## Lecture 20: Narrowbanding vs. Pole-Splitting

- **Announcements**:
  - Welcome back (from Spring Break)
  - Lab#3 in progress
  - HW#9 online
- **Lecture Topics**:
  - Review Last Lecture
  - Review of Pole/Zero Plots
  - Compensation
    - — Narrowbanding
    - — Pole-Splitting
  - Hand back graded midterm & discuss grading
- ------------------------------------
- **Last Time**: (it's been a long while, so we'll quickly review the whole last lecture)

**Stability & Compensation in Op Amps**

In general, op amps are used in neg FB loops. ← causes instability!

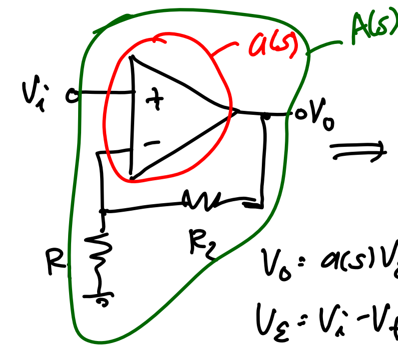
Reasons:

1. Feedback sets the biasing → no large coupling or bypass caps needed.
2. FB increases BW.
3. FB increases linearity or input range. (e.g., emitter degeneration is a type of FB)
4. Gain determined by external FB components → more accurate than op amp gain.
5. FB sets $R_i$ and $R_o$.
6. FB can improve temperature stability.

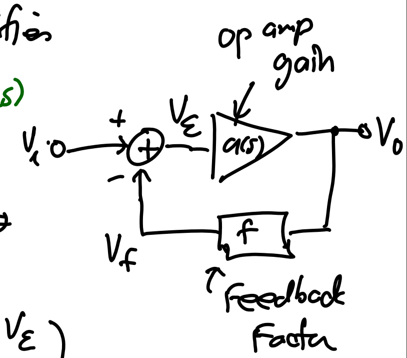
→ Problem: any FB loop can become unstable under certain conditions

↳ need to compensate the instabilities

**Ex.** Non-Inverting Amplifier




⇒




$$\left.\begin{aligned} V_o &= a(s)V_\epsilon \\ V_\epsilon &= V_i - V_f \\ V_f &= fV_o \end{aligned}\right\} \quad V_o = a(s)(V_i - fV_o)$$

$$A(s) = \frac{V_o(s)}{V_i(s)} = \frac{a(s)}{1+a(s)f} = \frac{a(s)}{1+T(s)}$$

$T_o = T(0)$

$T(s) = a(s)f$

$A(s)$: Closed Loop Voltage Gain

$T(s)$: Loop Transmission

Instability occurs when $A(s) \to \infty$.

$$\Rightarrow A(s) = \frac{a(s)}{1+a(s)f} \longrightarrow A(s) = \frac{a(s)}{1-1} \to \infty$$

$a(s)f = -1$

will also go unstable when denominator $= (-)$

## In General:

$$\boxed{\text{If } |a(s)f| \geq 1 \text{ when } \measuredangle a(s)f = -180^\circ \Big\} \Rightarrow \text{unstable!}}$$

This is just a simplified form of the Nyquist Criterion.

## Stability of FB Ckt. Using a Single Pole Op Amp

For a single pole op amp: $a(s) = \dfrac{a_o}{1 - \frac{s}{p_1}} \equiv$ op amp transfer function

Thus: closed loop Xfer fcn

$$A(s) = \frac{a(s)}{1 + a(s)f} = \underbrace{\frac{a_o}{1 + a_o f}}_{\text{feedback fact.}} \underbrace{\frac{1}{1 - \frac{s}{p_1(1 + a_o f)}}}_{\text{freq. shaping term}}$$

$A_o$ = closed loop dc gain → $(1 + a_o f) \approx a_o f \times$ smaller than $a_o$

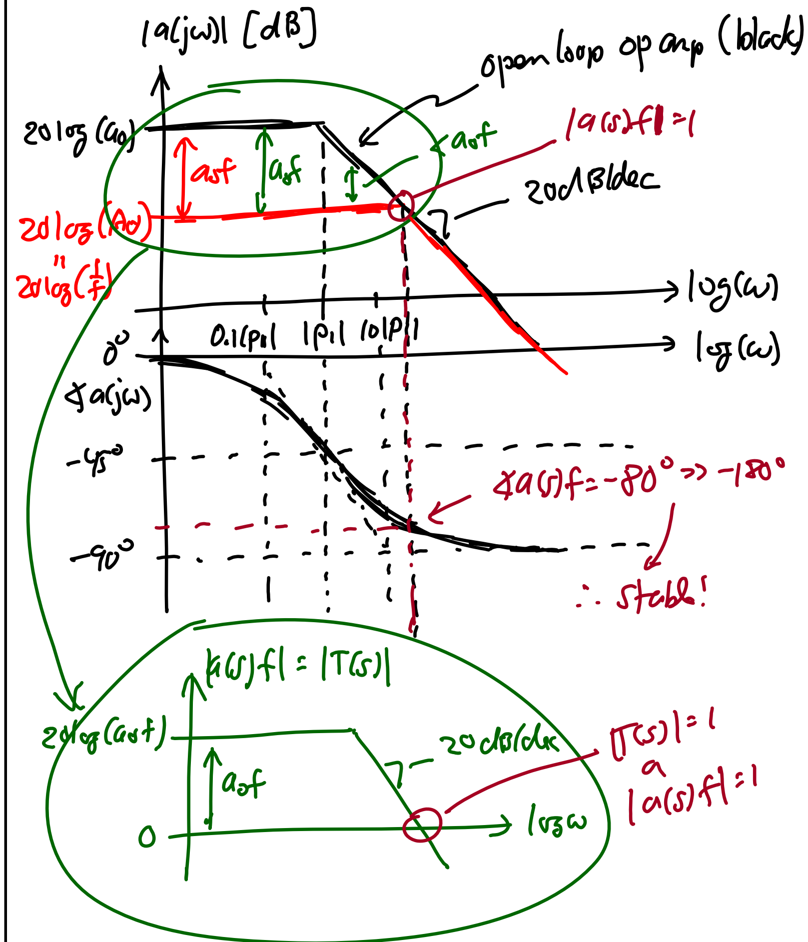
$$\frac{a_o}{1 + a_o f} \approx \frac{1}{f}$$

dc

$T_o = a_o f$ = loop gain (defined at dc)

$T(s) = a(s)f$ = loop transmission (defined for several freqs.)

## Bode Plot: → use to determine $\measuredangle a(s)f$ when $|a(s)f| = 1$ → then determine stability



Remarks:

① For the case of a single-pole op amp, FB can never reach $\measuredangle a(s)f = -180°$. (90° is the limit)

② Thus, an op amp FB ckt. w/ f = const. and using a single-pole op amp is always stable!

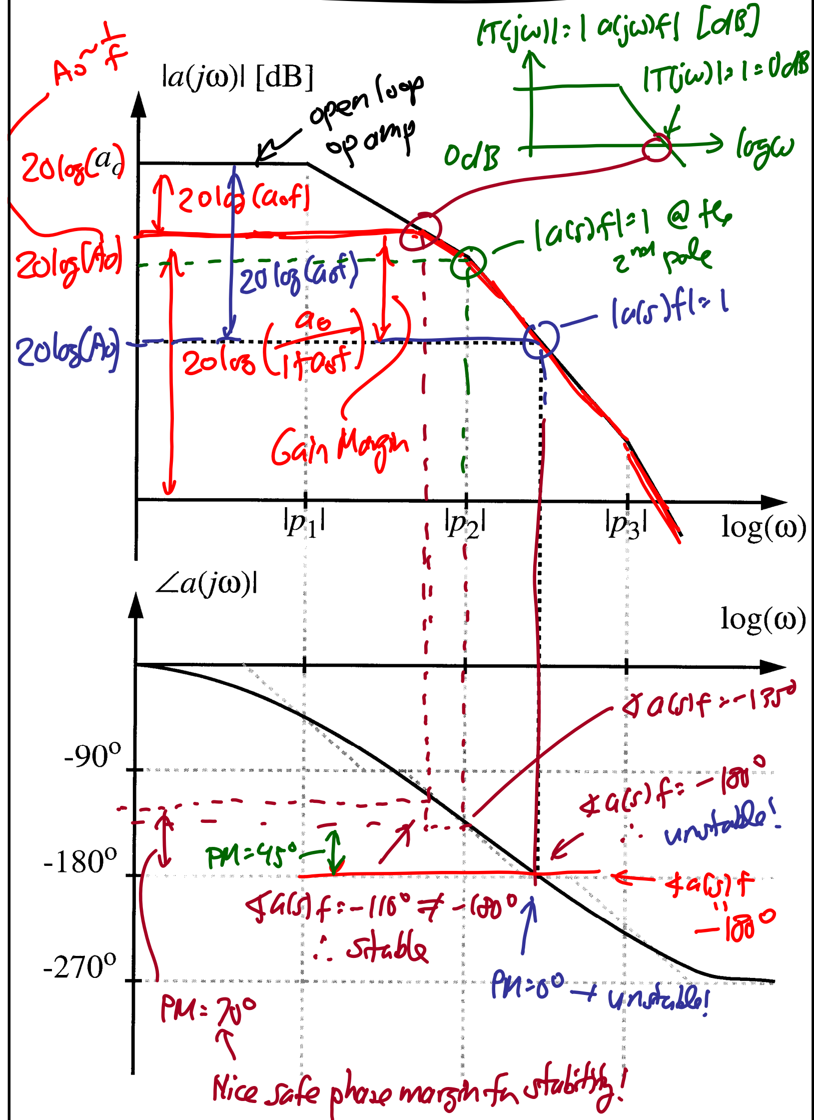
↓

But add a few non-dominant poles → then instability is possible! ∵ since now, $\measuredangle a(s)f$ can reach $-180°$!

↓

Can best visualize this via a Bode plot.

## Stability of a FB Ckt. Using a Multi-Pole Amp



For the more general case where a(s) has multiple poles:

⇒ A(s) has the same additional poles

⇒ i.e., @ freqs. > $|p_1|(1+a_o f)$, the A(s) curve just follow the a(s) curve

$$A(s) \cong \frac{A_0}{\left(1-\frac{s}{|p_1|(1+a_o f)}\right)\left(1-\frac{s}{p_2}\right)\left(1-\frac{s}{p_3}\right)}$$

makes sense, because @ freqs > $|p_1|(1+a_o f)$, the loop transmission $|a(s)f|<1$ → ∴ there really isn't much FB anymore

Definition:

Phase Margin = $180° + (\measuredangle a(j\omega)f$ @ freq. where $|a(j\omega)f| = 1)$

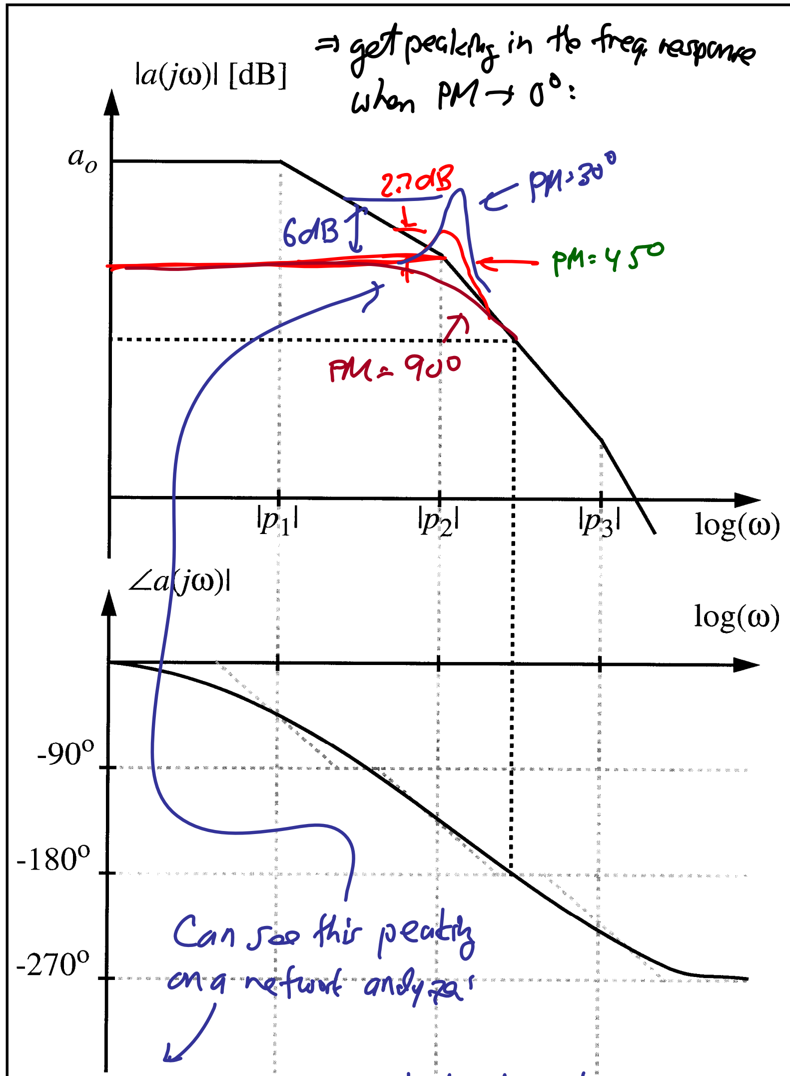
⇒ Phase Margin must be > 0° for stability

$\boxed{\text{For Stability: Phase Margin} > 0°}$

⇒ But for safety, design for

$\boxed{\text{Phase Margin} \geq 45°}$ (Design Criterion for Practical Design)

↳ oftentimes, use PM ≥ 60°.




As the PM continues to shrink, the peak grows ↳ when the peak gets sufficiently large, the ckt. will oscillate @ its freq. → instability!

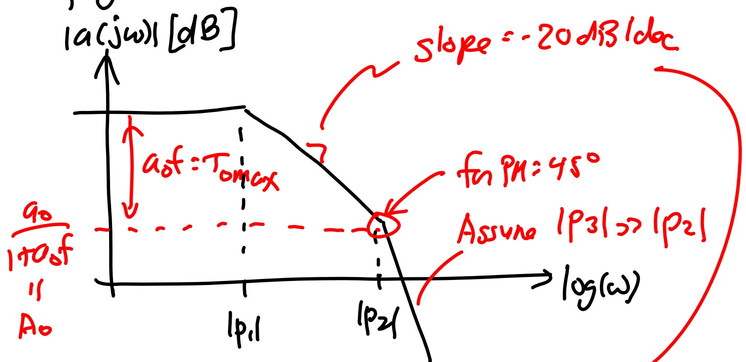
Definition:

Gain Margin = $|T(j\omega)|$ in dB @ freq. where $\measuredangle T(j\omega) = -180°$

↳ For Stability: $\boxed{\text{Gain Margin} < 0\,dB}$

Compensation of Op Amps

To compensate, need the distance between $p_1$ & $p_2$ to be large enough to encompass the largest desired loop gain!




$20(\log|p_2| - \log|p_1|) = 20\log(T_{omax})$

$\frac{|p_2|}{|p_1|} = T_{omax} \rightarrow |p_2| = |p_1| T_{omax}$

Largest expected loop gain

↳ For stability w/ PM = 45°

Two Ways to Compensate:

1. Narrowbanding
2. Pole-Splitting

Narrowbanding

dominant: "D"

∃ introduce a pole $p_D$ so that is sufficient separation between $p_D$ & $p_1$

which becomes the "new $p_2$"