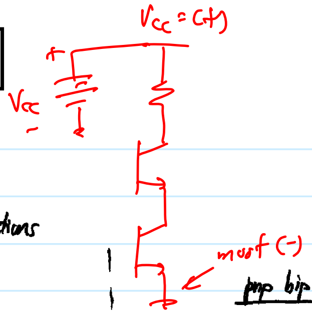
EE 140 **BJT Modeling** CTN ①

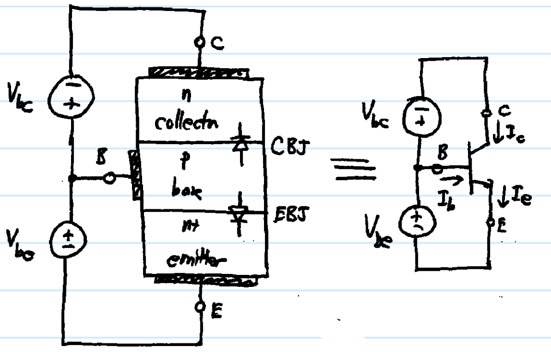


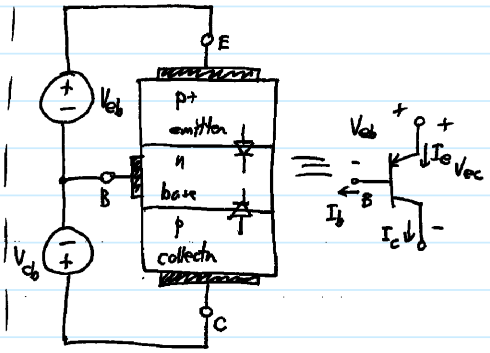
Modeling the Bipolar Junction Transistor (BJT)

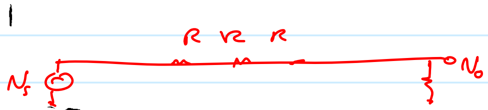
⇒ physically, BJT's are just back-to-back pn junctions

npn bipolar Xsistor

pnp bipolar Xsistor







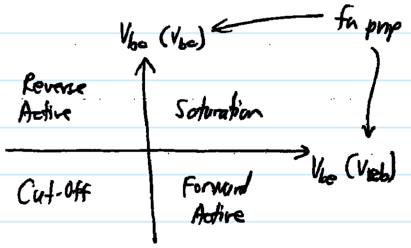
Regions of Bipolar Xsistor Operation

→ Key: R ≜ reverse-biased
F ≜ forward-biased

EBJ	CBJ	Mode
R	R	Cut-off (both diodes off)
F	R	Forward Active (widely used in analog amplifier ckts)
R	F	Reverse Active
F	F	Saturation

⇒ can also think of this in a convenient graphical sense:

→ for npn (pnp):

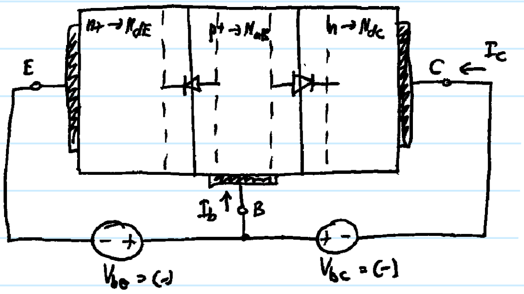


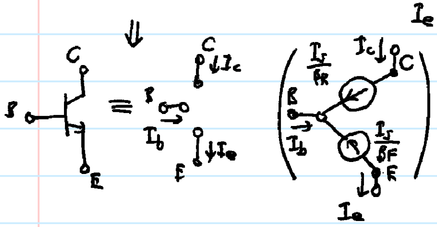
① Cut-Off Region - (npn transistor)

⇒ both diodes reverse-biased

⇒ no current flows:

$I_b=0, I_c=0, I_e=0$

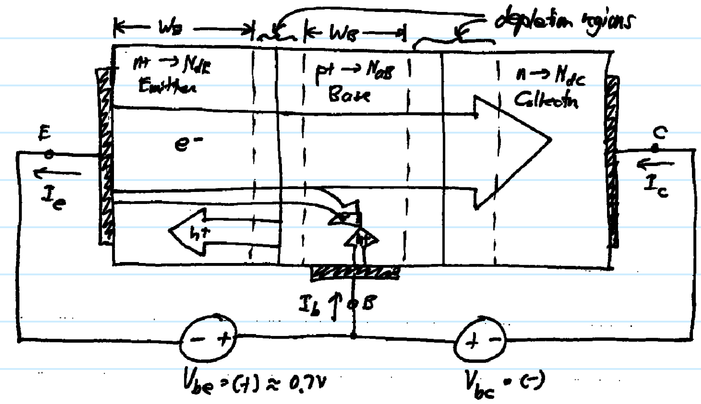


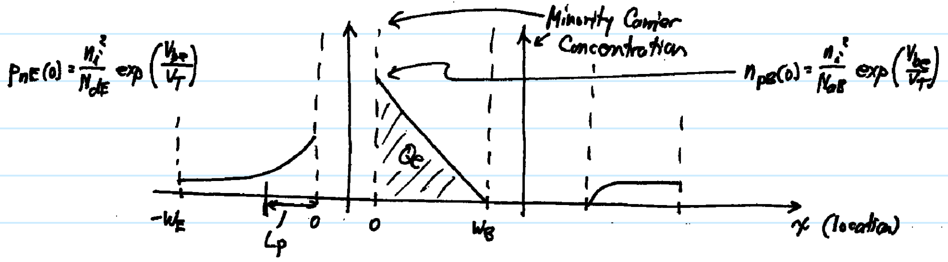


EE 140 | BJT Forward-Active | CTN | ②

② Forward-Active Region - (npn transistor)

⇒ BEJ Forward-Biased (i.e., diode on), BCJ Reverse-Biased (i.e., diode off)





Forward biasing of the BEJ generates three current components:

① e^-'s injected from emitter to base : $I_{nB} = -A J_{nB}^{diff}$

② h^+'s injected from base to emitter : $I_{pE} = A J_{pE}^{diff}$

③ recombination of e^-'s & h^+'s in base : I_{rB}

$I_c = I_{nB} = ①$

$I_e = I_{nB} + I_{pE} + I_{rB} = ① + ② + ③$

$I_b = I_{pE} + I_{rB} = ② + ③$

$$I_{nB} = -A J_{nB}^{diff} = -A q \underset{\text{diffusion constant for } e^-\text{'s in B}}{D_{nB}} \underbrace{\frac{dn_p(x)}{dx}}_{\text{slope}} = -qAD_{nB} \frac{[n_{pB}(W_B) - n_{pB}(0)]}{W_B} = \boxed{qAD_{nB} \frac{n_i^2}{N_{aB} W_B} \exp\left(\frac{V_{be}}{V_T}\right) = ①} *$$

$$\left[\begin{array}{l} n_{pB}(W_B) = \frac{n_i^2}{N_{aB}} \exp\left(\frac{V_{bc}}{V_T}\right) \approx 0 \\ n_{pB}(0) = \frac{n_i^2}{N_{aB}} \exp\left(\frac{V_{be}}{V_T}\right) \end{array}\right]$$

$$I_c = I_S \exp\left(\frac{V_{be}}{V_T}\right)$$

$$I_{pE} = A J_{pE}^{diff} = A q \underset{\text{diffusion constant for } h^+\text{'s in E}}{D_{pE}} \underbrace{\frac{dp_n(x)}{dx}}_{\text{slope}} = qAD_{pE} \frac{[p_{nE}(0) - p_{nE}(-W_E)]}{W_E} = \boxed{qAD_{pE} \frac{n_i^2}{N_{dE} W_E} \exp\left(\frac{V_{be}}{V_T}\right) = ②} *$$

$$\left[\begin{array}{l} p_{nE}(0) = \frac{n_i^2}{N_{dE}} \exp\left(\frac{V_{be}}{V_T}\right) \\ p_{nE}(-W_E) \approx 0 \end{array}\right]$$

W_E: could also replace by diffusion length, L_p (for h^+ in n-type material)

EE 140 — BJT Forward-Active

CTN ③

$$I_{rB} = \overset{\text{minority-carrier charge in base}}{\frac{Q_e}{\underset{\text{minority carrier lifetime in base}}{\tau_b}}} = \frac{1}{\tau_b}\left[\frac{1}{2} n_{pB}(0) W_B qA\right] = \boxed{\frac{1}{2}\frac{n_i^2 W_B qA}{N_B \tau_b} \exp\left(\frac{V_{be}}{V_T}\right) = ③} \quad *$$

<u>Define Forward Current Gain = β_F</u>:

$$\beta_F = \frac{I_c}{I_b} = \frac{①}{③+②} = \frac{\dfrac{qAD_{nB}n_i^2}{N_{aB}W_B}}{\dfrac{1}{2}\dfrac{n_i^2 W_B qA}{N_{aB}\tau_b} + \dfrac{qAD_{pE}n_i^2}{N_{dE}\underset{L_p}{W_E}}} = \left[\frac{W_B^2}{2\tau_b D_{nB}} + \frac{D_{pE}W_B \overset{N_{aB}}{N_A}}{D_{nB}\underset{L_p}{W_E}\underset{N_{dE}}{N_D}}\right]^{-1}$$

<u>To maximize β_F, want</u>: ① W_B = small

② $N_{dE} \gg N_{aB}$ (this is why emitter is n+) → also leads to $\overbrace{D_{pE} \ll D_{nB}}^{\text{which we also want}}$

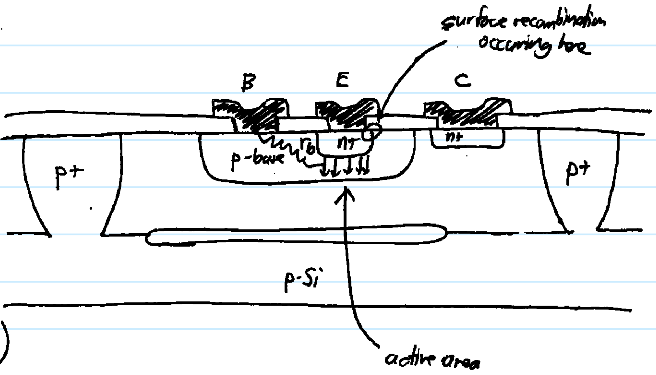
③ τ_b = large (base Si must be free of impurities/defects to prevent recombination)

<u>More Complete Expression for β_F</u>:

$$\beta_F = \underbrace{\frac{N_{aB}W_B}{D_{nB}}\cdot\frac{D_{pE}}{N_{dE}L_{pE}}}_{\text{Injection Efficiency}} + \underbrace{\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}_{\text{Volume Recombination}} + \underbrace{s\left(\frac{A_s}{A_E}\right)\left(\frac{W_B}{D_{nB}}\right)}_{\text{Surface Recombination}} + \underbrace{\frac{W_E N_{aB} W_B}{2D_{nB}\tau n_i} e^{-\frac{V_{be}}{2V_T}}}_{\text{Recombination in the BE Depletion Region}}$$

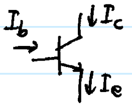
← significant @ low current levels

where:
s = Surface Recombination Velocity
D_i = Diffusion constant
n_i = intrinsic carrier concentration
N_i = carrier concentration
A_E = total emitter area
A_s = sidewall emitter area
τ = minority carrier lifetime
L_i = diffusion length ($L_i = \sqrt{D_i \tau}$)
W_B = active base width



Forward-Active LS Models

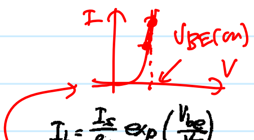
So β relates I_b & I_c. To relate I_c & I_e, use KCL:



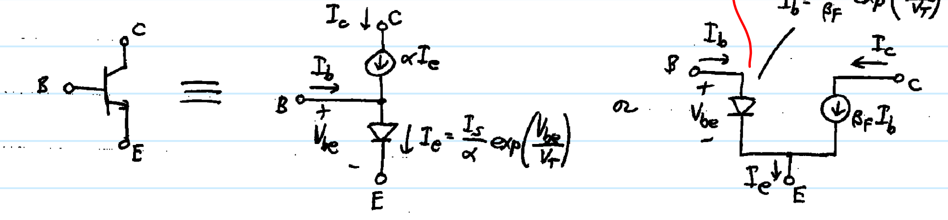
$$I_e = I_c + I_b = I_c + \frac{I_c}{\beta} = \left(1+\frac{1}{\beta}\right)I_c$$

$$\Rightarrow I_c = \left(\frac{1}{1+\frac{1}{\beta}}\right)I_e = \left(\frac{\beta}{\beta+1}\right)I_e = \alpha I_e, \text{ where } \boxed{\alpha = \frac{\beta}{\beta+1}} \Rightarrow \boxed{\beta = \frac{\alpha}{1-\alpha}}$$

Equivalent Large Signal Ckt. Models for Forward-Active BJT's



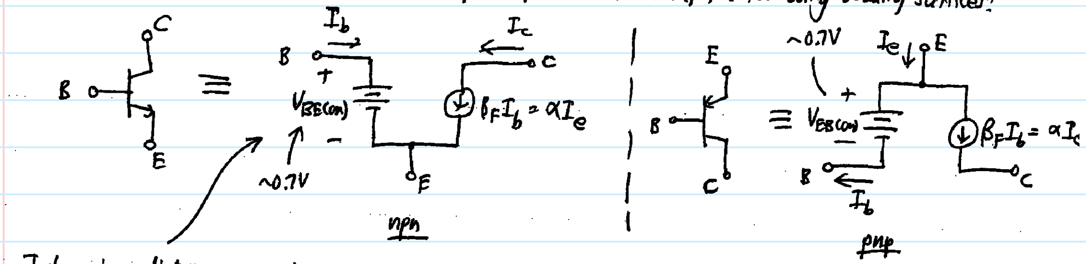
There are several of them. The most useful ones are:



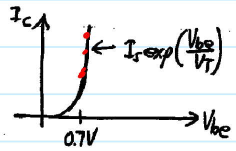
Common Base (CCCS)

Common Emitter (CCCS)

But usually one doesn't have to use those complicated models. Rather, the following usually suffices:



Just as in a diode:



You should already be used to using approximate models like this

⇒ the more complicated models are a waste of time in comparison

③ Reverse-Active Region –

⇒ very similar to forward-active region except now: BEJ reverse-biased
BCJ forward-biased

⇒ one important difference:

$$\beta_R \propto \frac{N_{dC}\overset{L_{pC}}{W_E}D_{nB}}{N_{aB}W_B D_{pE}} \rightarrow N_{dC} \ll N_{aB} \rightarrow D_{nB} \ll D_{pC}$$

since collector is n^-

∴ β_R is much smaller than β_F

⇒ poor device performance

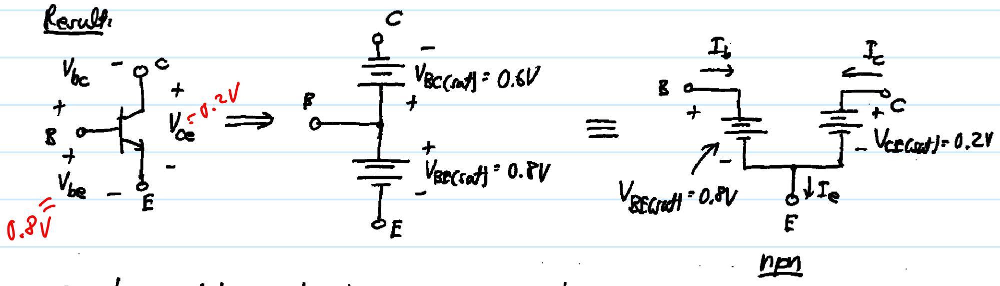
EE 140 — BJT Saturation LS Models — CTN ⑤

④ Saturation Region:-

BEJ forward-biased → $V_{BE(on)}^{sat} \sim 0.8V$ (higher than 0.7V in saturation)

BCJ forward-biased → $V_{BC(on)}^{sat} \sim 0.6V$

Result:



⇒ currents now determined by the attached elements & KCL:

$$I_e = I_b + I_c \quad ; \text{ no longer have } \underline{I_b = \frac{I_c}{\beta} \text{ or } I_c = \alpha I_e}$$

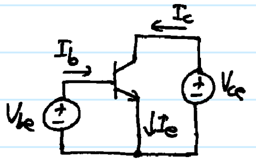
These no longer apply when BJT is in saturation.

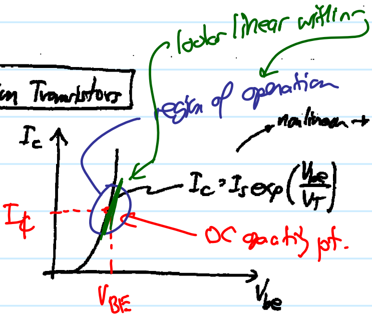
When determining DC operating point:

Pass {
① Assume forward-active → check for cut-off (enough V_{be}?)
② Determine V_{ce}.

③ If $V_{ce} > V_{CE(sat)} = 0.2V$, then ok (i.e., it's forward-active) ... otherwise, must do the analysis over assuming saturation.

IV Characteristics of Bipolar Junction Transistors



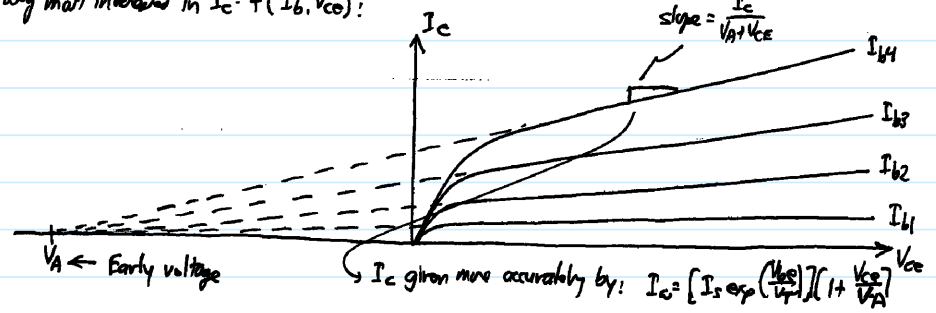


nonlinear → not easy to work w/ since we can't use linear system theory → thus need to linearize!

(a diode-like characteristic)

digital ckts. → piecewise linear
analog ckts. → small-signal models

→ really most interested in $I_c = f(I_b, V_{ce})$:



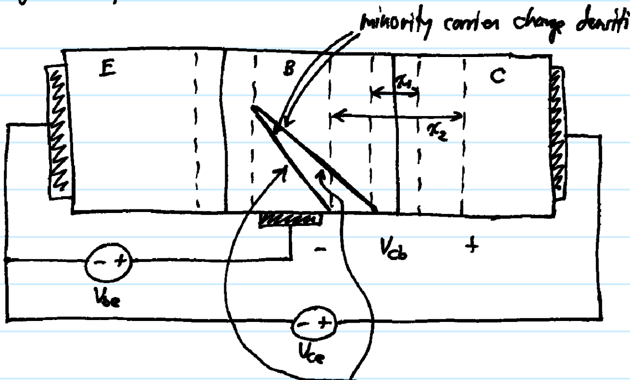
↳ I_c given more accurately by: $I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left(1 + \frac{V_{ce}}{V_A}\right)$

EE 140

BJT Early Effect

CTN ⑥

What is happening physically?



$x_1 \triangleq$ depl. region width for $V_{ce}=V_{ce1}$

$x_2 \triangleq$ depl. region width for $V_{ce}=V_{ce2}>V_{ce1}$

① Case: $V_{ce}=V_{ce1} \rightarrow x_1 \rightarrow I_{c1} \propto$ slope of this ~~curve~~ line

② Now, increase $V_{ce1} \rightarrow V_{ce2} \longrightarrow V_{cb}\uparrow \longrightarrow x\uparrow$ to $x_2 \longrightarrow I_{c2} \propto$ slope of this line

$\therefore I_{c2} > I_{c1}$

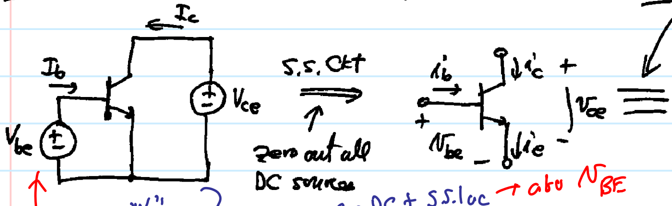
Thus, $V_{ce}\uparrow \longrightarrow I_c\uparrow$ due to $x_{depl.}\uparrow$

Result: $I_c = f(I_b, V_{ce})$ in forward-active!

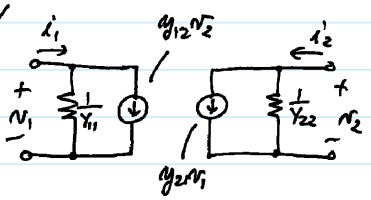
$$\boxed{I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1+\frac{V_{ce}}{V_A}\right]}$$

← This, then, is a more accurate I_c equation.

Small-Signal Models for Forward-Active Bipolar Xsistors

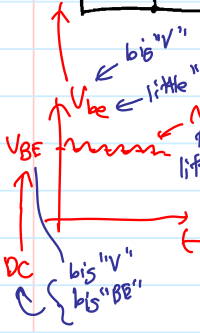


Y-Parameter Model



$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0}$ $y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$

$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$ $y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0}$

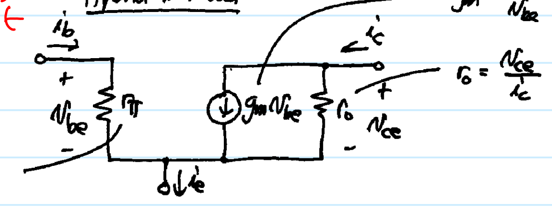


big "V", little "be" } total signal = DC + s.s. ac → also v_{BE}

v_{be}: little "v", little "be" } s.s. ac

If only interested in the forward direction

Hybrid-Π Model



$g_m = \frac{i_c}{v_{be}}$

$r_o = \frac{v_{ce}}{i_c}$

$r_\pi = \frac{v_{be}}{i_b}$

} Specified by the bias point.

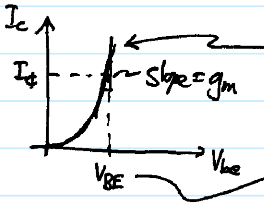
EE 140 | BJT Small-Signal Model

CTN ⑦

Determine the S.S. elements:

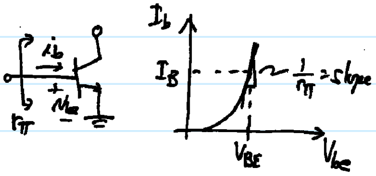
$$g_m = \frac{i_c}{v_{be}} = \frac{\partial I_c}{\partial v_{be}}\bigg|_{Q\text{-pt.}} = \frac{\partial}{\partial v_{be}}\Big[\underbrace{I_s\exp\left(\frac{v_{be}}{V_T}\right)}_{=\,I_c}\Big]\bigg|_{v_{be}=V_{BE}} = \overbrace{\frac{I_s}{V_T}\exp\left(\frac{V_{BE}}{V_T}\right)}^{I_C} \Rightarrow \boxed{g_m = \frac{I_C}{V_T}}$$

Note: function of the DC operating pt.



Bias pt.

$r_\pi = \frac{v_{be}}{i_b}$:

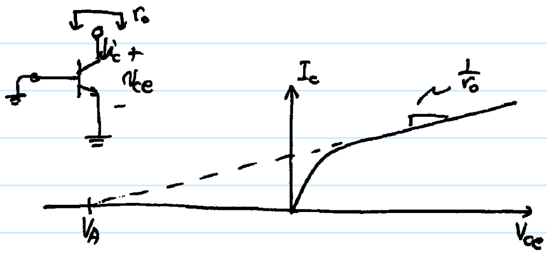


$$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{i_c}{\beta}} = \frac{\beta}{g_m} = \frac{\beta}{\frac{I_C}{V_T}} = \frac{V_T}{I_B}$$

$$\therefore \boxed{r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}}$$

← Again, function of the DC operating pt.

$r_o = \frac{v_{ce}}{i_c}$:



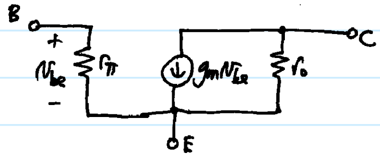
$$r_o = \frac{\partial V_{ce}}{\partial I_c}\bigg|_Q = \left[\frac{\partial I_c}{\partial V_{ce}}\bigg|_{Q\text{-pt.}}\right]^{-1}$$

$$= \left[\frac{\partial}{\partial V_{ce}}\left(\left[I_s\exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1+\frac{V_{ce}}{V_A}\right]\right)\bigg|_{V_{be}=V_{BE}}\right]^{-1}$$

$$= \left[\frac{\underbrace{I_s\exp\left(\frac{V_{BE}}{V_T}\right)}_{\frac{I_C}{1+\frac{V_{CE}}{V_A}}}}{V_A}\right]^{-1} = \left[\frac{I_C}{V_A+V_{CE}}\right]^{-1} = \frac{V_A+V_{CE}}{I_C}$$

$$\therefore \boxed{r_o = \frac{V_A+V_{CE}}{I_C} \underset{[V_A \gg V_{CE}]}{\approx} \frac{V_A}{I_C}}$$

... and thus, we have the hybrid-π model:



SPICE: BF

$$\boxed{\begin{aligned} r_\pi &= \frac{\overset{\downarrow}{\beta}}{g_m} = \frac{V_T}{I_B} \\ g_m &= \frac{I_C}{V_T} \\ r_o &= \frac{\overset{\uparrow}{V_A}+V_{CE}}{I_C} \approx \frac{I_C}{V_A} \end{aligned}}$$

SPICE: VAF

Remarks:

① g_m is independent of $\underbrace{\text{device specifics}}_{}$ (i.e., β, I_s); depends only on temperature (thru V_T) and biasing I_C → quite different from MOS, as we'll see

② small-signal model valid for $v_{be} \ll V_T$ ← ≈ 26mV @ 300K