EE 140

Input Offset Voltage

CTN

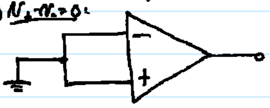
Device Mismatch Effects in Diff. Amplifiers

⇒ up to this point, we've assumed that Q_1 & Q_2 are perfectly matched

⇒ in actual ckt., get device mismatches due to processing variations

The Result:

① $v_+ - v_- = 0$: → Output not zero when Input is zero → $v_{od} \neq 0$ when $v_{id} = 0$!



$v_o = A_{dm}(v_+ - v_-)$

Ideal Case: $v_o = 0$

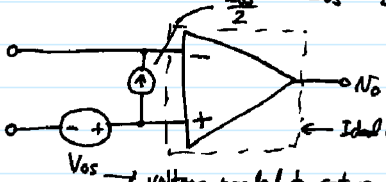
Reality: $v_o \neq 0$, even w/ $(v_+ - v_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if Q_1 & Q_2 not matched. (for BJT & JFET only.)

To model those effects, introduce:

① Input Offset Voltage, V_{os} → Typ. 1-5 mV for BJT

② Input Offset Current, I_{os} → Typ. I_{os} = 10 nA for BJT



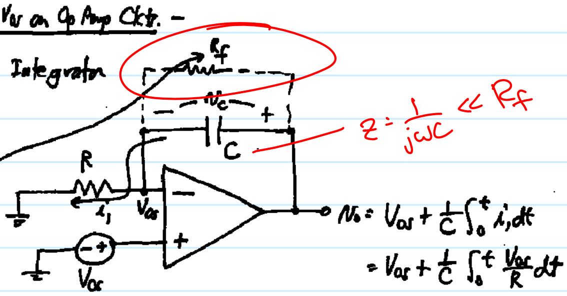
$\frac{I_{os}}{2}$ → $I_{os} \triangleq I_{B1} - I_{B2}$ (difference between input currents)

← Ideal Op Amp → well, it could have finite A_v, R_i, & R_o!

V_{os} → voltage needed to set $v_o = 0$ in this ckt. (negating the effect of mismatch)

Effect of V_{os} on Op Amp Ckt. –

Example. Integrator

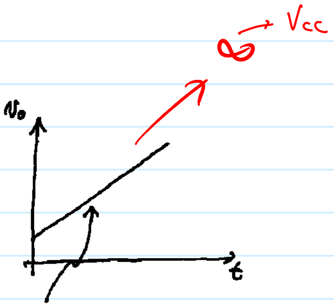


$Z = \frac{1}{j\omega C} \ll R_f$

$$v_o = V_{os} + \frac{1}{C}\int_0^t i_1 dt$$

$$= V_{os} + \frac{1}{C}\int_0^t \frac{V_{os}}{R} dt$$

$$= V_{os}\left(1 + \frac{t}{RC}\right) + v_c\big|_{t=0}$$



will continue to increase until op amp hits the voltage rails

Fix: Place an R_f in shunt w/ the C

→ then $v_o = V_{os}\left(1 + \frac{R_f}{R}\right)$, and railing doesn't happen

⇒ but, usually R_f = large to allow the C to dominate the integrator Xfer Function ∴ $v_o = V_{os}\left(1 + \frac{R_f}{R}\right)$ can be quite large → still want V_{os} = small

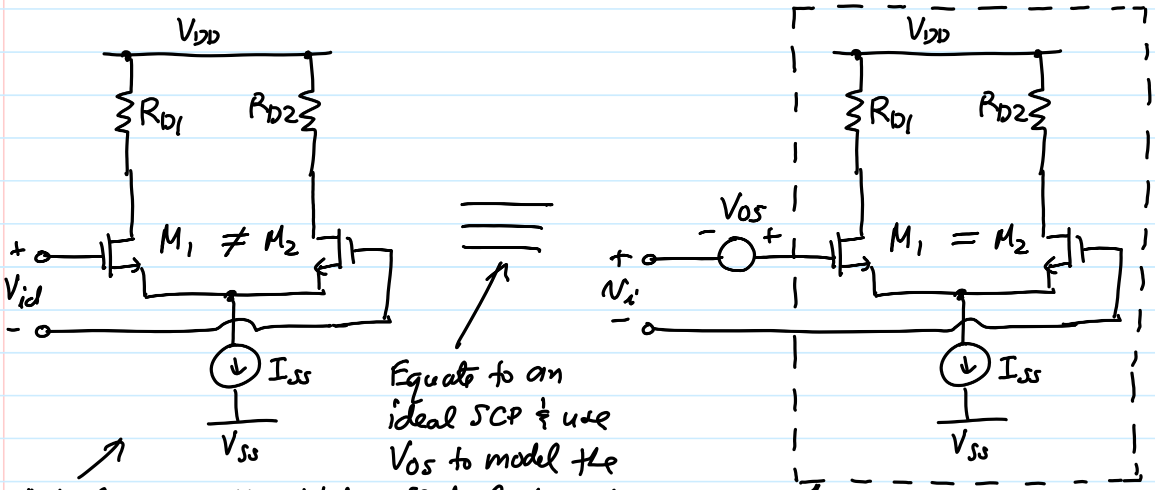
V_{os} is even more important in setting the resolution of A/D converters and other precision ckts.

EE 140 | V_{OS} of a Mismatched SCP

CTN

V_{OS} of a Mismatched SCP

Objective: Derive an expression for V_{OS}.



Actual SCP w/ Mismatched Xsistors & R's

Equate to an ideal SCP & use V_{OS} to model the effect of mismatches

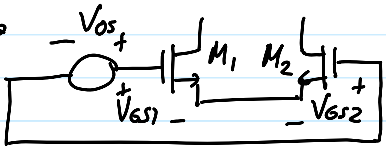
Ideal SCP w/ Matched M_1 & M_2 and $R_{D1} = R_{D2}$

Input offset voltage V_{OS} arises due to variations in:

① Xsistors, M_1 & M_2 → $\frac{W}{L}$ and V_t vary

② $R_{D1} \neq R_{D2}$ → causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$ in this ckt.



KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \quad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$	$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$	$V_t = \frac{V_{t1} + V_{t2}}{2}$	$R_D = \frac{R_{D1} + R_{D2}}{2}$

EE 140 | V_{OS} of a Mismatched SCP

CTN

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$V_{os} = \Delta V_t + \sqrt{\frac{\overset{2I_D\left(1+\frac{\Delta I_D}{2I_D}\right)}{2(I_D + \Delta I_D/2)}}{\underset{\frac{W}{L}\left[1+\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}} - \sqrt{\frac{\overset{2I_D\left(1-\frac{\Delta I_D}{2I_D}\right)}{2(I_D - \Delta I_D/2)}}{\underset{\frac{W}{L}\left[1-\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}}$$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

↳ Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\underbrace{\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancelto{0}{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancelto{0}{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore\ V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{os} \rightarrow V_{od} = 0$ $\therefore$ $I_{D1}R_{D1} = I_{D2}R_{D2}$ ⟶ mismatch in I_D must be opposite that of R_D

$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Mismatch

↑ bias independent

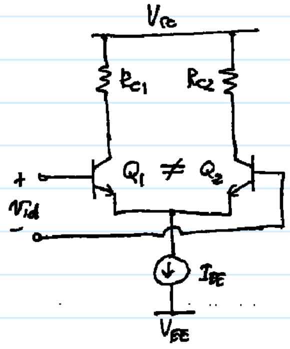
Geometric (i.e., Layout) Variation

↳ scale w/ overdrive

V_{OS} of a Mismatched ECP

V_{os} in a Mismatched ECP

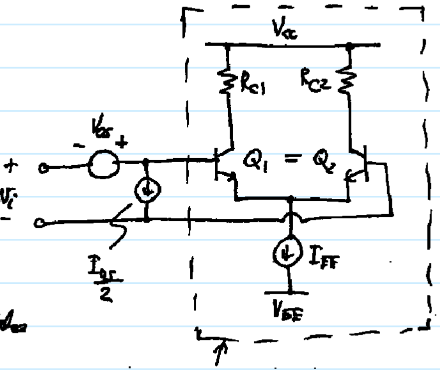
Objective: Derive an expression for V_{os}.



Actual ECP w/ Mismatched Xsistrs & R's

$\equiv$

Equals to an ideal ECP & one V_{os} & I_{os} to model the effect of mismatches



Ideal ECP w/ Matched Q_1 & Q_2 and $R_{C1} = R_{C2}$

Input Offset Voltage V_{os} arises due to variations in:

① Xsistrs, Q_1 & Q_2 → I_s & β vary: $I_s = \dfrac{q n_i^2 \overline{D_n} A}{N_A W_B(V_{CB})}$ ∴ $I_{S1} \neq I_{S2}$ can be caused by:

"function of"

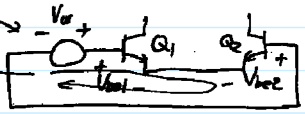
(i) $A_1 \neq A_2$ (etching tolerance limits)

(ii) $N_{A1} \neq N_{A2}$ (doping variations of base)

(iii) $W_B = f(V_{CB})$ (width variations exacerbated by V_{CB} diff.)

② $R_{C1} \neq R_{C2}$ → cause gain variation

Definition. $V_{os} = V_{id}$ to get $V_{od} = 0$, in this ckt. ~~which occurs when~~:



KVL: $V_{os} - V_{be1} + V_{be2} = 0$

$$V_{os} = V_{be1} - V_{be2} = V_T \ln \frac{I_{c1}}{I_{s1}} - V_T \ln \frac{I_{c2}}{I_{s2}} = V_T \ln\left(\frac{I_{c1}}{I_{c2}} \frac{I_{s2}}{I_{s1}}\right)$$

Find $\frac{I_{c1}}{I_{c2}}$ in terms of design elements:

$$\left[\text{When } V_{id} = V_{os} \rightarrow V_{od} = 0V\right] \rightarrow V_{od} = (\cancel{V_{cc}} - I_{c1}R_{c1}) - (\cancel{V_{cc}} - I_{c2}R_{c2}) = 0$$

$$I_{c1}R_{c1} = I_{c2}R_{c2} \longrightarrow \frac{I_{c1}}{I_{c2}} = \frac{R_{c2}}{R_{c1}}$$

$$\boxed{V_{os} = V_T \ln\left(\frac{R_{c2}}{R_{c1}} \frac{I_{s2}}{I_{s1}}\right)}$$

This is an exact equation for V_{os}. It's often more useful & intuitive to express this in terms of percent variations (and eventually standard deviations).

EE 140

V_{OS} of a Mismatched ECP

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Convert to Percent Variation Form –

Define.

$$\left.\begin{aligned} R_c &= \frac{R_{c1}+R_{c2}}{2}, & \Delta R_c &= R_{c1}-R_{c2} \\ I_s &= \frac{I_{s1}+I_{s2}}{2}, & \Delta I_s &= I_{s1}-I_{s2} \end{aligned}\right\}$$

Objective: Express V_{OS} in terms of percent variations $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$.

$$\Downarrow$$

In general: $\left.\begin{aligned} \Delta X &= X_1 - X_2 \\ X &= \frac{X_1+X_2}{2} \end{aligned}\right\} \begin{aligned} X_1 &= X + \frac{\Delta X}{2} \\ X_2 &= X - \frac{\Delta X}{2} \end{aligned} \Rightarrow$ Thus: $\begin{aligned} R_{c1} &= R_c + \frac{\Delta R_c}{2}, \; R_{c2} = R_c - \frac{\Delta R_c}{2} \\ I_{s1} &= I_s + \frac{\Delta I_s}{2}, \; I_{s2} = I_s - \frac{\Delta I_s}{2} \end{aligned}$

With these formulations:

$$V_{OS} = V_T \ln\left[\frac{R_{c2}}{R_{c1}}\frac{I_{s2}}{I_{s1}}\right] = V_T \ln\left\{\frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \; \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}}\right\} = V_T \ln\left\{\frac{1 - \frac{\Delta R_c}{2R_c}}{1 + \frac{\Delta R_c}{2R_c}} \; \frac{1 - \frac{\Delta I_s}{2I_s}}{1 + \frac{\Delta I_s}{2I_s}}\right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right] \Rightarrow V_{OS} \cong V_T\left\{-\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s}\right\}$$

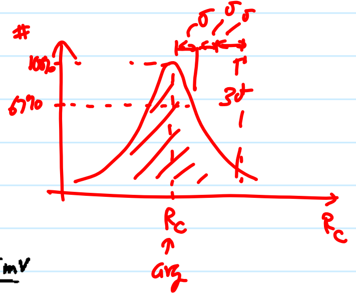
taking the first term assuming $\Delta R \ll R_c$ & $\Delta I_s \ll I_s$

$$\boxed{V_{OS} = V_T\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}$$

Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically varying parameters for a given process run & layout, one usually expresses terms in the form of variances when specifying V_{OS}:

→ since $\frac{\Delta R_c}{R_c}$ & $\frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like powers:

$$\sigma_{V_{OS}} = V_T\sqrt{\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta I_s/I_s}}$$



Ex. Typ. $\sigma_{\Delta R_c/R_c} \sim 0.01$, $\sigma_{\Delta I_s/I_s} \sim 0.05$

$$\therefore \sigma_{V_{OS}} = (26m)\sqrt{(0.01)^2 + (0.05)^2} = \underline{1.3mV}$$

Typ. V_{OS} for BJT ~ $\underline{1\text{-}5mV}$

V_{OS} Drift w/ Temperature

$$\frac{dV_{OS}}{dT} = \frac{kT}{q}\underbrace{\left\{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}\right\}}_{\text{indep. of } T}\frac{1}{T} = \frac{V_{OS}}{\underset{[\text{in kelvin}]}{T}}$$

Ex. $\frac{dV_{OS}}{dT} = \frac{1.3m}{300k} = 4.3\,\mu V/°C$ drift around $T = 300K$.

EE 140

I_{OS} of a Mismatched ECP

CTN

I_{os} in a Mismatched ECP

By Definition: $I_{os} = I_{B1} - I_{B2} = \boxed{\frac{I_{c1}}{\beta_1} - \frac{I_{c2}}{\beta_2} = I_{os}}$

To express in percent variations:

$$\begin{cases} I_{c1} = I_c + \frac{\Delta I_c}{2} \\ I_{c2} = I_c - \frac{\Delta I_c}{2} \end{cases} \qquad \begin{cases} \beta_1 = \beta + \frac{\Delta\beta}{2} \\ \beta_2 = \beta - \frac{\Delta\beta}{2} \end{cases}$$

$$\therefore I_{os} = \frac{I_c + \frac{\Delta I_c}{2}}{\beta + \frac{\Delta\beta}{2}} - \frac{I_c - \frac{\Delta I_c}{2}}{\beta - \frac{\Delta\beta}{2}} = \frac{I_c}{\beta}\left\{\frac{1 + \frac{\Delta I_c}{2I_c}}{1 + \frac{\Delta\beta}{2\beta}} - \frac{1 - \frac{\Delta I_c}{2I_c}}{1 - \frac{\Delta\beta}{2\beta}}\right\}$$

$$\left[\frac{1}{1+x} \approx 1 - x + x^2 - \cdots\right] \longrightarrow = \frac{I_c}{\beta}\left\{\left(1 + \frac{\Delta I_c}{2I_c}\right)\left(1 - \frac{\Delta\beta}{2\beta}\right) - \left(1 - \frac{\Delta I_c}{2I_c}\right)\left(1 + \frac{\Delta\beta}{2\beta}\right)\right\}$$

$$= \frac{I_c}{\beta}\left\{\cancel{1} + \frac{\Delta I_c}{2I_c} - \frac{\Delta\beta}{2\beta} - \cancel{\frac{\Delta I_c}{2I_c}\frac{\Delta\beta}{2\beta}} - \cancel{1} + \frac{\Delta I_c}{2I_c} - \frac{\Delta\beta}{2\beta} + \cancel{\frac{\Delta I_c}{2I_c}\frac{\Delta\beta}{2\beta}}\right\}$$

$$\boxed{I_{os} = \frac{I_c}{\beta}\left\{\frac{\Delta I_c}{I_c} - \frac{\Delta\beta}{\beta}\right\}}$$

But for $V_{od} = 0V \Longrightarrow \frac{I_{c1}}{I_{c2}} = \frac{R_{c2}}{R_{c1}} \longrightarrow \frac{\Delta I_c}{I_c} = -\frac{\Delta R_c}{R_c}$

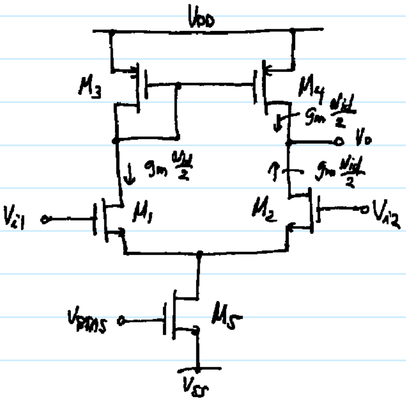
$$\therefore \boxed{I_{os} = -\frac{I_c}{\beta}\left(\frac{\Delta R_c}{R_c} + \frac{\Delta\beta}{\beta}\right)}$$

Ex. Typ: $\sigma_{\Delta\beta/\beta} = 0.1$, $\sigma_{\Delta R_c/R_c} = 0.01$

$$\longrightarrow I_{os} = -\frac{I_c}{\beta}\left[\sigma^2_{\Delta R_c/R_c} + \sigma^2_{\Delta\beta/\beta}\right]^{1/2} \cong -0.1\frac{I_c}{\beta} \approx -0.1 I_B = I_{os}$$

Diff. Pair w/ I-Mirror Load V_{OS}

MOS Differential Stage w/ Current Mirror Load



all the same λ effect, etc...

Small-Signal Gain: (similar to BJT)

$$\frac{v_o}{v_{id}} = g_{m2}(r_{o2} \| r_{o4}) = \frac{g_{m2}}{g_{ds2} + g_{ds4}} = \frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}}{\lambda_2 I_{D2} + \lambda_4 I_{D4}}$$

$$= \frac{\sqrt{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{SS}}}{\frac{I_{SS}}{2}(\lambda_2+\lambda_4)} \Rightarrow \boxed{\frac{v_o}{v_{id}} = \frac{2}{\lambda_2+\lambda_4}\sqrt{\frac{\mu_n C_{ox}(W/L)_2}{I_{SS}}}}$$

$$\left[\frac{\Delta(W/L)_{1,2}}{(W/L)_{1,2}} - \frac{\Delta(W/L)_{3,4}}{(W/L)_{3,4}}\right]$$

Offset Voltage – $V_{OS} = V_{GS1} - V_{GS2}$ when $V_{od} = 0V$

$$V_{OS} = \Delta V_{t1,2} + \Delta V_{t3,4}\left(\frac{g_{m3,4}}{g_{m1,2}}\right) + \frac{(V_{GS}-V_t)_{1,2}}{2}\underbrace{\left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}}\right]}$$

Via similar derivation to what we just did

For small V_{OS}: ① small $(V_{GS}-V_t)$

② $g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2}$ ∴ $\left(\frac{W}{L}\right)_{3,4} < \left(\frac{W}{L}\right)_{1,2}$