQ} - V_t = \Delta V$ then M_2 will cut-off

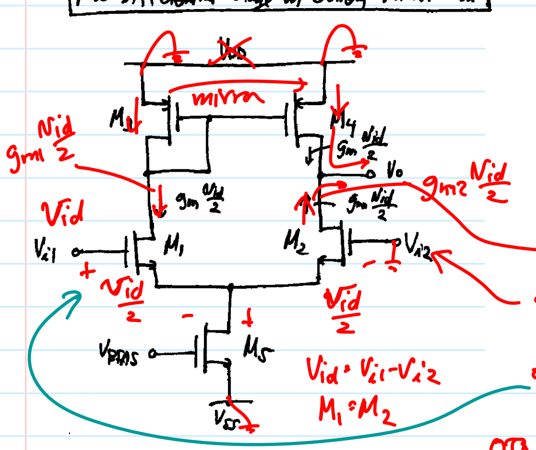
$\therefore\ V_{id} \le 2(V_{GSQ}-V_t)$ → to maintain saturation

$$V_{GSQ} - V_t = \sqrt{\frac{2I_{d2}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \frac{V_{id}}{2}$$

then plug in ΔI_d & solve for V_{id}

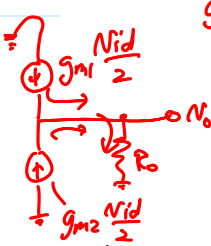
EE 140 | Diff. Pair w/ Current Mirror Load | CTN | 11

MOS Differential Stage w/ Current Mirror Load



$v_{id} = v_{i1} - v_{i2}$

$M_1 = M_2$



$g_{m1} = g_{m2} = g_m$

$v_o = \left(\frac{g_m}{2} + \frac{g_m}{2}\right) v_{id} R_o$

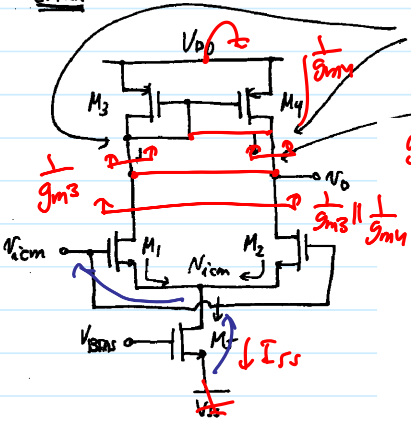
$\frac{v_o}{v_{id}} = g_m R_o$ → ? f/ main loading pole

$$\frac{v_o}{v_{id}} = g_m (r_{o2} \| r_{o4})$$

A_{dm} large!

OTA → operational transconductance amplifier

CMRR -

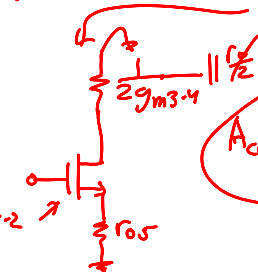


$V_{D3} = V_{D4}$ (they track each other) → provided the ckt. is completely symmetrical.

⇒ Thus, can short the drains of M_3 & M_4 → truly symmetric → ∴ we can draw half-ckt or just combine the two sides together!

$g_{m3} = g_{m4} = g_{m3\text{-}4}$

$\frac{1}{g_{m3}} \| \frac{1}{g_{m4}} = \frac{1}{2g_{m3\text{-}4}}$



$$A_{cm} \cong -\frac{2g_{m1\text{-}2}\left(\frac{1}{2g_{m3\text{-}4}}\right)}{1 + 2g_{m1\text{-}2} r_{o5}}$$

Thus:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \cancel{g_{m1\text{-}2}} (r_{o1\text{-}2} \| r_{o3\text{-}4})(1 + 2g_{m1\text{-}2} r_{o5})\left(\frac{g_{m3\text{-}4}}{\cancel{g_{m1\text{-}2}}}\right)$$

$$CMRR = (1 + 2g_{m1\text{-}2} r_{o5}) g_{m3\text{-}4} (r_{o1\text{-}2} \| r_{o3\text{-}4})$$

Common-Mode Input Range - Range of input voltages for which all devices remain in saturation.

Low End - must keep M_5 saturated

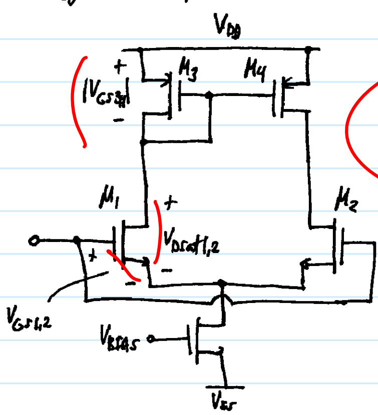
smallest (or lowest) common-mode voltage @ input w/o something goes wrong → triode

$V_{icm(min)} = CMR^- = V_{SS} + V_{OV5} + V_{GS1,2}$

$$CMR^- = V_{SS} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}(W/L)_5}} + V_{t1,2} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox}(W/L)_1}}$$

EE 140 | Diff. Pair w/ Current Mirror Load | CTN | 12

High End - keep M_1, M_2 saturated



$$\cancel{V_{ov1,2}} + V_{t1,2}$$

$$V_{icm(max)} = CMR+ = V_{DD} - |V_{GS3,4}| - \cancel{V_{ov1,2}} + V_{GS1,2}$$

$$V_{icm(max)} = CMR+ = V_{DD} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} (W/L)_{3,4}}} - |V_{t3,4}| + V_{t1,2}$$

EE 140 | Input Offset Voltage | CTN | 1

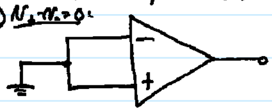
Device Mismatch Effects in Diff. Amplifiers

⇒ up to this point, we've assumed that Q_1 & Q_2 are perfectly matched

⇒ in actual ckt., get device mismatches due to processing variations

The Result:

① $v_+ - v_- = 0$: → Output not zero when input is zero → $v_{od} \neq 0$ when $v_{id} = 0$!



$v_o = A_{dm}(v_+ - v_-)$

Ideal Case: $v_o = 0$

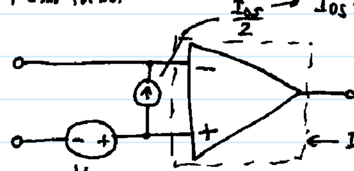
Reality: $v_o \neq 0$, even w/ $(v_+ - v_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if Q_1 & Q_2 not matched. (for BJT & JFET only.)

To model these effects, introduce:

① Input Offset Voltage, V_{os} ← Typ. 1-5mV for BJT

② Input Offset Current, I_{os} ← Typ. I_{os} = 10 nA for BJT



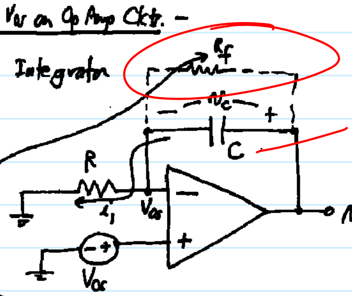
$\frac{I_{os}}{2}$ → $I_{os} \triangleq I_{B1} - I_{B2}$ (difference between input currents)

← Ideal Op Amp → well, it could have finite A_v, R_i, & R_o!

V_{os} → voltage needed to set $v_o = 0$ (negating the effect of mismatch) in this ckt.

Effect of V_{os} on Op Amp Ckt. –

Example. Integrator

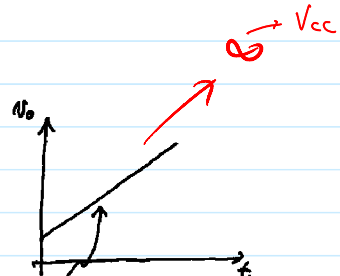


$Z = \frac{1}{j\omega C} \ll R_f$

$$v_o = V_{os} + \frac{1}{C}\int_0^t i_1 dt$$

$$= V_{os} + \frac{1}{C}\int_0^t \frac{V_{os}}{R} dt$$

$$= V_{os}\left(1 + \frac{t}{RC}\right) + v_C|_{t=0}$$



→ will continue to increase until op amp hits the voltage rails

Fix: Place an R_f in shunt w/ the C

→ then $v_o = V_{os}\left(1 + \frac{R_f}{R}\right)$, and railing doesn't happen

⇒ but, usually R_f = large to allow the C to dominate the integrator Xfer Function ∴ $v_o = V_{os}\left(1 + \frac{R_f}{R}\right)$ can be quite large → still want V_{os} = small

V_{os} is even more important in setting the resolution of A/D converters and other precision ckts.