Lecture 6: Frequency Response Inspection Analysis

- Announcements:
- Make-up lecture for next Tuesday
  - Friday: 6 p.m. in ???
- Lecture Topics:
  - Amplifier Bode plot
  - Open Circuit Time Constant (OCTC) Analysis
  - Frequency Response Inspection Analysis
  - Frequency Response Examples
- ------------------------------------
- Last Time:

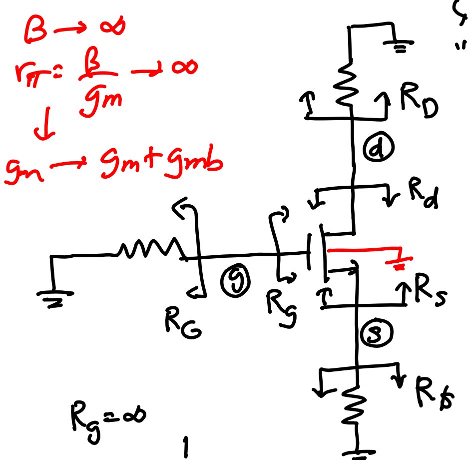
as well as tied to V_{DD}

Mos Inspection Formulas w/ Substrate Grounded

$\beta \to \infty$

$r_\pi = \frac{\beta}{g_m} \to \infty$

$g_m \to g_m + g_{mb}$



only difference from "substrate tied to source" case is that g_m is replaced by g_m+g_{mb} in some of the formulas

particularly ones where the source is involved!

$R_g = \infty$

$R_s = \frac{1}{g_m+g_{mb}}$

$R_d = r_o\left[1+(g_m+g_{mb})R_S\right]$

over

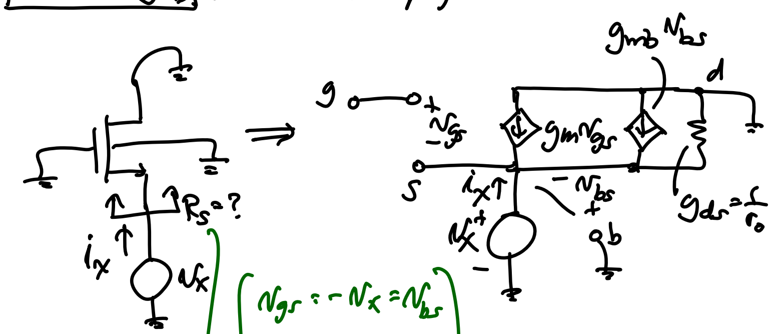
$\frac{v_d}{v_g} = -G_m R_d\ , \quad G_m = \frac{g_m}{1+(g_m+g_{mb})R_S}$

$\frac{v_d}{v_s} = -G_m R_d\ , \quad G_m = -(g_m+g_{mb})$

$\frac{v_s}{v_g} = \frac{g_m R_S}{1+(g_m+g_{mb})R_S}$

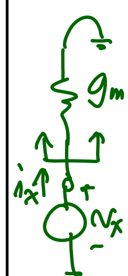
Remark. When the substrate is tied to the source, $g_{mb}=0$.

Effect of g_{mb} (one more example)



$v_{gs} = -v_x = v_{bs}$

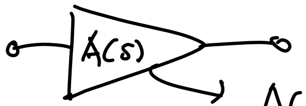
$v_{ds} = -v_x$



$g_m+g_{mb}+g_{ds} \to R_s = \frac{1}{g_m+g_{mb}+g_{ds}} = \frac{1}{g_m}\Big\|\frac{1}{g_{mb}}\Big\| r_o$

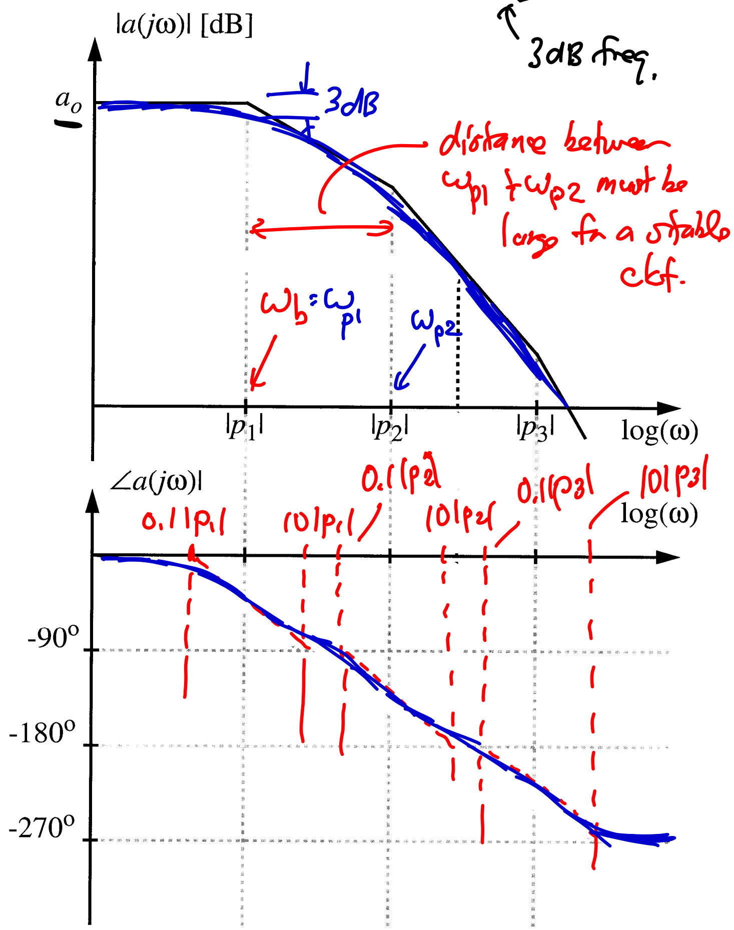
$\boxed{R_s \approx \frac{1}{g_m}\Big\|\frac{1}{g_{mb}} = \frac{1}{g_m+g_{mb}}}$

Poles & Zeros → Bode Plots



$$A(s) = \frac{a_o}{\left(1+\frac{s}{\omega_b}\right)\left(1+\frac{s}{\omega_2}\right)(\cdots}$$

3dB freq.



Freq. Response

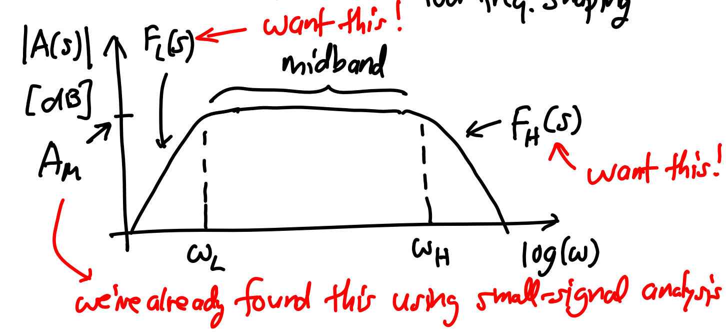
Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s)$$

midband gain

low freq. shaping

high frequency shaping

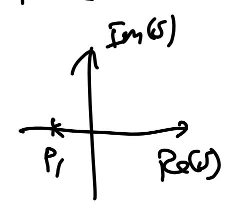


High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$



from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo}$$

where C_j are capacitors in the H.F. ckt., i.e., small ones

$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$

(ii) $F_H(s) \cong \dfrac{1}{1+\frac{s}{\omega_H}}$

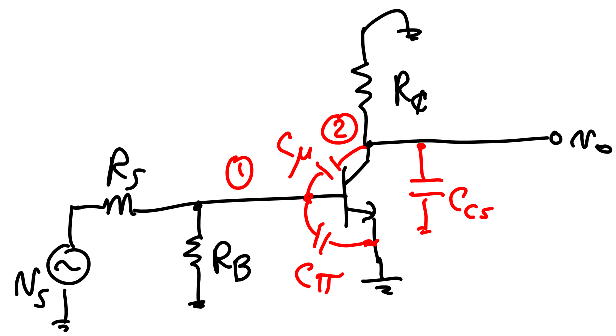
(iii) $b_1 \cong \dfrac{1}{\omega_{p1}} \rightarrow \boxed{\omega_H = \omega_{p1} \cong \dfrac{1}{b_1} = \dfrac{1}{\sum_j \tau_{jo}} = \dfrac{1}{\sum_j C_j R_{jo}}}$

When there is no dominant pole, an approximate expression for ω_H is:

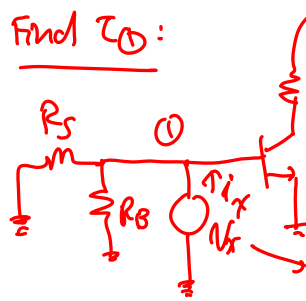
$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

(just FYI)

$\boxed{\text{Example HF Analysis}}$ C.E. Ckt.



Find τ_1:

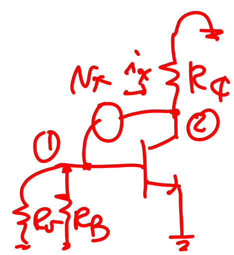


$C_1 = C_\pi$

$R_1 = \dfrac{v_x}{i_x} = r_\pi \| R_B \| R_s$

$\tau_1 = C_1 R_1 = C_\pi (r_\pi \| R_B \| R_s)$

Find $\tau_{\mu o}$:



$R_{\mu o} = \dfrac{v_x}{i_x} = R_1 + R_2 + g_m R_1 R_2$

neg!

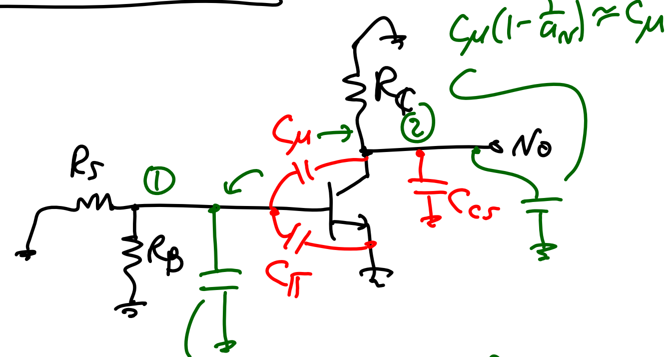
$R_{\mu o} = r_\pi \| R_B \| R_s + \cancel{r_o} \| R_C + g_m (r_\pi \| R_B \| R_s)(r_o \| R_C)$

$\tau_{\mu o} = R_{\mu o} C_\mu$

Find $\tau_{②}$: $\tau_{②} = (r_o \| R_C) C_{cs} \approx R_C C_{cs}$

$$\boxed{\omega_H = \frac{1}{\tau_{①} + \tau_{\mu o} + \tau_{②}}}$$

Now, use Miller's Theorem



$C_\mu(1-a_v) = C_\mu(1+g_m R_C)$

Find $\tau_{①}$:

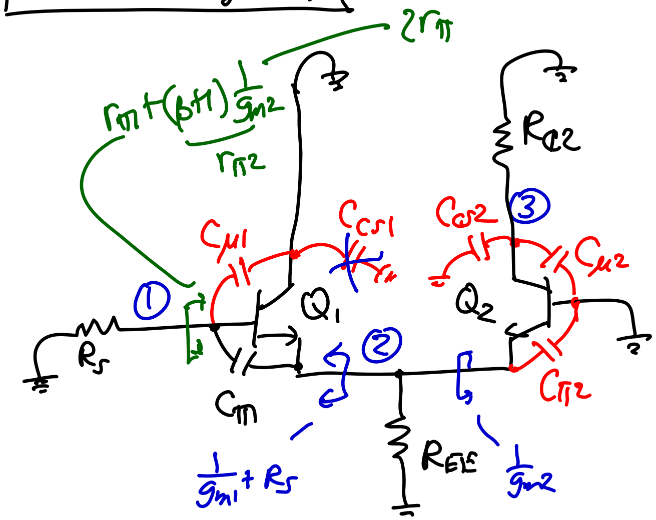
$\tau_{①} = (r_\pi \| R_B \| R_S)(C_\pi + C_\mu(1+g_m R_C))$

Find $\tau_{②}$:

$\tau_{②} = R_C(C_\mu + C_{cs})$

$$\boxed{\omega_H = \frac{1}{\tau_{①} + \tau_{②}}}$$

Ex. Multi-Stage Ckt.



$\tau_{①} = C_{\mu1}(2r_\pi \| R_S)$

$\tau_{\pi} = C_{\pi1}\left[r_\pi \,\Big\|\, \frac{R_S + \frac{1}{g_{m2}} \| R_{EE}}{1+g_{m1}\left(\frac{1}{g_{m2}} \| R_{EE}\right)}\right]$

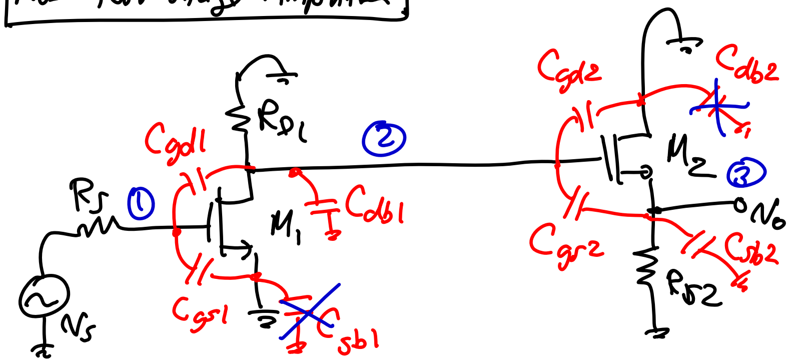
$\tau_{②} = C_{\pi2}\left[\left(\frac{1}{g_{m1}} + \frac{R_S}{\beta+1}\right) \Big\| \frac{1}{g_{m2}} \Big\| R_{EE}\right]$

$\tau_{③} = (C_{\mu2} + C_{cs2})(R_{C2} \| \cancel{(2r_{o2})})$ neg!

$$\boxed{\omega_H = \frac{1}{\tau_{①} + \tau_{\pi1} + \tau_{②} + \tau_{③}}}$$

$\frac{v_{②}}{v_{①}} = \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} = \frac{1}{2}$

MOS Two-Stage Amplifier



Step 1: Eliminate caps.

Step 2: Determine time constants.

$$\tau_{①} = [C_{gs1} + C_{gd1}(1 + g_{m1} R_{D1})] R_S$$

$$\tau_{②} = [C_{gd1} + C_{db1} + C_{gd2}](r_{o1} \| R_D) \qquad \sim R_D$$

Miller

$$\tau_{③} = C_{sb2}\left(R_{S2} \,\Big\|\, \frac{1}{g_m + g_{mb}}\right) \qquad \frac{1}{g_m+g_{mb}} + \cancel{\frac{R_{D1}}{\beta+1}} \; (\to 0,\ \beta \to \infty)$$

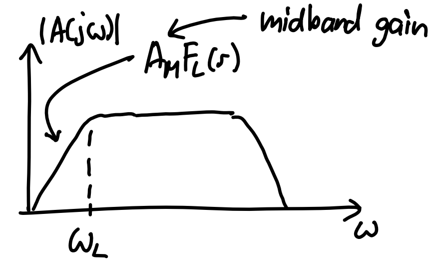
$$\cancel{\tau_{gs2} = C_{gs2}\left(\frac{R_{D1} + R_{S2}}{1 + (g_m + g_{mb})R_{S2}}\right)} \leftarrow \text{not quite right... (see G\&M)}$$

$$\boxed{\omega_H = \frac{1}{\tau_{①} + \tau_{②} + \tau_{③} + \tau_{gs2}}}$$

→ there's a zero, too!

Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall:



In general, for the low freq. response:

$$F_L(s) = \frac{s^{n_L} + d_1 s^{(n_L-1)} + \cdots}{s^{n_L} + e_1 s^{(n_L-1)} + \cdots}, \quad n_L = \#\text{poles} = \#\text{zeros}$$

We can express the coefficient e_1 by:

$$e_1 = \omega_{p1} + \omega_{p2} + \cdots + \omega_{pn_L}$$

Similar analysis to that used for OCTC...

For the case of a dominant pole:
↳ i.e., the highest freq. pole

$$F_L(s) \cong \frac{s}{s + \omega_L} = \frac{s}{s + e_1} \rightarrow e_1 \cong \omega_{p1} = \omega_L$$

$$\therefore \boxed{\omega_L \cong e_1 = \sum_j \omega_{pj} = \sum_j \frac{1}{C_j R_{js}} = \sum_j \frac{1}{\tau_{js}}}$$

where $C_j \triangleq$ various large (>10nF) capacitors in the ckt. (e.g., the bypass caps.)

$R_{js} \triangleq$ driving point resistance seen between the terminals of C_j determined with:

① all large capacitors short-circuited, except C_j, which is replaced by the test voltage source for R determination

For reading, can go to Sedra & Smith

2. all independent sources eliminated
(i.e., short voltage sources, open current sources)
3. open all H.F. capacitors (i.e., small caps in the pF range, or < 1nF)

Again, for the case where there are no dominant poles, a reasonable approximation is:

$$\omega_L \cong \sqrt{\omega_{P1}^2 + \omega_{P2}^2 - 2\omega_{Z1}^2 - 2\omega_{Z2}^2}$$