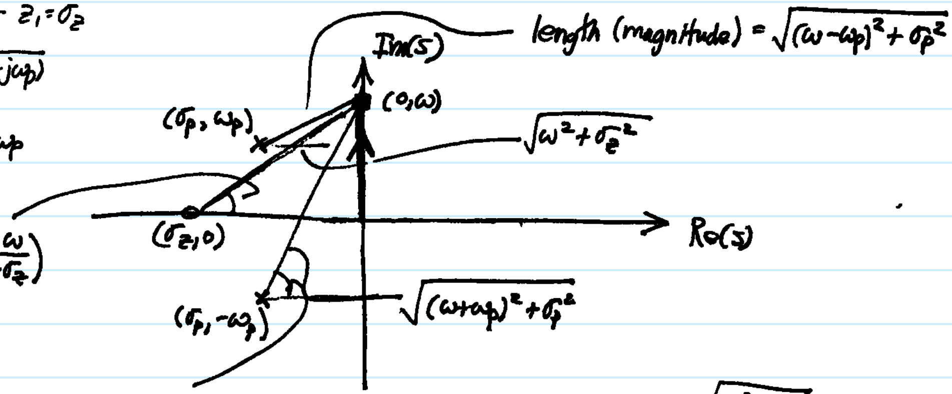
Review of Pole/Zero Plots

The frequency response of a given system is completely characterized by knowledge of the poles, zeros, and dc gain factor (H_0) of the system in question. In fact, the magnitude & phase of the network can be determined graphically from a pole/zero diagram.

$$H(s) = \frac{(s-\sigma_z)}{(s-\sigma_p-j\omega_p)(s-\sigma_p+j\omega_p)} \qquad z_1 = \sigma_z$$

$p_1 = \sigma_p + j\omega_p \qquad p_2 = \sigma_p - j\omega_p$



$\angle = \tan^{-1}\left(\frac{\omega}{-\sigma_z}\right)$

$\angle = \tan^{-1}\left(\frac{\omega+\omega_p}{-\sigma_p}\right) = \tan^{-1}\left(\frac{\omega+|\omega_p|}{|\sigma_p|}\right)$

$$\therefore |H(j\omega)| = \frac{\sqrt{\omega^2+\sigma_z^2}}{\sqrt{(\omega-\omega_p)^2+\sigma_p^2}\sqrt{(\omega+\omega_p)^2+\sigma_p^2}}$$

$$\therefore \angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{|\sigma_z|}\right) - \tan^{-1}\left(\frac{\omega-|\omega_p|}{|\sigma_p|}\right) - \tan^{-1}\left(\frac{\omega+|\omega_p|}{|\sigma_p|}\right)$$

In general:

$$|H(j\omega)| = H_0 \frac{\Pi\,(\text{mag. of vectors from zeros to } j\omega)}{\Pi\,(\text{mag. of vectors from poles to } j\omega)} = H_0 \frac{\prod_{j=1}^{m} |j(\omega-\omega_{zj})-\sigma_{zj}|}{\prod_{i=1}^{n} |j(\omega-\omega_{pi})-\sigma_{pi}|}$$

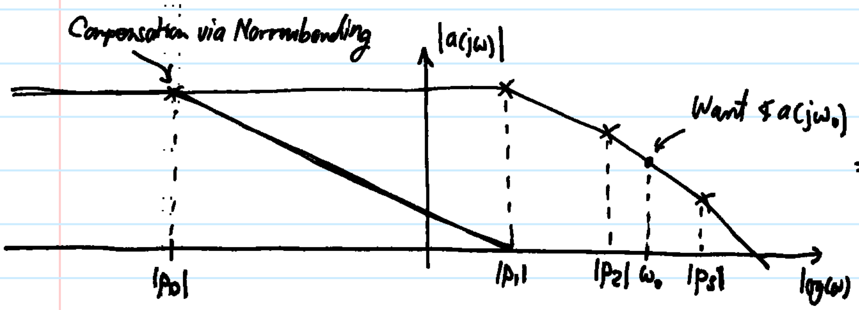
$z_j = \sigma_{zj} + j\omega_{zj}$

$p_i = \sigma_{pi} + j\omega_{pi}$

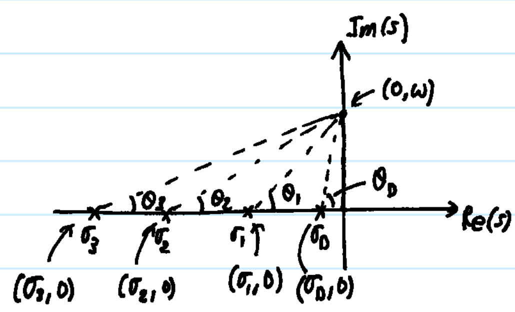
$$\angle H(j\omega) = \Sigma \text{ angles from zeros} - \Sigma \text{ angles from poles} + \angle H_0$$

$\angle H_0$ could be 0° (−) or 180° (+)

Precise Determination of Phase



$\Longrightarrow$



$\sigma_D = (-),\ \sigma_1 = (-),\ \sigma_2 (-),\ \sigma_3 = (-)$

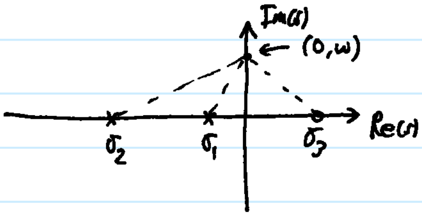
$$\measuredangle a(j\omega) = -\theta_D - \theta_1 - \theta_2 - \theta_3$$

$$= -\tan^{-1}\left(\frac{\omega}{-\sigma_D}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right)$$

$\tan^{-1}\left(\frac{\omega-0}{0-\sigma_3}\right)$

$$\therefore \measuredangle a(j\omega_0) = \underbrace{-90° - 90°}_{\text{since } \omega_0 \gg |p_D| \,\&\, |p_1|} - \tan^{-1}\left(\frac{\omega_0}{|p_2|}\right) - \tan^{-1}\left(\frac{\omega_0}{|p_3|}\right)$$

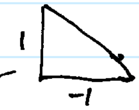
Ex: System w/ 2 poles & 1 RHP zero



σ1 = (-) σ2 = (-) since it's a zero!

$$\boxed{\begin{aligned}\measuredangle H(j\omega) &= -\tan^{-1}\left(\frac{\omega}{-\sigma_1}\right) - \tan^{-1}\left(\frac{\omega}{-\sigma_2}\right) + \tan^{-1}\left(\frac{\omega}{-\sigma_3}\right) \\ &= -\tan^{-1}\left(\frac{\omega}{|\sigma_1|}\right) - \tan^{-1}\left(\frac{\omega}{|\sigma_2|}\right) + \tan^{-1}\left(-\frac{\omega}{|\sigma_3|}\right)\end{aligned}}$$

σ3 = (+)



$$\measuredangle H(j\omega) = -90^\circ - 90^\circ + (-90^\circ) \Rightarrow \underbrace{\measuredangle H(j\omega)}_{\omega=\infty} = -270^\circ$$

$$\boxed{|H(j\omega)| = \frac{\sqrt{\omega^2+\sigma_3^2}}{\sqrt{\omega^2+\sigma_1^2}\sqrt{\omega^2+\sigma_2^2}}}$$

Note: The RHP zero contributes a (-) phase shift!