

Lecture 12: Op Amps & Emitter Coupled Pair

Announcements:

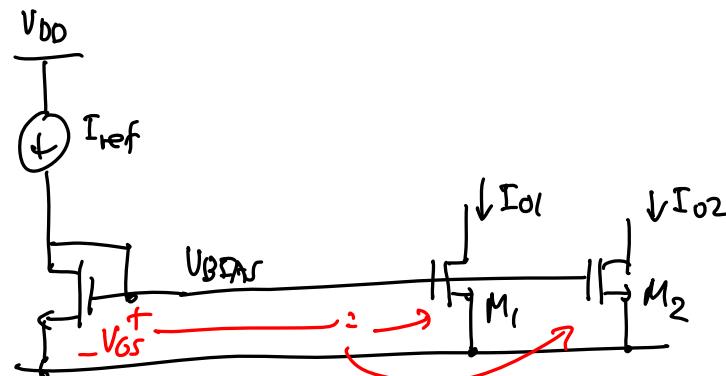
- ↳ Pre-Lecture materials online
- ↳ HW#1A online for 240A folks
- ↳ Lab#2 starts tomorrow
- ↳ Lab#1 due tomorrow at the beginning of your lab section; turn it in to your TA
- ↳ HW#5 due tomorrow
- ↳ Midterm will be on the date specified in your syllabus: Thursday, March 21, 3:30-5 p.m. in 390 Hearst Mining Building

Lecture Topics:

- ↳ Op Amp Review
- ↳ Emitter Coupled Pair (ECP)
- ↳ Half Circuits

Last Time:

Current Source Matching Considerations

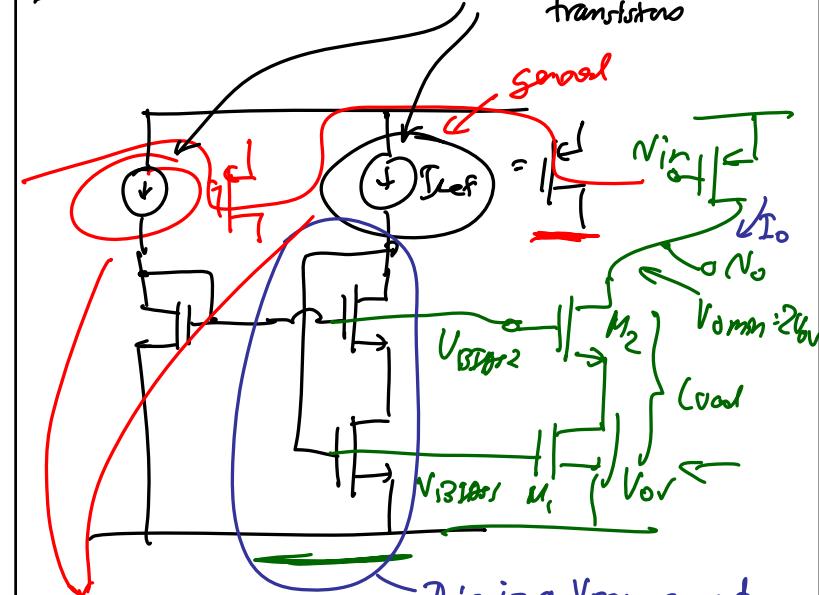


$$\therefore \frac{\Delta I_D}{I_D} = \frac{\Delta(W/L)}{(W/L)} - \frac{\Delta V_F}{(V_{DD}/2)}$$

this could be (-), so this doesn't necessarily help!

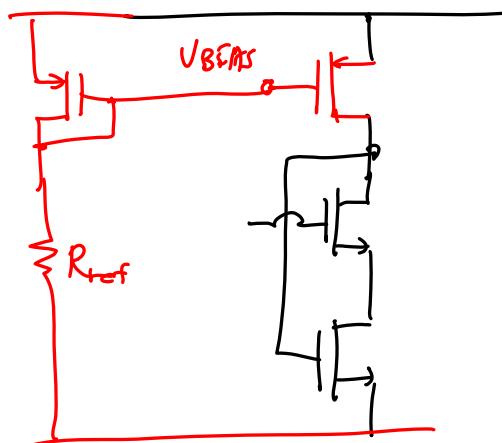
Fractional Current Mismatch  
Geometric Based Compensation helps  
Independent of Bias Pt.  
Increases CS Work  
Today: VDD + T work  
Read new Xorith  
generation, Must make  $(W/L)M$

Aside: When I draw current sources, they're just biased transistors

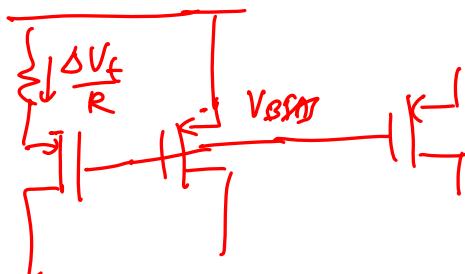


These are circuits that set to current value.  
e.g., \*  
This is a VBIAS generator that takes their current to generate VBIAS1 & VBIAS2 so F0: Iref + VI02

\* simple current source ...



... or  $V_t$  reference ...

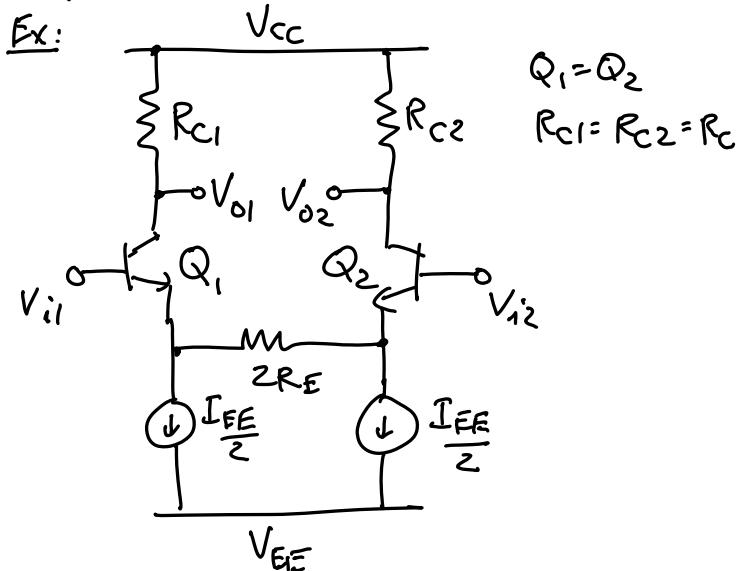


... or whatever is needed to generate  $I_{ref}$  w/ the needed supply + temperature independence

- Now, start on Op Amps & ECP using the Pre-Lecture handout

- Half Circuits:

Ex:



$$Q_1 = Q_2$$

$$R_{C1} = R_{C2} = R_C$$

**EE 140/240A**

**Ideal Op Amps**

CTN 1

Ideal Voltage Amplifier

→ ideal when  $\frac{R_o}{R_s} = A_{nr}$ ; i.e., when source and load R's do not influence the gain of the amplifier.

For this to occur, the voltage division at the input & output must be eliminated.  
This happens when:

$$\begin{cases} R_i = \infty \\ R_o = 0 \end{cases} \quad \begin{array}{l} \text{These resistance values define an} \\ \text{ideal voltage amplifier.} \end{array}$$

We'll look at other amplifier types later.

→ This, then, naturally leads us to:

Ideal Operational Amplifiers (Op Amps)

→ The work horse of analog electronics → combination of op amps w/ feedback components allow the implementation of analog computers, sampled-data systems, analog filters, A/D converters, DAC's, instrumentation amplifiers

In general, have a minimum of 5 terminals:

Simplify

Perhaps the best way to define an op amp is thru its equivalent circuit:

Equivalent Ckt. of an Ideal Op Amp:

$N_0 = A(N_2 - N_1) + A(N_1 - N_2)$

Voltage-Controlled Voltage Source (VCVS)

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Ideal Op Amps
CTN
(2)

Properties of Ideal Op Amps:

- ①  $R_{in} = \infty$  leads to ④  $i_+ = i_- = 0$
- ②  $R_o = 0$
- ③  $A = \infty$  leads to ⑤  $N_+ = N_-$ , assuming  $N_o = \text{finite}$   
 ↳ Why? Because  $f_{in} \propto (N_+ - N_-) \Rightarrow N_o = \text{finite}$

**Big assumption! ( $N_o = \text{finite}$ )**  
 How can we assume this?  $\Rightarrow$  only when there is an appropriate negative feedback path!

**Do perturbative analysis to determine whether it's (1) PB or not**

**Negative Feedback acts to oppose or subtract from input.**

$S_o = a S_i$        $S_o = a(S_i - \beta S_o)$   
 $S_o = S_i - \beta S_o$        $S_o(1 + a\beta) = a S_i \rightarrow \frac{S_o}{S_i} = \frac{a}{1 + a\beta}$

**overall transfer function**

$[a \rightarrow \infty] \Rightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$

$\therefore S_o = \frac{1}{\beta} S_i = \text{finite!}$   
 (when there is neg. FB around the amplifier)

**In Summary:**

- ① Neg. FB can insure  $S_o = \text{finite}$  even with  $a = \infty$ .
- ② Gain dependent (or overall T.F.) dependent only on external components. (e.g.,  $\beta$ )
- ③ Overall (closed-loop) gain  $\frac{S_o}{S_i}$  is independent of amplifier gain  $a$ .  
 ↳ very important!  $\Rightarrow$  as you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain.  
 i.e., if you're shooting for  $a = 50,000$ , you might get  $47,000$  or  $60,000$  instead.

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Op Amp Circuits
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*Contract w/ Positive Feedback*

*Blows Up!*

$S_i \uparrow$     $S_F \uparrow$     $\alpha \beta \uparrow$

$N_o \Rightarrow \text{output blows up!}$   
(for  $\alpha \beta > 1$ )

will be the case for  $\alpha = \infty$   
 $\Rightarrow$  usually get oscillation!

Thus, for a bounded, controllable function, need negative FB around an op amp.

*Op Amp Ckt.*

Example: Inverting Amplifier

$i = \frac{N_i}{R_1}$

$N_o = -\frac{R_2}{R_1} N_i$

*neg. PB ✓*

*virtual ground*

① Verify that there is negative FB.

②  $\therefore N_o = \text{finite} \rightarrow N_+ = N_- \rightarrow$  node attached to (-) terminal is virtual ground

③  $i_1 = 0 \therefore i_1 = i_2$

$$i_1 = \frac{N_i - 0}{R_1} = \frac{N_i}{R_1} = i_2 \quad \left. \begin{array}{l} \Rightarrow N_o = -\left(\frac{N_i}{R_1}\right) R_2 = -\frac{R_2}{R_1} N_i \\ N_o = 0 - i_2 R_2 = -i_2 R_2 \end{array} \right\} \therefore \boxed{\frac{N_o}{N_i} = -\frac{R_2}{R_1}}$$

*Note: Gain dependent only on  $R_1 + R_2$  (external components), not on the op amp gain.*

Example:

*(+)* FB X

$N_o = L^+ \text{ or } L^- \text{ depending on initial condns.}$

*cannot analyze using ideal or common method!*

$\therefore N_o \neq \text{finite}, N_+ \neq N_-$   
 $\Rightarrow$  this oft. will "rail out"

$N_+ = (+) \rightarrow L^+$   
 $N_+ = (-) \rightarrow L^-$

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Basic Op Amp Design
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How does one make an op amp? (It turns out, you already know!)

⇒ Basic Needed Attributes:

- ① Gain (voltage gain).
- ② Two inputs, (+) and (-).
- ③ One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE 105 can by itself serve as an op amp!  
(need only decide which of the inputs is the (+) and (-) terminal)

Let's now look at the diff. pair in more detail! ... but first

Why have 2 inputs?

- ① To get a virtual short for op amp diff.
- ② To suppress common-mode noise:

Can avoid this w/ a differential input:

$A(N_{nt} + N_{rt}) = 0$

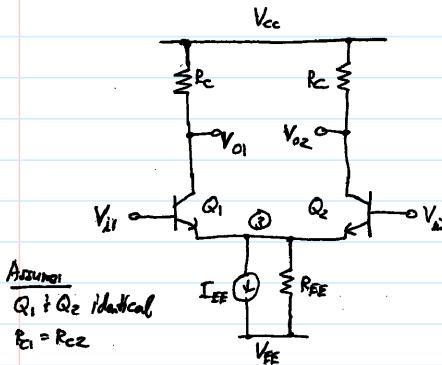
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Differential Pair (Bipolar)

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Differential Pair (Emitter-Coupled Pair)



Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

Definition:  $V_{id} = V_{i1} - V_{i2}$  (differential input)  
 $V_{icm} = \frac{V_{i1} + V_{i2}}{2}$  (common-mode input)

Assume:  
Q<sub>1</sub> & Q<sub>2</sub> identical  
R<sub>C1</sub> = R<sub>C2</sub>

$$\Rightarrow \begin{cases} V_{i1} = V_{icm} + \frac{V_{id}}{2} \\ V_{i2} = V_{icm} - \frac{V_{id}}{2} \end{cases}$$

Differential Gain =  $A_d = \frac{V_{o1} - V_{o2}}{V_{id}} = \frac{V_{od}}{V_{id}}$  (want this to be large for this differential amplification)

Common-Mode Gain =  $A_{cm} = \frac{V_{o1}}{V_{icm}} \approx \frac{V_{o2}}{V_{icm}}$  (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio = CMRR =  $\frac{A_{dm}}{A_{cm}}$  (should be very high to favor the differential mode and reject the common-mode)

$\Rightarrow$  we also want a high Common-Mode Input Range to reject DC input offsets

$\Rightarrow$  Note: No need for bypass capacitors (large) to the inputs or outputs  $\rightarrow$  can just use direct coupling!

Biasing & Large Signal Common-Mode Behavior

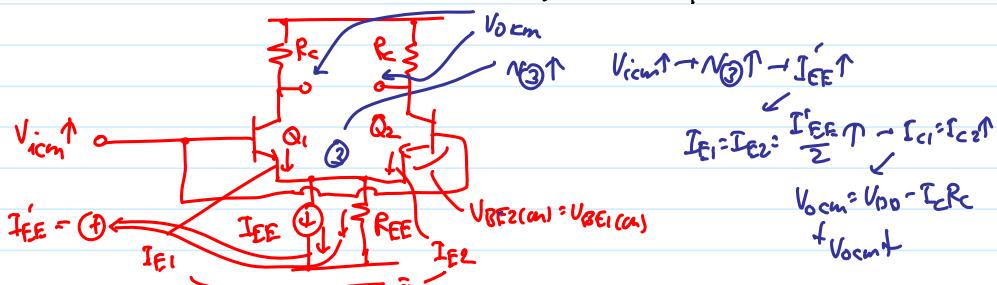
Case:  $R_{EE} = \infty \rightarrow$  ideal current source biasing  $\rightarrow I_{E1} = I_{E2} = \frac{I_{EE}}{2} \rightarrow V_{o1} = V_{o2} \Rightarrow V_{od} = 0$

If  $V_{icm} \uparrow \rightarrow V_{(2)} \uparrow$ , but current draw from  $I_{EE}$  stays constant  $\rightarrow I_{c1} \& I_{ce1}$  stay constant  $\rightarrow$  bias pt. doesn't change

Case:  $R_{EE}$  finite  $\rightarrow V_{(2)} = V_{i1} - V_{BE(cm)}$

If  $V_{icm} \uparrow \rightarrow V_{(2)} \uparrow \rightarrow I_{E1} = I_{E2} \uparrow$  (current draw =  $I_{EE} + \frac{V_{icm}}{R_{EE}}$ )

$\Rightarrow$  in general,  $R_{EE}$  will be large, so this component will be large, and the bias pt. won't change much



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Differential Mode Analysis
CTN

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**Small-Signal Analysis of Diff. Pair**

$$N_{o1} = -\frac{1}{2} g_m N_{i1} R_C$$

$$N_{o2} = -\frac{1}{2} g_m N_{i2} R_C$$

$$N_{o1} = +\frac{1}{2} g_m N_{i2} R_C$$

$$N_{o2} = +\frac{1}{2} g_m N_{i1} R_C$$

$$N_{od} = N_{o1} - N_{o2} = -g_m R_C (N_{i1} - N_{i2})$$

$$\therefore \frac{N_{od}}{N_{i1}} = A_{dm} = -g_m R_C$$

*If current source is long, then current will go into this!*

$\Rightarrow$  Easiest to see this happening using the T-model! (for those who must see the model stuff)

*This time also get the  $\frac{N_{o1}-N_{o2}}{N_{i1}}$  gain!*

$$N_{o1} = -\frac{1}{2} g_m R_C N_{i1}$$

$$N_{o2} = +\frac{1}{2} g_m R_C N_{i2}$$

$$N_{o1} - N_{o2} = -g_m R_C N_{i1}$$

$$\therefore \frac{N_{od}}{N_{i1}} = -g_m R_C$$

$$\frac{N_{o2}}{N_{i1}} = \frac{1}{2} g_m R_C$$

$$\frac{N_{o1}}{N_{i1}} = -\frac{1}{2} g_m R_C$$

*Ground to get only the contributions from  $N_{i1}$*

**[Diff. Mode Analysis]**

Assume a diff. w/ only diff. input:

Total current thru  $I_{EE}$  = const.

$\rightarrow V_E = \text{const. or input changes}$

$\rightarrow$  (3) acts as an incremental ground!  $\rightarrow V_3 = 0V$  (always!)

s. we can ground (3), and then, have

a **Differential Half Ckt.**

*Note: Can really only make this for a purely symmetrical ckt!*

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Common-Mode Analysis
CTN 7

**Differential Half Ckt.**

By inspection:  $\frac{V_{d1}/2}{Middle} = \frac{V_{d1}}{Middle} = A_{dm} = -g_m R_c$

$$\frac{V_{d1}/2}{I_b} = r_T \rightarrow R_{id} = 2r_T > R_{id}$$

S.S. parameters determined w/  $I_C = \frac{I_{FE}}{2}$

$\frac{V_{d1}/2}{I_b} = r_T \parallel R_c \rightarrow R_{id} = \frac{V_{d1}}{I_b} = 2(r_T \parallel R_c) \approx 2R_c = R_{id}$

First define  $r_T + \frac{1}{g_m(2+1)} - 2r_T$

$R_{id} = \frac{V_{d1}}{I_{FE}} = 2r_T$

thoroughly ground, so can import this

**Common-Mode Analysis**

Assume a pure CM input  $\rightarrow$  tie inputs together

By symmetry,  $i_{x1} = 0 \Rightarrow$  thus, totally have to equivalent of an open ckt. here

$\therefore \Rightarrow$  can split the ckt. into CM half-ckt's!

S.S. CM Half-Ckt.

$R_{ic} = r_T + (g_m)^2 (2R_{EE})$  @ each input

$A_{cm} = \frac{V_{d1}}{V_{ic}} = -\frac{g_m R_c}{1 + g_m (2R_{EE})} \approx -\frac{R_c}{2R_{EE}}$

Want small for large CMRR  $\therefore$  want  $R_{EE}$  large!

Common-Mode Rejection Ratio =  $CMRR = \frac{A_{dm}}{A_{cm}} = \frac{-g_m R_c}{-g_m R_c + 1 + g_m (2R_{EE})} \Rightarrow CMRR = 1 + 2g_m R_{EE}$

want as large as possible  
need a current source that is as ideal or possible!

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the EOP.

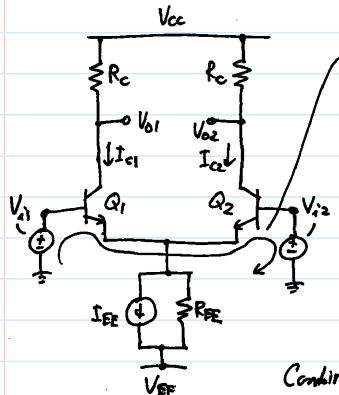
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Large Signal ECP Performance

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Large Signal ECP performance



Find  $I_{c1}$  &  $I_{c2}$ :

$$KVL: V_{c1} - V_{be1} + V_{be2} - V_{i2} = 0$$

$$I_{c1} = I_{T1} \exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{c1}}{I_{T1}}\right), V_{be2} = V_T \ln\left(\frac{I_{c2}}{I_{T2}}\right)$$

$$V_{i1} = V_T \ln\left(\frac{I_{c1}}{I_{T1}} \frac{I_{c2}}{I_{T2}}\right) - V_{i2} = 0 \rightarrow \ln \frac{I_{c1}}{I_{c2}} = \frac{V_{i1} - V_{i2}}{V_T} = \frac{V_{i1}}{V_T}$$

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{i1}}{V_T}\right) \quad (1)$$

$$I_{EE} = I_{c1} + I_{c2} = \frac{1}{\alpha} (I_{c1} + I_{c2}) \quad (2)$$

Combine (1) & (2) to get:

$$I_{c1} = \frac{\alpha I_{EE}}{1 + \exp\left(-\frac{V_{i1}}{V_T}\right)}, \quad I_{c2} = \frac{\alpha I_{EE}}{1 + \exp\left(\frac{V_{i1}}{V_T}\right)} \quad (3)$$

Find  $V_{od}$ :

$$\begin{aligned} V_{o1} &= V_{cc} - I_{c1} R_C \\ V_{o2} &= V_{cc} - I_{c2} R_C \end{aligned} \quad \left. \begin{aligned} V_{od} &= V_{o1} - V_{o2} = (I_{c2} - I_{c1}) R_C \\ &= \alpha_F I_{EE} R_C \left\{ \frac{1}{1 + \exp\left(\frac{V_{i1}}{V_T}\right)} - \frac{1}{1 + \exp\left(-\frac{V_{i1}}{V_T}\right)} \right\} \\ &\quad \times \frac{\exp\left(-\frac{V_{i1}}{2V_T}\right)}{\exp\left(\frac{V_{i1}}{2V_T}\right)} \quad \times \frac{\exp\left(\frac{V_{i1}}{2V_T}\right)}{\exp\left(-\frac{V_{i1}}{2V_T}\right)} \end{aligned} \right\}$$

$$\begin{aligned} &= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{i1}}{2V_T}\right)}{\exp\left(-\frac{V_{i1}}{2V_T}\right) + \exp\left(\frac{V_{i1}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{i1}}{2V_T}\right)}{\exp\left(\frac{V_{i1}}{2V_T}\right) + \exp\left(-\frac{V_{i1}}{2V_T}\right)} \right\} \\ &= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{i1}}{2V_T}\right) - \exp\left(\frac{V_{i1}}{2V_T}\right)}{\exp\left(-\frac{V_{i1}}{2V_T}\right) + \exp\left(\frac{V_{i1}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_C \frac{\sinh\left(-\frac{V_{i1}}{2V_T}\right)}{\cosh\left(-\frac{V_{i1}}{2V_T}\right)} \end{aligned}$$

$$\begin{cases} \sinh u = \frac{1}{2}(e^u - e^{-u}) \\ \cosh u = \frac{1}{2}(e^u + e^{-u}) \end{cases} \quad u = -\frac{V_{i1}}{2V_T}$$

$$\therefore V_{od} = \alpha_F I_{EE} R_C \tanh\left(-\frac{V_{i1}}{2V_T}\right)$$

From our knowledge of the Taylor series for  
 $\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \dots$

this is fairly linear for small  $V_{i1}$ , but gets nonlinear  
 abruptly when  $V_{i1}$  approaches a threshold value!

