

EE 140

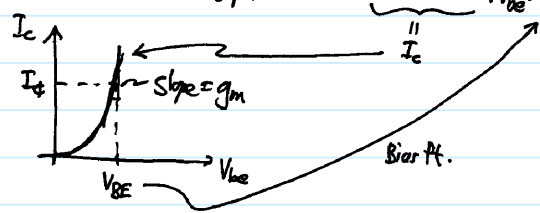
BJT Small-Signal Model

CTN

7

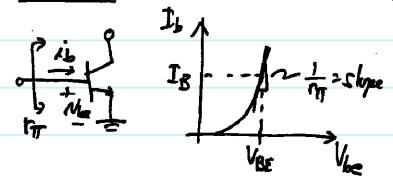
Determine the S.S. elements

$$g_m = \frac{i_c}{v_{be}} = \frac{\partial I_c}{\partial v_{be}} \Big|_{Q.pt.} = \frac{\partial}{\partial v_{be}} \left[I_s \exp\left(\frac{v_{be}}{V_T}\right) \right] \Big|_{v_{be}=V_{BE}} = \frac{I_c}{V_T} \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow g_m = \frac{I_c}{V_T}$$



Note: function of the DC operating pt.

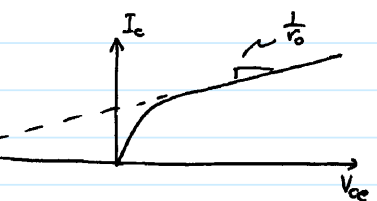
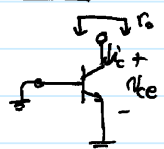
$$r_{\pi} = \frac{v_{be}}{i_b}$$



$$r_{\pi} = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{I_c}{\beta}} = \frac{\beta}{g_m} = \frac{\beta}{\frac{I_c}{V_T}} = \frac{\beta V_T}{I_c}$$

∴ $r_{\pi} = \frac{\beta}{g_m} = \frac{\beta V_T}{I_c}$ Again, function of the DC operating pt.

$$r_o = \frac{v_{ce}}{i_c}$$

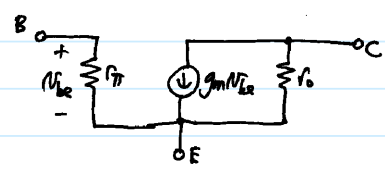


$$r_o = \frac{\partial v_{ce}}{\partial I_c} \Big|_Q = \left[\frac{\partial I_c}{\partial v_{ce}} \Big|_{Q.pt.} \right]^{-1} = \left[\frac{\partial}{\partial v_{ce}} \left(I_s \exp\left(\frac{v_{be}}{V_T}\right) \left[1 + \frac{v_{ce}}{V_A} \right] \right) \Big|_{v_{be}=V_{BE}} \right]^{-1}$$

$$= \left[\frac{I_s \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A} \right]^{-1} = \left[\frac{I_c}{V_A + V_{CE}} \right]^{-1} = \frac{V_A + V_{CE}}{I_c}$$

$$r_o = \frac{V_A + V_{CE}}{I_c} \approx \frac{V_A}{I_c} \quad [V_A \gg V_{CE}]$$

... and thus, we have the hybrid-π model:



SPICE: BJT

$$r_{\pi} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$g_m = \frac{I_c}{V_T}$$

$$r_o = \frac{V_A + V_{CE}}{I_c} \approx \frac{I_c}{V_A}$$

SPICE: VAF

Remarks:

- ① g_m is independent of device specifics; depends only on temperature (thru V_T) and biasing I_c
- ② small-signal model valid for $v_{be} \ll V_T \leftarrow \approx 26mV @ 300K$

quite different from MOS, as we'll see

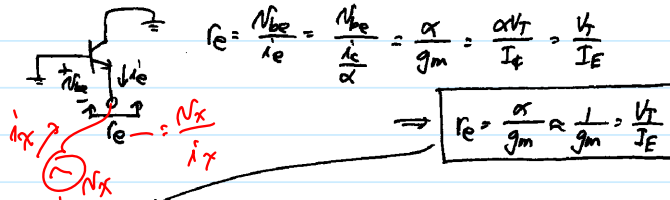
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BJT SS Model

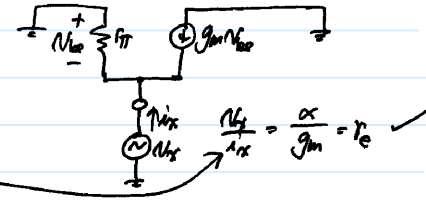
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8

What about emitter resistance?

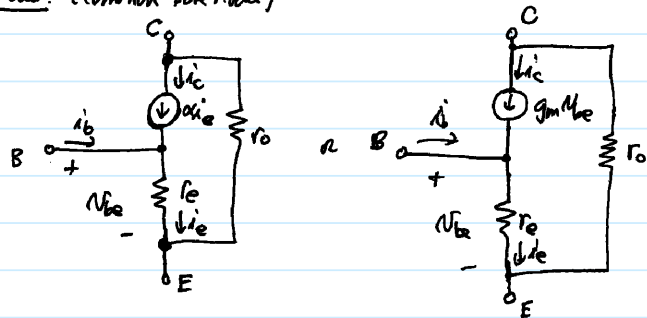


Note that although it's not explicitly shown in the hybrid- π model, r_e is present.
 ⇒ i.e., if you analyze this, you find that



To explicitly show emitter resistance, use the T-model:

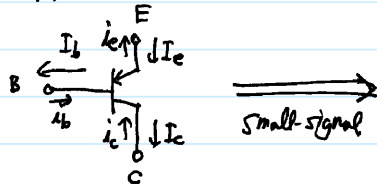
T-Model: (Common Base Model)



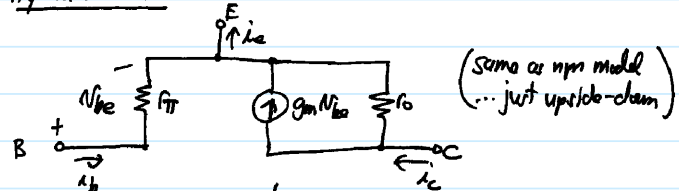
where as before:
 $g_m = \frac{I_F}{V_T}$
 $r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$
 relative

Small-Signal Models for pnp Transistors

For pnp transistors, use the same small-signal models as npn with no change in polarities!

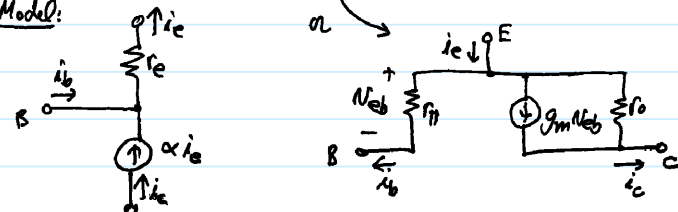


Hybrid- π Model:



Note that in these S.S. models, the same current directions as used for npn are used too ⇒ i.e., no change in S.S. polarities (large-signal directions, however, can be as before)

T-Model:



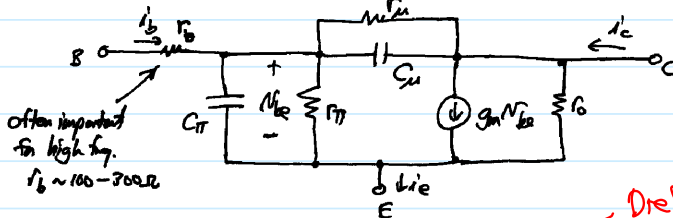
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C_{μ}, C_{π}

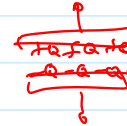
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9

More Complete Hybrid- π Model (adding frequency effects & 2nd order effects)

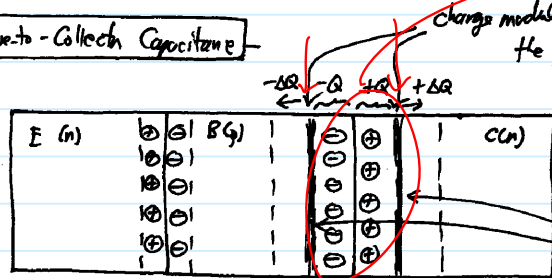


often important for high freq.
 $r_{\pi} \sim 100 - 200 \Omega$



Dielectric $f \propto C \rightarrow C_{\mu}$

C_{μ} - Base-to-Collector Capacitance



charge modulates how \therefore these are the plates of a capacitor $\rightarrow C_{\mu}$

if have $V_{cb1} = V_{cb1} + N_{cb1}$, then eq gets modulated over these small regions \Rightarrow thus, these are effectively the plates of a capacitor!

if change to a different bias V_{cb2} , get new capacitor plate locations $\rightarrow C_{\mu} = \frac{\epsilon_r}{x_d(V_{cb2})}$

$C_{\mu} = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{\phi_j}}}$ where $C_{\mu 0} =$ capacitance for $V_{CB} = 0$
 $\phi_j =$ function of the built-in potential between p and n-type semiconductors $\left. \begin{matrix} \\ \end{matrix} \right\} = \frac{kT}{q} \ln \left(\frac{N_B N_C}{n_i^2} \right)$

In general: $C_{\mu} = \frac{C_{\mu 0}}{(1 + \frac{V_{CB}}{\phi_j})^m}$, where $m = \frac{1}{2}$ or $\frac{1}{3}$ depending upon how abrupt the junction is

Detailed Derivation: [PTI]

$x_d \approx x_n = \left[\frac{2\epsilon(V_0 + V_{CB})}{qN_A(1 + \frac{N_A}{N_D})} \right]^{1/2} \rightarrow Q = qN_A A x_d = A \left[\frac{2\epsilon q N_A (V_0 + V_{CB})}{1 + \frac{N_A}{N_D}} \right]^{1/2}$

$C_j = \frac{dQ}{dV_0} \Big|_{V_{CB}} = \left[\frac{2\epsilon q N_A}{1 + \frac{N_A}{N_D}} \right]^{1/2} \frac{1}{2} A (V_0 + V_{CB})^{-1/2} = A \left[\frac{q\epsilon N_A N_D}{2(N_A + N_D)} \right]^{1/2} \frac{1}{\sqrt{V_0 + V_{CB}}} = C_j |_{V_{CB}}$

$C_j = \frac{\epsilon_s A}{x_d(V_{CB})}$

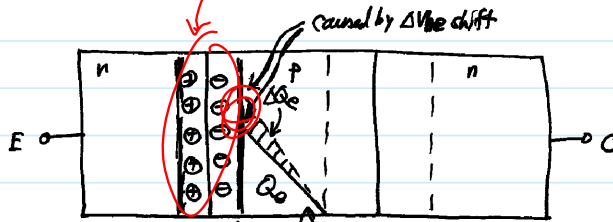
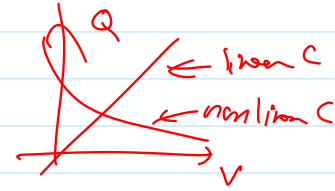
EE 140 C_{π}

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10

C_{π} - Base-to-Emitter Capacitance

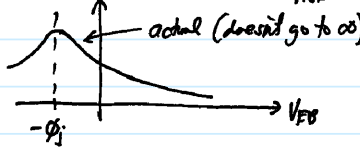
Two components comprise C_{π} :
 ① Junction capacitance, C_{je}
 ② Diffusion capacitance, C_b



Plates of a junction capacitor:
 bias level determines what the plates are

$$C_{je} = \frac{C_{je0}}{(1 + \frac{V_{EB}}{V_0})^m}$$

C_{je0} : CJE, VJE, MJE
 V_0 : I_{S1} STICE
 m : bias level determines what the plates are



Diffusion capacitance: (or Base Charging Capacitance)
 \Rightarrow can define a base transit time:

$$\tau_F = \frac{Q_e}{I_c} = \frac{x_B^2}{2D_n}$$

} avg. time spent by carrier in crossing base
 think of I_c as the rate of x_{sf} of charge through the base

$$Q_e = \tau_F I_c$$

$$\Delta Q_e = \tau_F \Delta I_c$$

Switch to S.S. parameters (variables):

$Q = CV$
 $\Delta Q = C \Delta V$
 $C = \frac{Q}{V}$

$$q_e = \tau_F i_c$$

$$Q_e = C_b \Delta V_{be} \rightarrow C_b = \frac{q_e}{\Delta V_{be}} = \tau_F \frac{i_c}{\Delta V_{be}} = \tau_F g_m = \tau_F \frac{I_c}{V_T} = C_b$$

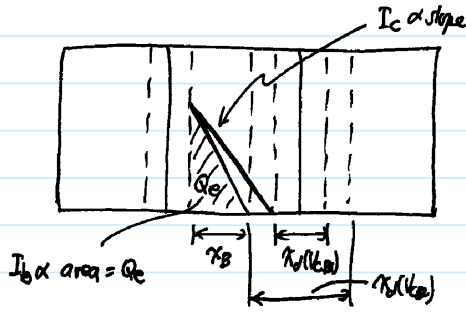
$\therefore C_b \propto I_c$

$$C_{\pi} = C_b + C_{je} \approx 2C_{je0}$$

$$C_{\pi} = \tau_F g_m + \frac{C_{je0}}{(1 + \frac{V_{EB}}{V_0})^m}$$

Mixing Carrier C in the Base \rightarrow Base-Emitter Junction Capacitance

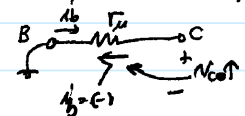
Collector-to-Base Feedback Resistor, r_u



Remember, recombination base current $I_{RB} \approx \frac{Q_e}{\tau_b}$!

$\therefore N_{ce} \uparrow \rightarrow x_{Bd} \downarrow \rightarrow Q_e \downarrow \rightarrow i_b \downarrow$
 $\rightarrow i_c \uparrow$ (due to Early effect)

$N_{ce} \uparrow \rightarrow i_b \downarrow$ can be modeled by an r_u connect C-to-B

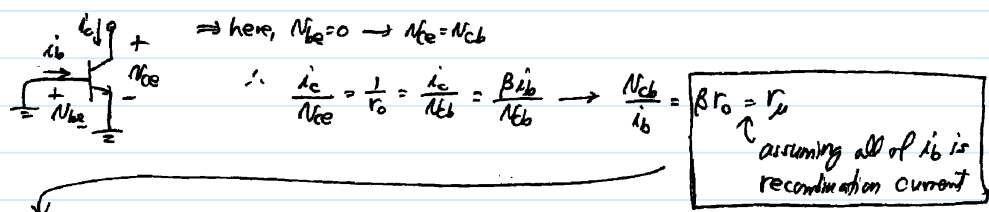


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r_{π}

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11



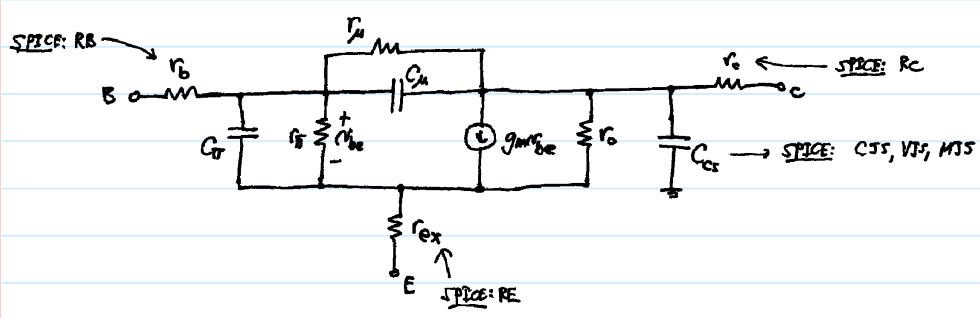
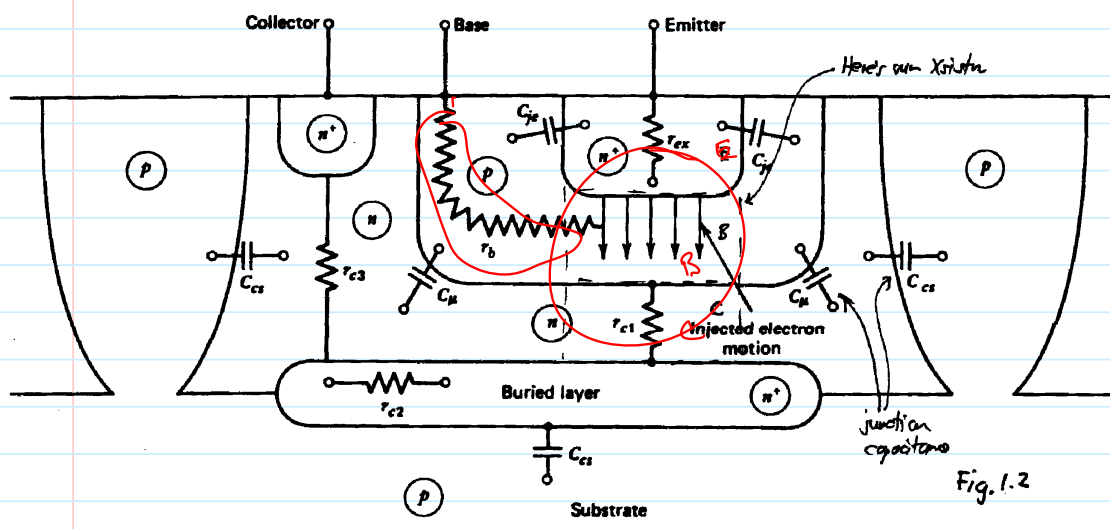
In general, base recombination current is only part of the total base current and is the only component dependent on $V_{bc} \Rightarrow$ thus,

$r_{\mu} > \beta_0 r_o \rightarrow r_{\mu} = 2-10\beta_0 r_o$
 (total pnp npn $\rightarrow I_b$ is 10% recomb. where base recomb. more significant)

Complete Forward-Active BJT S.S. Model (including parasitics)

\Rightarrow Actual integrated BJT:

should draw this on the board



EE 140 BJT Layout CTN 12

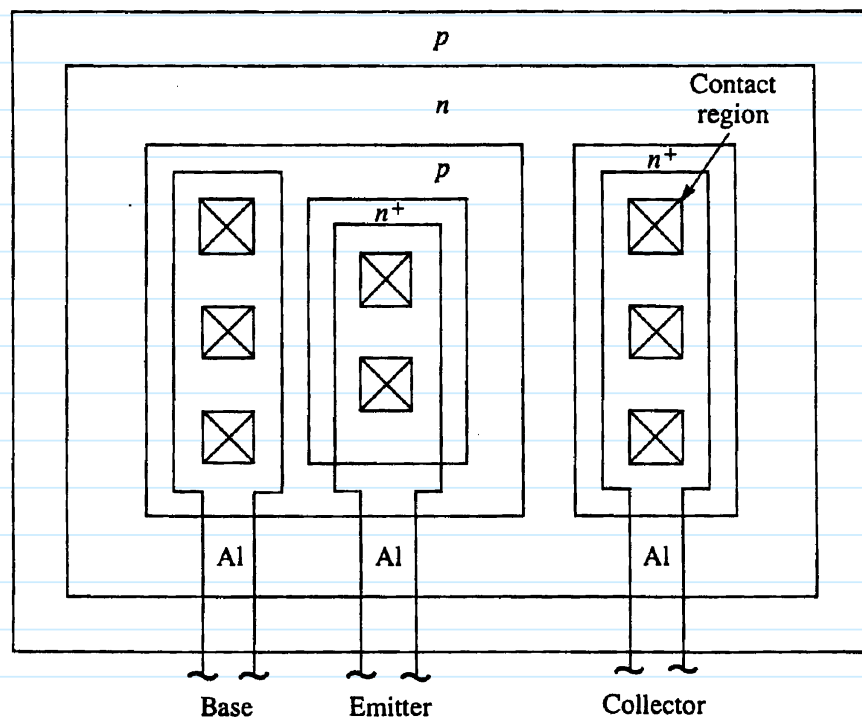
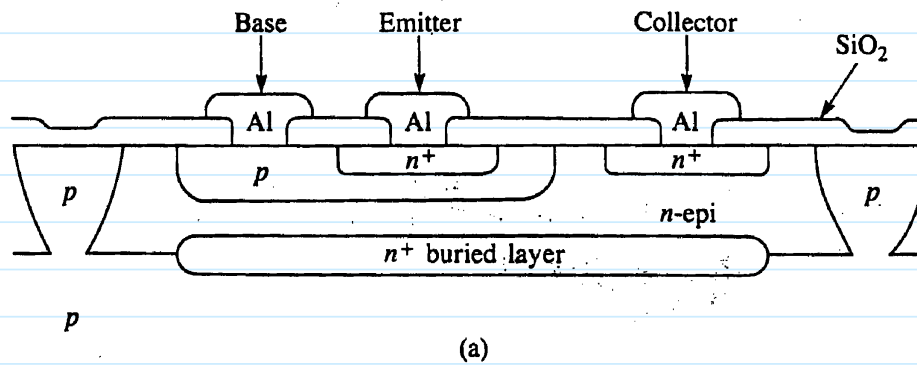


Fig. 1.1

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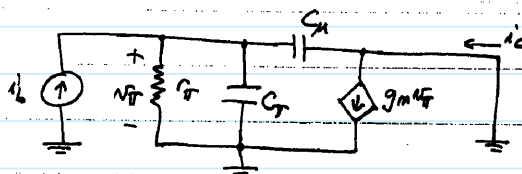
f_T

CTN

13

f_T (unity gain freq. for β)

Find $\beta(j\omega)$: (β as a function of freq.)



Find $\frac{i_c}{i_b} |_{\omega \rightarrow 0}$:

$$v_{\pi} = i_b \left(r_{\pi} \parallel \frac{1}{sC_{\pi}} \parallel \frac{1}{sC_{\mu}} \right)$$

$[g_m \gg sC_{\mu}]$

$$i_c = g_m v_{\pi} - sC_{\mu} v_{\pi} = (g_m - sC_{\mu}) v_{\pi} \approx g_m v_{\pi}$$

$$i_c = g_m \left(r_{\pi} \parallel \frac{1}{sC_{\pi}} \parallel \frac{1}{sC_{\mu}} \right) i_b$$

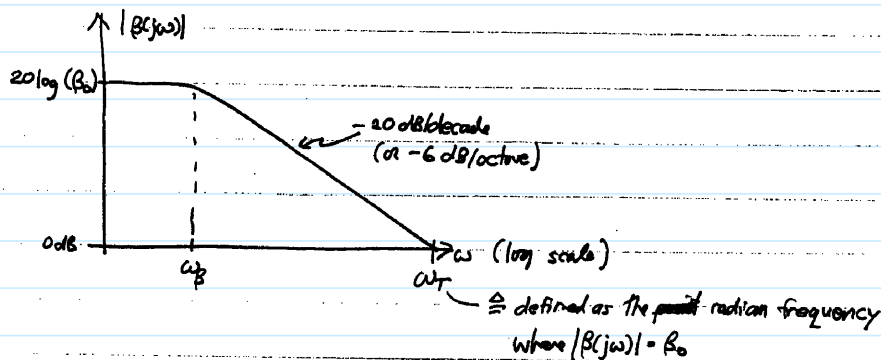
$$\frac{i_c}{i_b} = \frac{g_m}{\frac{1}{r_{\pi}} + s(C_{\pi} + C_{\mu})} = \frac{g_m r_{\pi}}{1 + s r_{\pi} (C_{\pi} + C_{\mu})} = \frac{\beta_0}{1 + \frac{j\omega}{\omega_p}} \quad [\beta_0 = g_m r_{\pi}]$$

(low freq. β)

$$\beta(j\omega) = \frac{\beta_0}{1 + \frac{j\omega}{\omega_p}}$$

$$\omega_p = \frac{1}{r_{\pi} (C_{\pi} + C_{\mu})}$$

Plot $|\beta(j\omega)|$: (Bode plot)



For ω large: (e.g. ω close to ω_T)

$$|\beta(j\omega)| \approx \frac{\beta_0}{\omega r_{\pi} (C_{\pi} + C_{\mu})} = 1 \rightarrow \omega_T = \frac{g_m}{C_{\pi} + C_{\mu}} \Rightarrow f_T = \frac{\omega_T}{2\pi}$$

is a figure of merit for the frequency performance of a transistor.

Also, note that $\omega_T = \beta_0 \omega_p$

$$C_{\pi} = \frac{g_m}{\omega_T} - C_{\mu}$$

$f_T = 100 \text{ MHz} \rightarrow 15 \text{ GHz}$ for bipolar Xistors.

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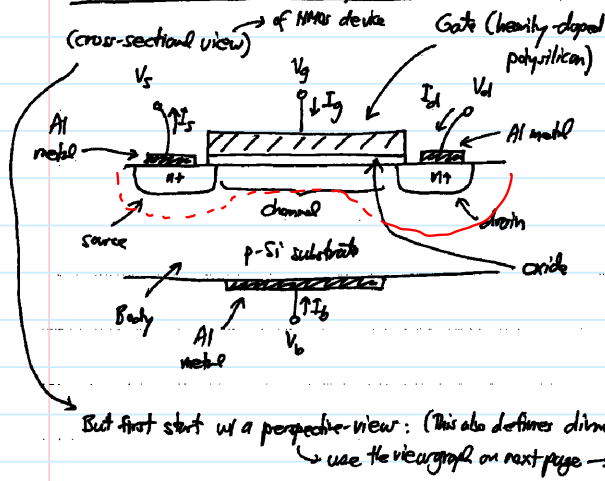
MOS Transistors

CTN

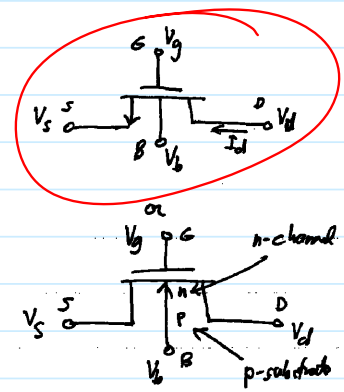
14

MOS Transistors

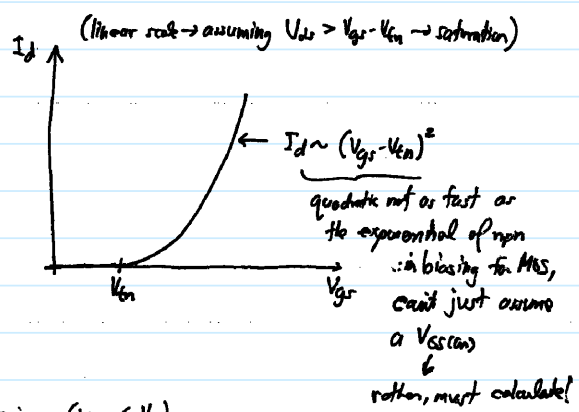
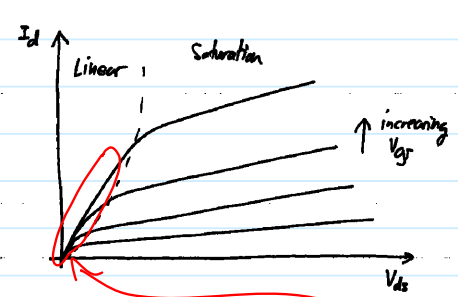
Physical Structure & Device Symbols -



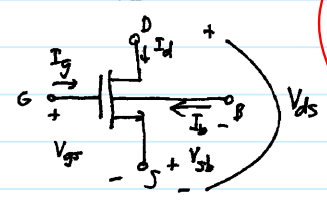
NMOS X-resistor Device Symbol



IV Characteristics (NMOS)



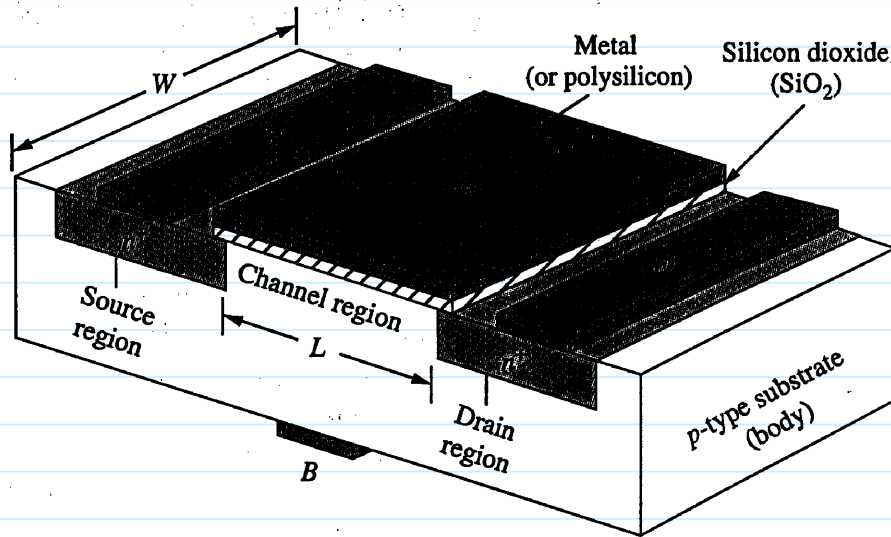
NMOS X-resistor Mathematical Model



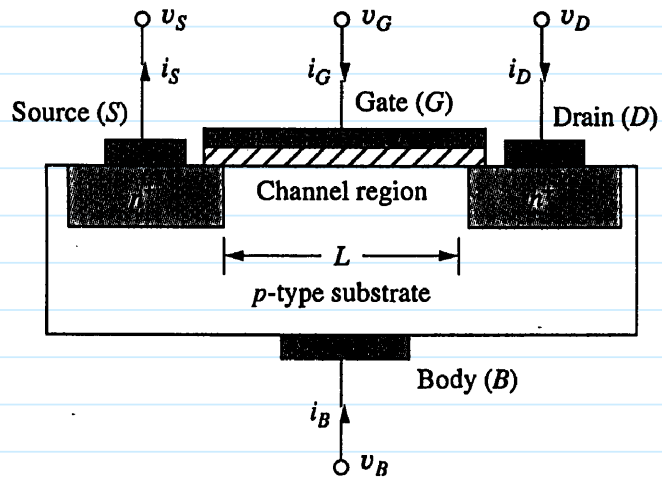
- ① Cut-off Region: ($V_{gs} \leq V_{t1}$)
 $I_g = I_b = 0$; $I_d = 0$
- ② Linear (or Triode) Region: ($V_{gs} - V_{t1} \geq V_{ds} \geq 0$)
 $I_g = I_b = 0$; $I_d = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_{t1} - \frac{V_{ds}}{2}) V_{ds}$
 $= k_n (V_{gs} - V_{t1} - \frac{V_{ds}}{2}) V_{ds}$
- ③ Saturation Region: ($V_{ds} \geq V_{gs} - V_{t1} \geq 0$)
 $I_g = I_b = 0$; $I_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_{t1})^2 (1 + \lambda V_{ds})$
 $= \frac{1}{2} k_n (V_{gs} - V_{t1})^2 (1 + \lambda V_{ds})$

Body Factor $\rightarrow \beta = \frac{1}{C_{ox}} \sqrt{2q\epsilon_s N_{sub}}$ ← substrate doping conc.
permittivity in Si
General:
 $k_n = k_n' \frac{W}{L} = \mu_n C_{ox} \frac{W}{L}$
 $I_g = I_b = 0$ for all regions (at least for dc)
 $V_{t1} = f(V_{t0}) = V_{t0} + \gamma (\sqrt{|V_{t0} + 2\phi_{fp}|} - \sqrt{|2\phi_{fp}|})$

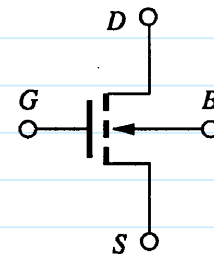
$\mu_n \hat{=} e^-$ mobility in the channel
 $C_{ox} \hat{=} \text{gate oxide capacitance per unit area}$



(a)



(b)



(c)

Fig. 2.1

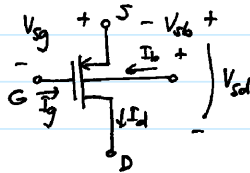
EE 140

PMOS

CTN

16

PMOS Xirta Mathematical Model



① Cut-off Region: $(V_{sg} \leq -V_{tp})$ or $(|V_{gs}| \geq |V_{tp}|)$
 $I_{sd} = 0$

② Linear (or Triode) Region: $(V_{sg} + V_{tp} \geq V_{sd} \geq 0; \text{ or } |V_{gs}| - |V_{tp}| \geq |V_{ds}| \geq 0)$
 $I_{sd} = k_p (V_{sg} + V_{tp} - \frac{V_{sd}}{2}) V_{sd} = \mu_p C_{ox} \frac{W}{L} (V_{sg} + V_{tp} - \frac{V_{sd}}{2}) V_{sd}$
 $= \mu_p C_{ox} \frac{W}{L} (|V_{gs}| - |V_{tp}| - \frac{|V_{ds}|}{2}) |V_{ds}|$

For all regions:

$k_p = k_p' \frac{W}{L} = \mu_p C_{ox} \frac{W}{L}$

$I_{gs} = 0$ and $I_{bs} = 0$ (at dc)

$V_{tp} = V_{t0} - \gamma (\sqrt{|V_{gs}| + 2\phi_f} - \sqrt{2\phi_f})$

③ Saturation Region: $(V_{sd} \geq V_{sg} + V_{tp} \geq 0; |V_{ds}| \geq |V_{gs}| - |V_{tp}| \geq 0)$

$I_{sd} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{sg} + V_{tp})^2 (1 + \lambda |V_{sd}|) = \frac{1}{2} k_p (V_{sg} + V_{tp})^2 (1 + \lambda |V_{sd}|)$

$= \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (|V_{gs}| - |V_{tp}|)^2 (1 + \lambda |V_{ds}|)$

$\mu_p \hat{=}$ h^+ mobility in the channel

$C_{ox} \hat{=}$ gate oxide capacitance per unit area

Threshold Voltage

$$V_t = \phi_{ms} - \psi_s - \frac{Q_B}{C_{ox}} - \frac{Q_{ss}}{C_{ox}}$$

where ϕ_{ms} = work function difference [in V] between gate material and bulk Si

ψ_s = surface potential in the Si @ onset of strong inversion

= $2\phi_f$ for uniformly doped substrate ($\phi_f \sim 0.3$ V)

Q_{ss} = oxide charge per unit area at the oxide-Si interface [C/cm^2]

Q_B = charge stored per unit area in the depletion region (at onset of inversion)

$\Rightarrow |Q_B| = \sqrt{2q\epsilon_s N_B (2|\phi_f| + |V_{SB}|)}$ [C/cm^2]

↑ conc. in bulk ↑ reverse bias

C_{ox} = gate oxide capacitance per unit area [F/cm^2]

EE 140

Threshold Voltage

CTN

17

Case: $V_{SB} = 0 \Rightarrow V_t(V_{SB} = 0) = V_{t0} = \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}}$, where

Then:

$$V_t = \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_B}{C_{ox}}$$

$$Q_{B0} = \sqrt{2q\epsilon_{si}N_B(2|\phi_f| + |V_{SB}|)}$$

$$= \phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}} - \frac{Q_B - Q_{B0}}{C_{ox}}$$

$$\underbrace{\phi_{ms} - 2\phi_f - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}}}_{V_{t0}}$$

$$V_t = V_{t0} - \gamma(\sqrt{2|\phi_f| + |V_{SB}|} - \sqrt{2|\phi_f|}), \quad \gamma = \frac{1}{C_{ox}}\sqrt{2q\epsilon_{si}N_B}$$

Signs in the V_t Equation:

Parameter	NMOS	PMOS
Substrate	p-type	n-type
ϕ_{ms} : metal gate	-	-
n+ Si gate	-	-
p+ Si gate	+	+
ϕ_f	-	+
Q_{B0} (or Q_B)	-	+
Q_{ss}	+	+
γ	-	+
C_{ox}	+	+

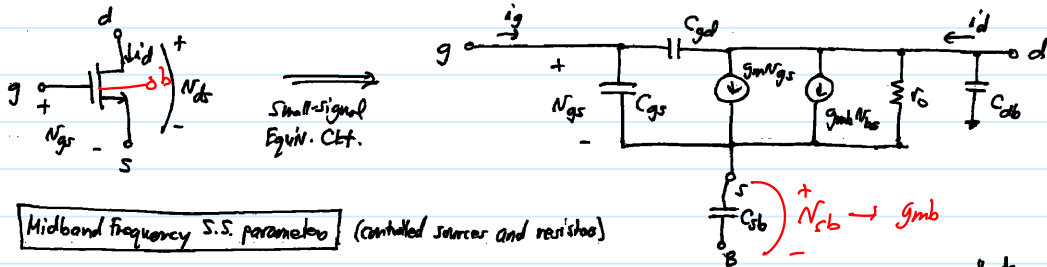
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MOS Small-Signal Model

CTN

18

MOS Small-Signal Model (for NMOS) in saturation



Midband frequency S.S. parameters (controlled sources and resistors)

Transconductance, gm:

$$g_m = \frac{i_d}{v_{gs}} = \left. \frac{\partial I_D}{\partial v_{gs}} \right|_{Q_{pt}} = \frac{\partial}{\partial v_{gs}} \left(\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (v_{gs} - v_{tn})^2 \right) \Big|_{Q_{pt}} = \mu_n C_{ox} \frac{W}{L} (v_{gs} - v_{tn}) \Big|_{Q_{pt}} = \mu_n C_{ox} \frac{W}{L} I_D$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (v_{gs} - v_{tn}) = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$g_{mb} = \frac{i_d}{v_{sb}} = - \frac{\partial I_D}{\partial v_{sb}} = - \left(\frac{\partial I_D}{\partial v_{tn}} \cdot \frac{\partial v_{tn}}{\partial v_{sb}} \right) \Big|_{Q_{pt}}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (v_{gs} - v_{tn})^2 \rightarrow (v_{gs} - v_{tn}) = \sqrt{\frac{2 I_D}{\mu_n C_{ox} \frac{W}{L}}}$$

$$\frac{\partial I_D}{\partial v_{tn}} \Big|_{Q_{pt}} = - \frac{\partial I_D}{\partial v_{gs}} = -g_m \quad ; \quad \frac{\partial v_{tn}}{\partial v_{sb}} \Big|_{Q_{pt}} = \frac{\partial}{\partial v_{sb}} \left[V_{t0} + \gamma \left(\sqrt{V_{t0} + 2|\phi_{s0}|} - \sqrt{2|\phi_{f1}|} \right) \right] = \frac{\gamma}{2\sqrt{V_{t0} + 2|\phi_{s0}|}} \equiv \eta$$

$$g_{mb} = \eta g_m$$

often neglected!

Note $v_{sb} \uparrow \rightarrow v_t \uparrow \rightarrow \eta \downarrow \rightarrow I_D \downarrow$

g_{mb} is minimized by maximizing λ !

Output Resistance, r_o : ($= \frac{1}{g_{ds}}$)

$$\Rightarrow \text{output conductance} = g_{ds} = \frac{i_d}{v_{ds}} = \left. \frac{\partial I_D}{\partial v_{ds}} \right|_{Q_{pt}} = \frac{\partial}{\partial v_{ds}} \left(\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (v_{gs} - v_{tn})^2 (1 + \lambda v_{ds}) \right) \Big|_{Q_{pt}}$$

$$= \lambda I_{Dsat} = \frac{\lambda I_D}{1 + \lambda v_{ds}} \approx \lambda I_D = g_{ds}$$

if v_{ds} is very large

$$r_o = g_{ds}^{-1} = \frac{1}{\lambda I_D} = \frac{1}{\lambda} + \frac{v_{ds}}{I_D}$$

High Frequency S.S. Parameters (capacitors)

C_{gs} = gate-to-source overlap capacitance

C_g = gate capacitance = $W L \epsilon_{ox} C_{ox}$

C_{gd} = gate-to-drain overlap capacitance

