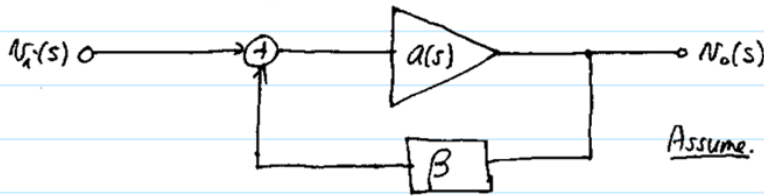


Obtain expressions for overshoot ($V_{\text{overshoot}}$) and settling time (T_s) as functions of phase margin, ϕ_m :



Assume $\beta = \text{const. w/ freq.}$

$$A(s) = \frac{N_o(s)}{N_i(s)} = \frac{a(s)}{1+a(s)\beta} \xrightarrow{\omega_0 = \omega_n} \frac{a_0 \omega_1 \omega_2}{s^2 + (\omega_1 + \omega_2)s + \omega_1 \omega_2 (1+a_0\beta)} = \frac{A_0 \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \frac{A_0 \omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

$$\left[a(s) = \frac{a_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} = \frac{a_0 \omega_1 \omega_2}{(s + \omega_1)(s + \omega_2)} \right]$$

General Lowpass Biquad Transfer Functions

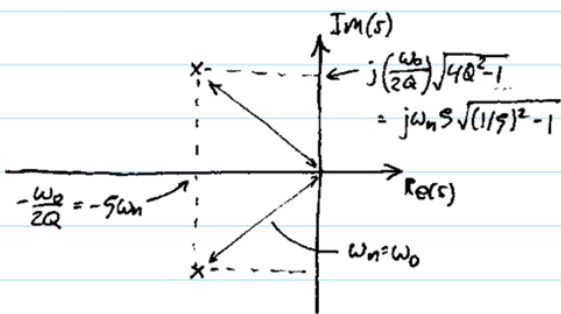
where by direct comparison:

Frequency response spectra for various values of ζ on next page

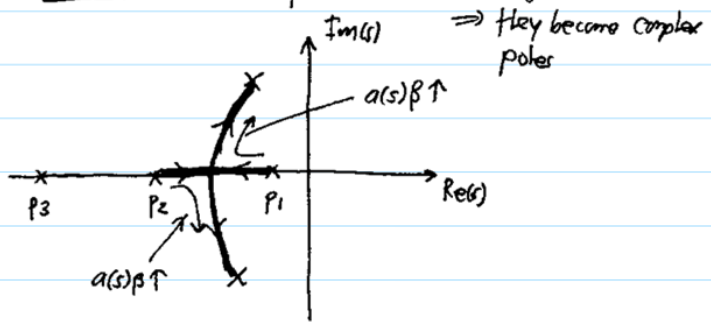
$$\begin{cases} \omega_n = \sqrt{\omega_1 \omega_2 (1+a_0\beta)} \\ 2\zeta \omega_n = \omega_1 + \omega_2 \rightarrow \zeta = \frac{\omega_1 + \omega_2}{2\omega_n} = \frac{1}{2} \frac{\omega_1 + \omega_2}{\sqrt{\omega_1 \omega_2 (1+a_0\beta)}} \\ A_0 \omega_n^2 = a_0 \omega_1 \omega_2 \rightarrow A_0 = \frac{a_0}{1+a_0\beta} \end{cases}$$

Properties of the General Lowpass Biquad Transfer Function (a very well-studied function!)

Pole-Zero Diagram -



Root Locus for FB Ckt: poles move as the loop gain increases



Time Domain Behavior -

$$N_o(t) = A_0 V_i \left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\sqrt{1-\zeta^2} \omega_n t + \phi) \right], \text{ where } \phi = \tan^{-1} \left[\frac{\sqrt{1-\zeta^2}}{\zeta} \right]$$

For $\zeta = \text{small} \rightarrow \phi = \frac{\pi}{2}$

For $\zeta < 1$: (for PM $< 90^\circ$)

$$\% \text{ Overshoot} = \frac{\text{Peak Value} - \text{Final Value}}{\text{Final Value}} = \exp \left[\frac{-\pi \zeta}{\sqrt{1-\zeta^2}} \right] \Rightarrow V_{\text{overshoot}} = V_0 \exp \left[\frac{-\pi \zeta}{\sqrt{1-\zeta^2}} \right]$$

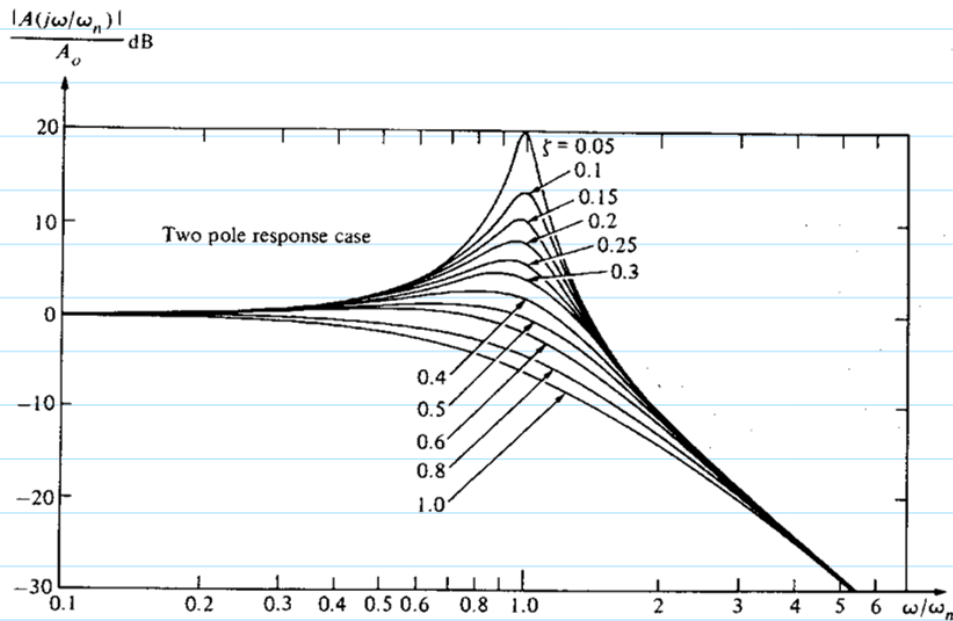
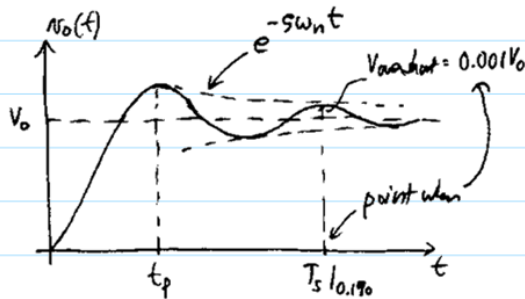


Figure C-2 Gain magnitude response for various values of ζ for a second-order, low-pass system.



Find t_p :

$$\sqrt{1-s^2} w_n t_p + \phi = \frac{3\pi}{2}$$

$$[s = \text{small} \rightarrow \phi \sim \frac{\pi}{2}] \Rightarrow$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-s^2}}$$

Approximate Determination of 0.190 Settling Time -

$$V_{\text{overshoot}} e^{-s w_n (T_s - t_p)} = 0.001 V_0$$

$$T_s = t_p - \frac{1}{s w_n} \ln \left[\frac{0.001 V_0}{V_{\text{overshoot}}} \right]$$

$$s = \frac{1}{w_n (T_s - t_p)} \ln \left[\frac{0.001 V_0}{V_{\text{overshoot}}} \right]$$

Determine $\phi_n = f(s) =$

\Rightarrow first get an expression for loop transmission:

$$A(s) = \frac{a(s)}{1+a(s)\beta} \rightarrow a(s) = A(s) + a(s)A(s)\beta \rightarrow a(s) = \frac{A(s)}{1-BA(s)}$$

$$\Rightarrow a(s)\beta = \frac{\beta A(s)}{1-BA(s)} = \frac{\beta A_0 \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2 - \beta A_0 \omega_n^2} \approx \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta \omega_n)}$$

$\left[\beta \approx \frac{1}{A_0} \right]$
↑ pole @ origin
↑ LHP pole

⇒ get freq. at which $|a(s)\beta| = 1 \rightarrow \omega_u$ = ω_{ult}

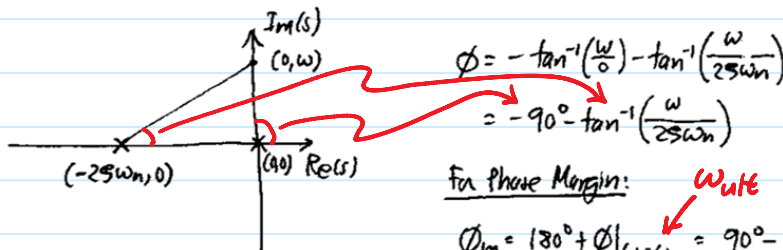
$$a(j\omega)\beta = \frac{\omega_n^2}{-\omega^2 + j2S\omega_n\omega} \Rightarrow |a(j\omega)\beta| = \frac{\omega_n^2}{\sqrt{\omega^4 + \omega^2 4S^2\omega_n^2}}$$

$$[|a(j\omega_u)\beta| = 1] \Rightarrow \frac{\omega_n^4}{\omega_u^4 + 4S^2\omega_n^2\omega_u^2} = 1 \rightarrow \omega_u^4 + 4S^2\omega_n^2\omega_u^2 - \omega_n^4 = 0$$

Solve quadratic: $\omega_u = \omega_n \left[\sqrt{4S^2 + 1} - 2S^2 \right]^{1/2}$

↑
Freq. for phase margin determination.

Find expression for phase:



For Phase Margin:

$$\phi_m = 180^\circ + \phi|_{\omega=\omega_u} = 90^\circ - \tan^{-1}\left(\frac{\omega_u}{2S\omega_n}\right)$$

$$\tan(90^\circ - \phi_m) = \frac{\omega_u}{2S\omega_n}$$

$$\left[\tan(A+90^\circ) = -\frac{1}{\tan A} \right] \Rightarrow -\frac{1}{\tan(-\phi_m)} = \frac{\omega_u}{2S\omega_n}$$

$$\Rightarrow \phi_m = \tan^{-1}\left(\frac{2S\omega_n}{\omega_u}\right)$$

$$\text{or } \phi_m = \cos^{-1}\left[\sqrt{4S^2 + 1} - 2S^2\right]$$

Thus:

$$S = \frac{1}{2} \frac{\omega_u}{\omega_n} \tan \phi_m$$

$$\Rightarrow T_s = t_p - \frac{2}{\omega_u \tan \phi_m} \ln \left[\frac{0.001V_o}{V_{error}} \right]$$

(ignoring effects of slew rate)

⇒ But there's more to it than this. See the following papers:

- ① B.V. Kamath, R.G. Meyer, and P.R. Gray, "Relationship between frequency response and settling time of operational amplifiers," IEEE J. of Solid-State Ckts., vol. SC-9, no. 6, pp. 347-352, Dec. 1974.
- ② R.J. Apfel and P.R. Gray, "A fast-settling monolithic operational amplifier using doublet compression techniques," IEEE J. of Solid-State Ckts, vol. SC-9, no. 6, pp. 332-340, Dec. 1974.