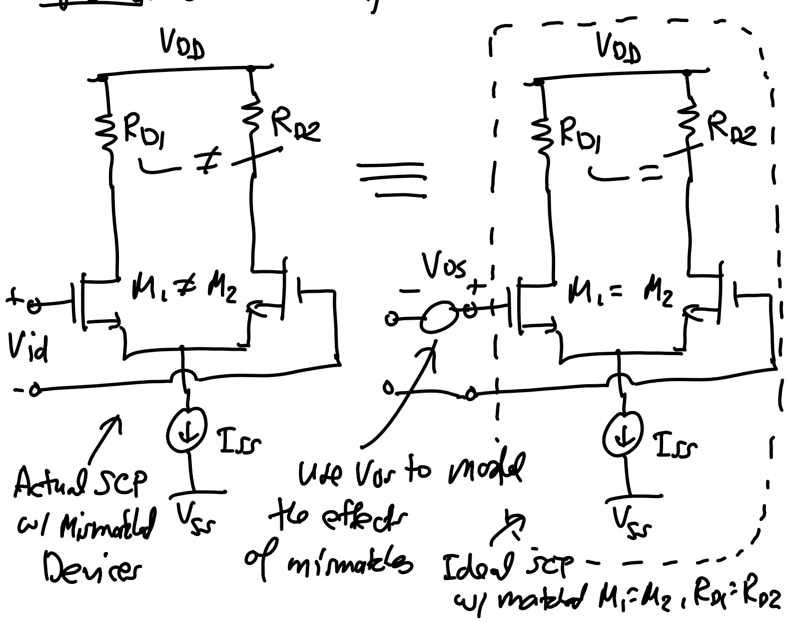
Lecture 15: Finite Gain-BW

- **Announcements:**
  - HW#6 due today; HW#7 online
  - Lab#2 extended one week; due week of 3/19
  - Reminder: No lecture this coming Thursday
  - Makeup lecture Friday, 3/9, 3-4:30 p.m., in 277 Cory
- **Lecture Topics:**
  - Input Offset Voltage (Vos)
  - Op Amp Finite Gain-BW
  - High Gain Op Amps
- --
- **Last Time:**

$\boxed{\text{Vos of a mismatched SCP}}$

Objective: Derive an expression for V_{os}.

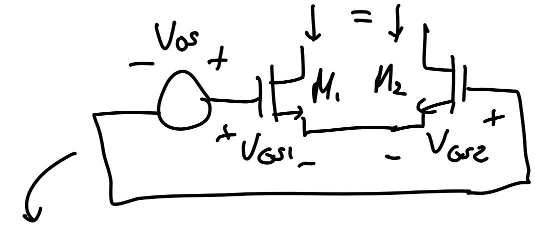


V_{os} arises due to variations in:

① Xsistors, M_1 & M_2 → $\frac{W}{L}$ & V_t vary

② $R_{D1} \neq R_{D2}$ → causes variations in gain

Define: V_{os} = V_{id} needed to get $V_{od} = 0V$ in this ckt.



KVL: $V_{os} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{os} = V_{GS1} - V_{GS2}$$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{os} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \quad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$
$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$V_t = \frac{1}{2}(V_{t1} + V_{t2})$	$R_D = \frac{1}{2}(R_{D1} + R_{D2})$

Reorranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1): $2I_D\left(1+\frac{\Delta I_D}{2I_D}\right)$

$$V_{os} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}$$

$\frac{W}{L}\left[1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{os} \rightarrow V_{od} = 0 \quad \therefore I_{D1}R_{D1} = I_{D2}R_{D2}$

↳ mismatch in I_D must be opposite that of R_D

$$\therefore \frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{os} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{+\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Voltage Mismatch

Bias Dependent

Geometric Variations (i.e., layout & litho)

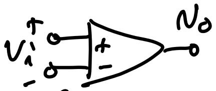
scale all cnadmp

↳ $\frac{W}{L}\uparrow \rightarrow V_{os}\downarrow$

These (-)'s don't help: $\frac{\Delta R_D}{R_D} = (+)$ or $(-)$

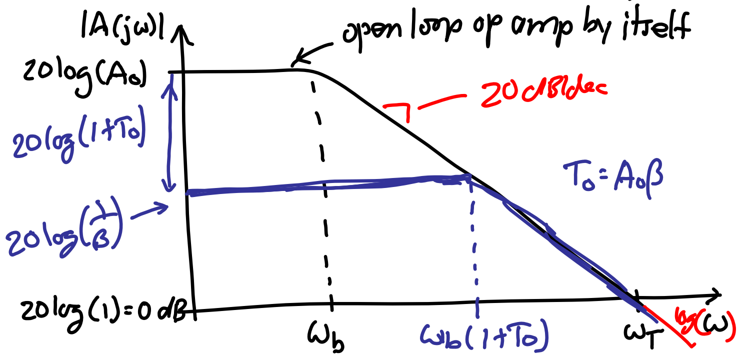
- Now, go through bipolar mismatch prepared material

Finite Op Amp Gain & Bandwidth



For an ideal op amp, $A = \infty$.

In reality, the gain is given by: $A(s) = \dfrac{A_o}{1 + s/\omega_b}$



$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$ $(= 0\,dB)$

At ω_T:

$$|A(j\omega_T)| = 1 = \frac{A_o}{\sqrt{1 + \left(\frac{\omega_T}{\omega_b}\right)^2}}$$

dc gain

$$[\omega_T \gg \omega_b] \Rightarrow \frac{A_o}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = A_o\omega_b}$$

Gain-Bandwidth Product

For $\omega \gg \omega_b$:

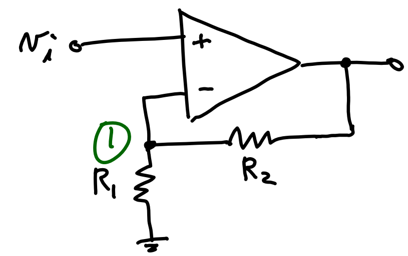
$$A(s) \approx \frac{A_o}{\frac{s}{\omega_b}} = \frac{A_o\omega_b}{s} = \frac{\omega_T}{s} = \frac{f_T}{f}$$

[Integrator w/ time constant $\tau = \frac{1}{\omega_T}$]

The unity gain bandwidth f_T is usually specified on op amp data sheets. Knowing f_T, one can easily determine the op amp gain at a given frequency f.

Frequency Response of Closed Loop Amplifiers

Example. Non-Inverting Amplifier



$$v_o = A(s)(v_+ - v_-), \quad A(s) = \frac{A_o}{1+s/\omega_b}$$

$$v_- = v_+ - \frac{v_o}{A(s)}$$

$$v_- = v_i - \frac{v_o}{A(s)}$$

Find an expression for the gain as a function of frequency.

① Brute force derivation:

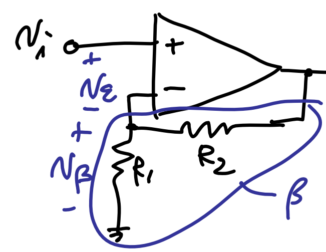
KCL @①: $\dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} \rightarrow \dfrac{v_o}{R_2} = v_-\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$$\frac{v_o}{R_2} = \left(v_i - \frac{v_o}{A(s)}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow \frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)}\left(1 + \frac{R_2}{R_1}\right)}$$

$$\left[A(s) = \frac{A_o}{1+s/\omega_b}\right] \xrightarrow{\text{algebra}} \boxed{\frac{v_o}{v_i}(s) = \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{\text{dc gain}} \underbrace{\frac{1}{1 + \frac{s}{A_o\omega_b\left(\frac{R_1}{R_1+R_2}\right)}}}_{\text{freq. shaping term}}}$$

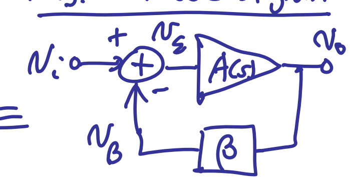
($A_o\omega_b = \omega_T$)

② More insightful way to do this:



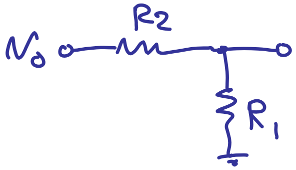
=

Neg. FB Block Diagram



$$v_\varepsilon = v_i - v_\beta$$

$$v_i = v_\varepsilon + v_\beta$$



$v_\beta = \left(\frac{R_1}{R_1+R_2}\right) v_o$

$$\boxed{\beta = \frac{v_\beta}{v_o} = \frac{R_1}{R_1+R_2}}$$

Recall from previous FB analysis:

$$\frac{v_o}{v_i}(s) = \frac{A(s)}{1+\beta A(s)}$$

$$\left[A(s) = \frac{A_o}{1+s/\omega_b}\right] \rightarrow \frac{v_o}{v_i}(s) = \frac{\frac{A_o}{1+s/\omega_b}}{1+\beta\left(\frac{A_o}{1+s/\omega_b}\right)}$$

$$\frac{v_o}{v_i}(s) = \underbrace{\frac{A_o}{1+\beta A_o}}_{\text{closed loop dc gain}} \underbrace{\frac{1}{1+\frac{s}{\omega_b(1+\beta A_o)}}}_{\text{freq. shaping term}}$$

$\omega_b(1+T_o)$

$\frac{\cancel{A_o}}{\cancel{A_o}\beta} \approx \frac{1}{\beta}$

$\beta A_o = T_o \triangleq$ "loop gain" at $\omega = 0$ (i.e., at dc)

Plug in β:

$$\frac{v_o}{v_i}(s) \approx \frac{1}{\beta}\,\frac{1}{1+\frac{s}{\omega_b A_o \beta}} = \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{s}{\omega_b A_o\left(\frac{R_1}{R_1+R_2}\right)}}$$

What if $A_o \neq$ large?

↳ can't say $\frac{A_o}{1+A_o\beta} \not\approx \frac{1}{\beta}$

Observations:

① Closed loop DC gain $= \frac{A_o}{1+\beta A_o} = \frac{A_o}{1+T_o} \approx \frac{A_o}{T_o}$ [$T_o \gg 1$]

i.e., the closed loop gain is reduced from the open loop gain by $1+T_o$ → show this on graph

② Alternatively, closed loop DC gain $\approx \frac{A_o}{\beta A_o} = \frac{1}{\beta}$ [$T_o \gg 1$]

③ ω_{-3dB} has increased from $\omega_b \rightarrow \omega_b(1+A_o\beta) = \omega_b(1+T_o)$

↳ To draw the Bode plot, just find the dc gain, draw a horizontal line across, then follow the open loop response after running into it!

④ Gain-BW Product $= \frac{A_o}{1+\beta A_o}\,\omega_b(1+\beta A_o) = A_o\omega_b = \omega_T$

∴ the Gain-BW product remains the same for the open & closed loop FB cases!

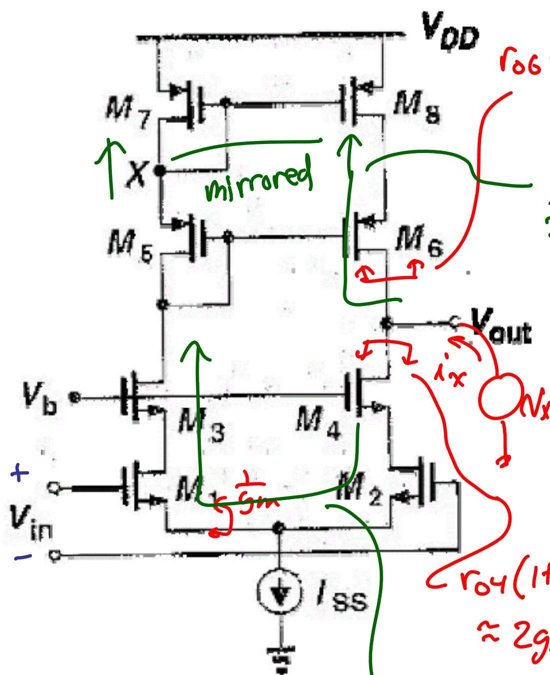
High Gain Op Amps

How can we increase gain?

① Cascode

② Cascade of Amplifiers

Telescopic Op Amp w/ Single-Ended Output



$r_{o6}(1+g_{m6}r_{o8})$

$\approx g_{m6}r_{o6}r_{o8}$

$\frac{v_x}{2g_{m4}r_{o2}r_{o4}}$

$r_{o2}\left(1+\frac{g_m}{g_m}\right)$

$r_{o4}(1+g_{m4}(2r_{o2}))$

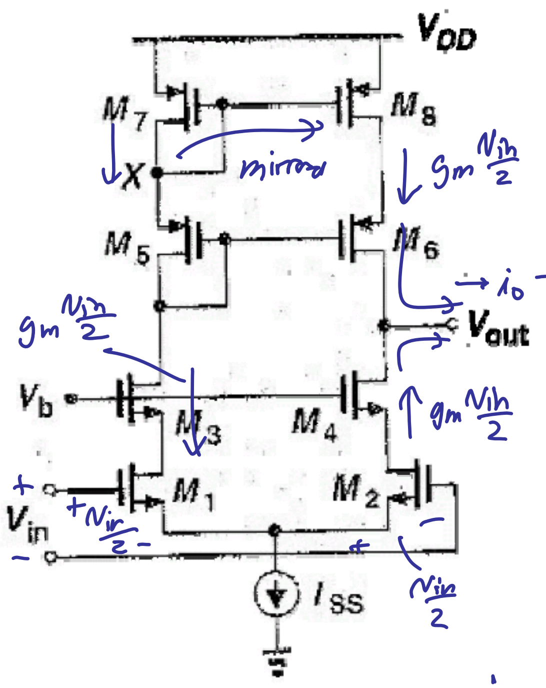
$\approx 2g_{m4}r_{o2}r_{o4}$

$\frac{v_x}{2g_{m4}r_{o2}r_{o4}}$

$$i_x = \frac{v_x}{g_{m6}r_{o6}r_{o8}} + \frac{v_x}{2g_{m4}r_{o2}r_{o4}} + \frac{v_x}{2g_{m4}r_{o2}r_{o4}}$$

$$R_o = \frac{v_x}{i_x} = (g_{m6}r_{o6}r_{o8}) \| (g_{m4}r_{o2}r_{o4})$$

$$\simeq (g_{mp}r_{op}^2) \| (g_{mN}r_{oN}^2)$$

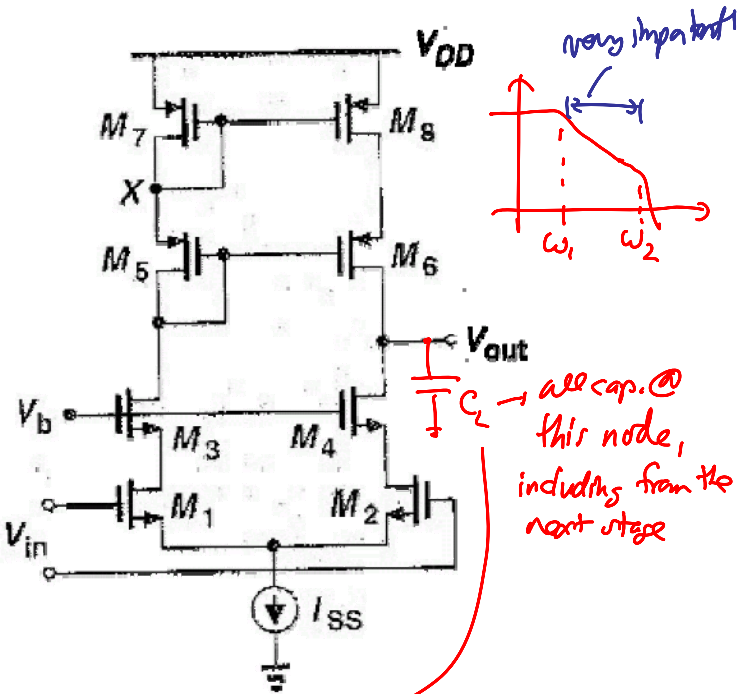


$$i_o = g_m\frac{v_{in}}{2} + g_m\frac{v_{in}}{2} \to G_m = \frac{i_o}{v_{in}} = g_m$$

$$\therefore \text{Gain}: A_v = g_{mN}\left[(g_{mN}r_{oN}^2) \| (g_{mp}r_{op}^2)\right]$$

huge R! huge R!

gain will be big!



This contributes the dominant pole:

$$\boxed{\omega_H = \frac{1}{R_o C_L}}$$