

EE 140

Input Offset Voltage

CTN

1

Device Mismatch Effect in Diff. Amplifiers

- ⇒ up to this point, we assumed that $Q_1 \neq Q_2$ are perfectly matched
- ⇒ in actual ckt, get device mismatch due to processing variations

The Result:

① $N_o = A_{dm}(n_f - n_r)$ → Output not zero when Input is zero → $N_o \neq 0$ when $N_{id} = 0$!

$n_f, n_r \neq 0$

$$N_o = A_{dm}(n_f - n_r) \quad \text{Ideal Case: } N_o = 0$$

Reality: $N_o \neq 0$, even w/ $(n_f - n_r) = 0$!

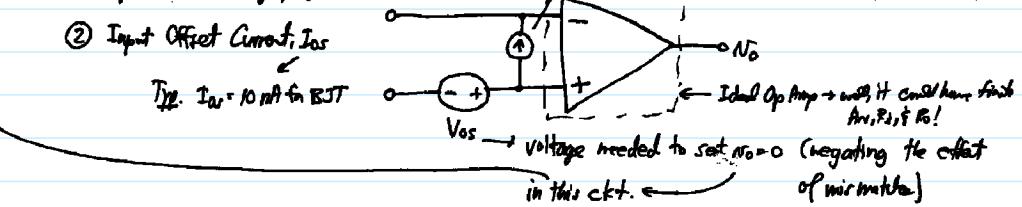
- ② Input $I_{B1} \neq I_{B2}$ if $Q_1 \neq Q_2$ not matched. (fn BJT + JFET only.)

To model those effects, introduce:

① Input Offset Voltage, V_{os}

② Input Offset Current, I_{os}

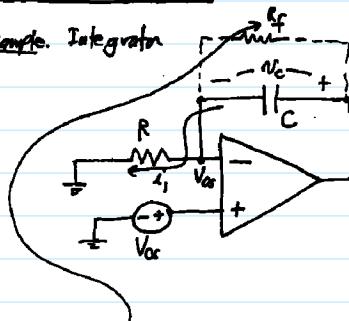
Typ. $I_{os} = 10 \text{ nA}$ fn BJT



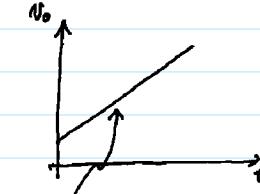
V_{os} → voltage needed to set $n_f = n_r$ (regarding the effect of mismatch)

Effect of V_{os} on Op Amp Ckt. -

Example: Integrator



$$\begin{aligned} N_o &= V_{os} + \frac{1}{C} \int_0^t i_o dt \\ &= V_{os} + \frac{1}{C} \int_0^t V_{os} \frac{dt}{R} \\ &= V_{os} \left(1 + \frac{t}{RC} \right) + N_e |_{t=0} \end{aligned}$$



Fix: Place an RF in shunt w/ the C

→ then $A_o = V_{os} \left(1 + \frac{R_F}{R} \right)$, and rail-hitting doesn't happen

→ but, usually $R_F = \text{large}$ to allow the C to dominate

the integrator Xfer Function. $\therefore N_o = V_{os} \left(1 + \frac{R_F}{R} \right)$ can be quite large \Rightarrow still want $V_{os} = \text{small}$

will continue to increase until op amp hits the voltage rails

V_{os} is even more important in setting the resolution of AD converters and other precision ckt.

EE 140

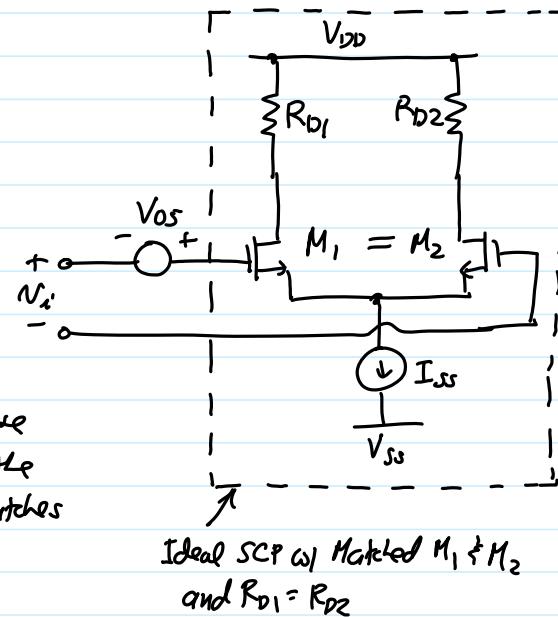
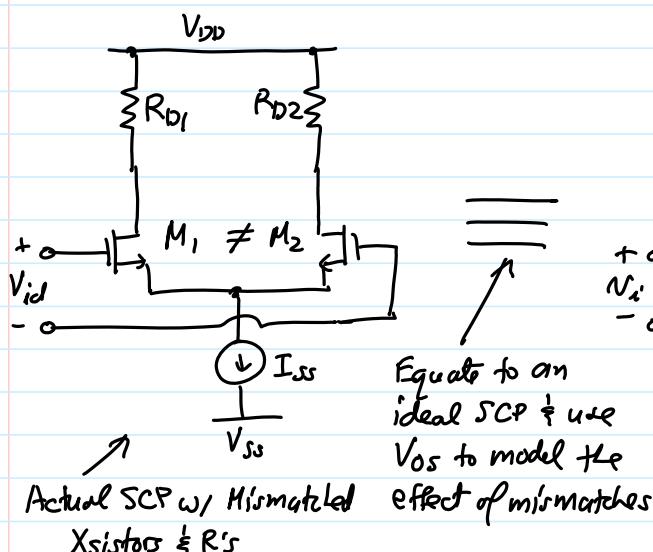
V_{OS} of a Mismatched SCP

CTN

2

V_{OS} of a Mismatched SCP

Objective: Derive an expression for V_{OS} .



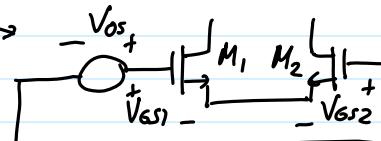
Input offset voltage V_{OS} arises due to variations in:

① Xsistors, M₁ & M₂ → $\frac{W}{L}$ and V_t vary

② R_{D1} ≠ R_{D2} → causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$ in this ckt.

KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$



$$\therefore V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} (W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} (W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} (W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} (W/L)_2}} \quad (1)$$

Define difference and average quantities:

$$\begin{aligned} \Delta I_D &= I_{D1} - I_{D2} & \Delta \left(\frac{W}{L}\right) &= \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2 & \Delta V_t &= V_{t1} - V_{t2} & \Delta R_D &= R_{D1} - R_{D2} \\ I_0 &= \frac{I_{D1} + I_{D2}}{2} & \left(\frac{W}{L}\right) &= \frac{1}{2} \left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2 \right] & V_t &= \frac{V_{t1} + V_{t2}}{2} & R_D &= \frac{R_{D1} + R_{D2}}{2} \end{aligned}$$

EE 140

V_{OS} of a Mismatched SCP

CTN

3

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2}$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2}$$

$$V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2}$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2}$$

$$V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$V_{OS} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{M_n C_{ox} \left[\left(\frac{W}{L}\right) + \frac{1}{2} \Delta \left(\frac{W}{L}\right) \right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{M_n C_{ox} \left[\left(\frac{W}{L}\right) - \frac{1}{2} \Delta \left(\frac{W}{L}\right) \right]}}$$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{M_n C_{ox} (W/L)}} \right] \quad \left[\frac{W}{L} \left[1 + \frac{1}{2} \frac{\Delta(W/L)}{(W/L)} \right] \right] \quad \left[\frac{W}{L} \left[1 - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)} \right] \right]$$

$$= \Delta V_t + (V_{GS} - V_t) \left\{ \sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2} \frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)}}} \right\}$$

→ Binomial Theorem:

$$(1 + nx)^m \xrightarrow{n=\text{small}} 1 + mn x$$

$$V_{OS} = \Delta V_t + (V_{GS} - V_t) \left\{ \left(1 + \frac{1}{4} \frac{\Delta I_D}{I_D} \right) \left(1 - \frac{1}{4} \frac{\Delta(W/L)}{(W/L)} \right) - \left(1 - \frac{1}{4} \frac{\Delta I_D}{I_D} \right) \left(1 + \frac{1}{4} \frac{\Delta(W/L)}{(W/L)} \right) \right\}$$

$$= \Delta V_t + (V_{GS} - V_t) \left(\frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)} \right)$$

$$\therefore V_{OS} = \Delta V_t + \frac{1}{2} (V_{GS} - V_t) \left\{ \frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

When $V_{id} = V_{OS} \rightarrow V_{od} = 0 \therefore I_{D1}R_{D1} = I_{D2}R_{D2} \rightarrow$ mismatch in I_D must be opposite

$$V_{OS} = \Delta V_t + \frac{1}{2} (V_{GS} - V_t) \left\{ -\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R}{R}$$

Threshold
Mismatch
bias independent

Geometric (i.e., Layout)
Variation
scale w/ overdrive

EE 140

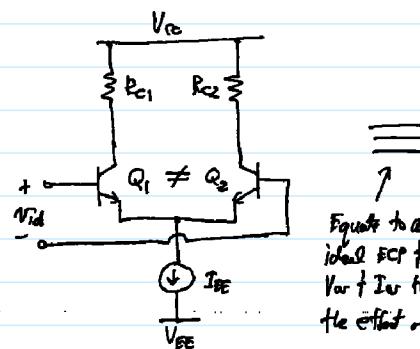
V_{OS} of a Mismatched ECP

CTN

4

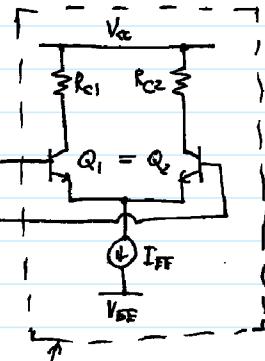
V_{OS} in a Mismatched ECP

Objectives: Derive an expression for V_{OS} .



Actual ECP w/ Mismatched Transistors & R's

Equivalent to an ideal ECP + one Var & I_{DS} to model the effect of mismatch



Ideal ECP w/ Mismatched Q₁ & Q₂ and R_{c1}=R_{c2}

Input Offset Voltage Var arises due to variations in:

- (1) Transistor, Q₁ & Q₂ \rightarrow I_s & β vary: $I_s = \frac{qN_A^2 D_n A}{N_B W_B(V_{CB})}$ $I_{S1} \neq I_{S2}$ can be caused by:
 ↗ "function of"
 (i) $A_1 \neq A_2$ (circuit tolerance limits)
 (ii) $N_{A1} \neq N_{A2}$ (doping variations of base)
 (iii) $W_B = f(V_{CB})$ (width variations exacerbated by V_{CB} diff.)
- (2) $R_{c1} \neq R_{c2} \rightarrow$ cause gain variation

Definition. $V_{OS} = V_{ID}$ to get $V_{OD} = 0$, which occurs when:

$$\text{Kv: } V_{OS} - V_{BE1} + V_{BE2} = 0$$



$$V_{OS} = V_{BE1} - V_{BE2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} = V_T \ln \left(\frac{I_{C1}}{I_{C2}} \frac{I_{S2}}{I_{S1}} \right)$$

Find $\frac{I_{C1}}{I_{C2}}$ in terms of design elements:

$$[\text{When } V_{ID} = V_{OS} \rightarrow V_{OD} = 0] \rightarrow V_{ID} = (V_{CE} - I_{C1}R_{c1}) - (V_{CE} - I_{C2}R_{c2}) = 0$$

$$I_{C1}R_{c1} = I_{C2}R_{c2} \rightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{c2}}{R_{c1}}$$

$$V_{OS} = V_T \ln \left(\frac{R_{c2}}{R_{c1}} \frac{I_{S2}}{I_{S1}} \right)$$

This is an exact equation for V_{OS} . It's often more useful & intuitive to express this in terms of percent variations (and eventually standard deviations).

EE 140

V_{OS} of a Mismatched ECP

CTN

5

Convert to percent variation form -

$$\text{Define. } R_c = \frac{R_{c1} + R_{c2}}{2}, \Delta R_c = R_{c1} - R_{c2} \quad \left. \begin{array}{l} \text{Objective: Express Var in terms of percent} \\ \text{variations } \frac{\Delta R_c}{R_c} \neq \frac{\Delta I_r}{I_r} \end{array} \right\}$$

$$I_s = \frac{I_{s1} + I_{s2}}{2}, \Delta I_s = I_{s1} - I_{s2}$$

$$\downarrow$$

$$\text{In general: } \left. \begin{array}{l} \Delta X = X_1 - X_2 \\ X = \frac{X_1 + X_2}{2} \end{array} \right\} \quad \left. \begin{array}{l} X_1 = X + \frac{\Delta X}{2} \\ X_2 = X - \frac{\Delta X}{2} \end{array} \right\} \Rightarrow \text{Thus: } R_{c1} = R_c + \frac{\Delta R_c}{2}, R_{c2} = R_c - \frac{\Delta R_c}{2}$$

$$I_{s1} = I_s + \frac{\Delta I_s}{2}, I_{s2} = I_s - \frac{\Delta I_s}{2}$$

With these formulations:

$$V_{OS} = V_T \ln \left[\frac{R_{c1} I_{s2}}{R_{c2} I_{s1}} \right] = V_T \ln \left\{ \frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}} \right\} = V_T \ln \left\{ \frac{1 - \frac{\Delta R_c}{2R_c}}{1 + \frac{\Delta R_c}{2R_c}} \frac{1 - \frac{\Delta I_s}{2I_s}}{1 + \frac{\Delta I_s}{2I_s}} \right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right] \Rightarrow V_{OS} \approx V_T \left\{ -\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s} \right\}$$

taking the first term assuming $\Delta R_c \ll R_c$ & $\Delta I_s \ll I_s$

$$V_{OS} = V_T \left\{ -\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s} \right\}$$

Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically varying

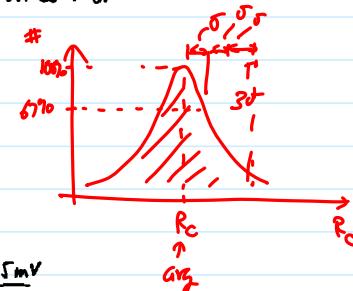
given process run & layout, one usually expresses terms in the form of variances when specifying V_{OS} :

\rightarrow since $\frac{\Delta R_c}{R_c} \neq \frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like powers:

$$\sigma_{V_{OS}}^2 = V_T^2 \sqrt{\sigma_{\Delta R_c / R_c}^2 + \sigma_{\Delta I_s / I_s}^2}$$

Ex: Typ. $\sigma_{\Delta R_c / R_c} \sim 0.01$, $\sigma_{\Delta I_s / I_s} \sim 0.05$

$$\therefore \sigma_{V_{OS}} = (26m) \sqrt{(0.01)^2 + (0.05)^2} = 1.3 \mu V \quad \text{Typ. Var for BJT} \sim 1-5 mV$$



V_{OS} Drift w/ Temperature

$$\frac{dV_{OS}}{dT} = \frac{kT}{q} \left\{ -\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s} \right\} \frac{1}{T} = \frac{V_{os}}{T} \quad \text{Ex. } \frac{dV_{os}}{dT} = \frac{1.3m}{300k} = 4.3 \mu V/C \text{ around } T=300K.$$

$\underbrace{\text{indep. w/ T}}$ \uparrow [in Kelvin]

EE 140

I_{OS} of a Mismatched ECP

CTN

6

I_{OS} in a Mismatched ECP

$$\text{By Definition: } I_{OS} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2} = I_{OS}$$

To express in percent variation:

$$\begin{cases} I_{C1} = I_c + \frac{\Delta I_c}{2} \\ I_{C2} = I_c - \frac{\Delta I_c}{2} \end{cases} \quad \begin{cases} \beta_1 = \beta + \frac{\Delta \beta}{2} \\ \beta_2 = \beta - \frac{\Delta \beta}{2} \end{cases}$$

$$\therefore I_{OS} = \frac{I_c + \frac{\Delta I_c}{2}}{\beta + \frac{\Delta \beta}{2}} - \frac{I_c - \frac{\Delta I_c}{2}}{\beta - \frac{\Delta \beta}{2}} = \frac{I_c}{\beta} \left\{ \frac{1 + \frac{\Delta I_c}{2I_c}}{1 + \frac{\Delta \beta}{2\beta}} - \frac{1 - \frac{\Delta I_c}{2I_c}}{1 - \frac{\Delta \beta}{2\beta}} \right\}$$

$$\begin{aligned} \left[\frac{1}{1+x} \approx 1 - x + x^2 - \dots \right] \rightarrow &= \frac{I_c}{\beta} \left\{ \left(1 + \frac{\Delta I_c}{2I_c} \right) \left(1 - \frac{\Delta \beta}{2\beta} \right) - \left(1 - \frac{\Delta I_c}{2I_c} \right) \left(1 + \frac{\Delta \beta}{2\beta} \right) \right\} \\ &= \frac{I_c}{\beta} \left\{ 1 + \frac{\Delta I_c}{2I_c} - \frac{\Delta \beta}{2\beta} - \frac{I_c}{2I_c} \frac{\Delta I_c}{2\beta} - 1 + \frac{\Delta I_c}{2I_c} - \frac{\Delta \beta}{2\beta} + \frac{\Delta I_c}{2I_c} \frac{\Delta \beta}{2\beta} \right\} \end{aligned}$$

$$I_{OS} = \frac{I_c}{\beta} \left\{ \frac{\Delta I_c}{I_c} - \frac{\Delta \beta}{\beta} \right\}$$

$$\text{But for } V_{od}=0V \Rightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{c2}}{R_{c1}} \rightarrow \frac{\Delta I_c}{I_c} = - \frac{\Delta R_c}{R_c}$$

$$\therefore I_{OS} = - \frac{I_c}{\beta} \left(\frac{\Delta R_c}{R_c} + \frac{\Delta \beta}{\beta} \right)$$

Ex. Typ: $\sigma_{\Delta R_B} = 0.1$, $\sigma_{\Delta R_{Refc}} = 0.01$

$$\rightarrow I_{OS} = - \frac{I_c}{\beta} \left[\sigma_{\Delta R_{Refc}}^2 + \sigma_{\Delta R_B}^2 \right]^{\frac{1}{2}} \approx - 0.1 \frac{I_c}{\beta} \approx - 0.1 I_B > I_{OS}$$

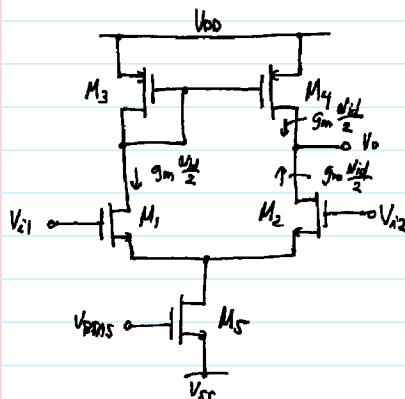
EE 140

Diff. Pair w/ I-Mirror Load V_{OS}

CTN

7

MOS Differential Stage w/ Current Mirror Load:



Small-Signal Gain: (similar to BJT)

$$\frac{A_v}{A_{vd}} = \frac{g_m r_o (V_{DD})}{g_m r_o + g_m r_s} = \frac{g_{m2}}{g_{m2} + g_{m4}} = \frac{\sqrt{2\mu_n C_{ox} (\frac{W}{L})_{2,4} I_{D2}}}{\lambda_L I_{D2} + \lambda_V I_{D2}}$$

$$= \frac{\sqrt{4\mu_n C_{ox} (\frac{W}{L})_{2,4} I_{D2}}}{I_{DS} (n_L + n_V)} \Rightarrow \frac{A_v}{A_{vd}} = \frac{2}{n_L + n_V} \sqrt{\frac{4\mu_n C_{ox} (\frac{W}{L})_{2,4}}{I_{DS}}}$$

$$\left[\frac{\Delta (W/L)_{1,2}}{(W/L)_{1,2}} - \frac{\Delta (W/L)_{3,4}}{(W/L)_{3,4}} \right]$$

Via similar derivation
to what we just did

Offset Voltage $V_{OS} = V_{GS1} - V_{GS2}$ when $V_{DD} = 0V$

$$V_{OS} = \Delta V_{GS1} + \Delta V_{GS2} \left(\frac{g_{m3,4}}{g_{m1,2}} \right) + \frac{(V_{GS} - V_t)_{1,2}}{Z} \left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}} \right]$$

For small V_{GS} :

$$\textcircled{1} \text{ small } (V_{GS} - V_t) \\ \textcircled{2} \text{ } g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2} \quad \frac{1}{2} (\frac{W}{L})_{3,4} < (\frac{W}{L})_{1,2}$$