Lecture 14: Vos & Finite Gain-BW

- **Announcements:**
  - ↳ HW#6 due tomorrow
  - ↳ HW#7 online soon
  - ↳ Pre-Lecture materials online
  - ↳ Midterm will be on the date specified in your syllabus: Thursday, March 21, 3:30-5 p.m. in 390 Hearst Mining Building
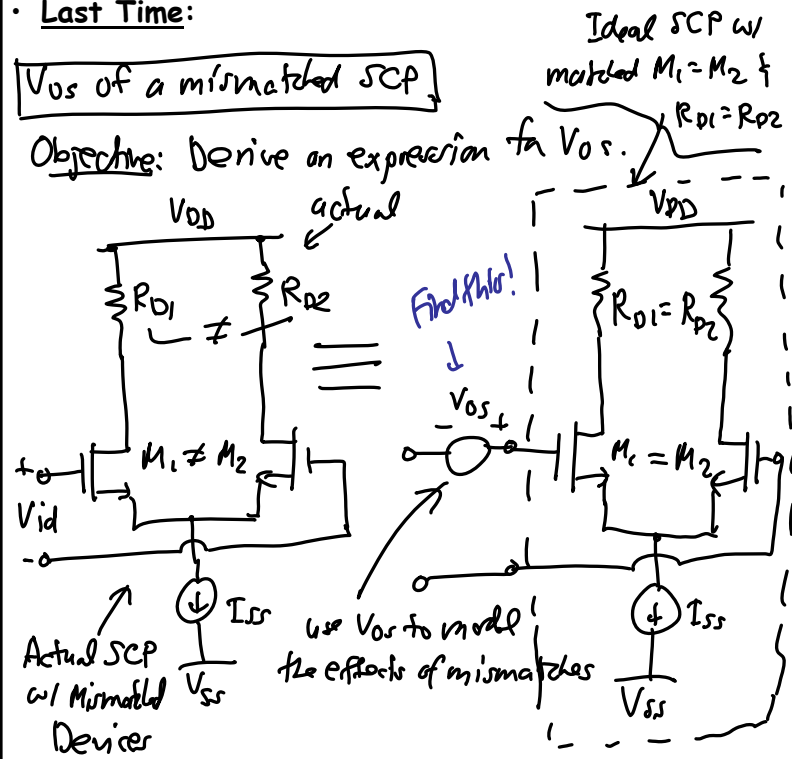- **Lecture Topics:**
  - ↳ Input Offset Voltage (Vos)
  - ↳ Op Amp Finite Gain-BW
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- **Last Time:**

Vos of a mismatched SCP

Objective: Derive an expression for Vos.

Ideal SCP w/ matched $M_1 \approx M_2$ & $R_{D1} \approx R_{D2}$

Actual SCP w/ Mismatched Devices

use Vos to model the effects of mismatches

Vos arises due variations in:

① X-sistor M_1; $M_2 \longrightarrow \frac{W}{L}$ & V_t vary

② $R_{D1} \neq R_{D2} \longrightarrow$ cause variation in gain

Definition. $V_{OS} = V_{id}$ needed to get $V_{od} = 0V$ when applying 0V to both inputs
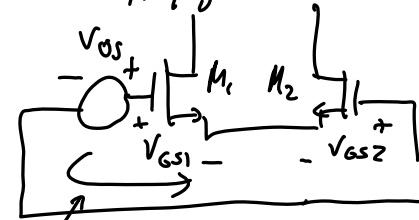
KVL: $V_{os} - V_{GS1} + V_{GS2} = 0$

$\therefore V_{os} = V_{GS1} - V_{GS2}$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{os} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \quad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$
$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$V_t = \frac{1}{2}(V_{t1} + V_{t2})$	$R_D = \frac{1}{2}(R_{D1} + R_{D2})$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1): $2I_D\left(1 + \frac{\Delta I_D}{2I_D}\right)$

$$V_{OS} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}$$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right] \qquad \frac{W}{L}\left[1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1 + mx)^m \xrightarrow{n = small} 1 + mnx$$

$$V_{OS} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right.$$
$$\left. - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D}$$
$$- \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{OS} \rightarrow V_{od} = 0$ $\therefore I_{D1}R_{D1} = I_{D2}R_{D2}$

$\hookrightarrow$ mismatch in I_D must be opposite that of R_D

$$\therefore \frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D} \qquad \text{Pay no attention to those signs}$$

$$\boxed{V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R_D}{R_D} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Mismatch $\qquad$ Scale nodure $\qquad$ Geometric Variations (i.e., layout)

Bias Independent $\qquad$ $\downarrow$ small $\downarrow$ $I_D \downarrow \alpha \frac{W}{L}T$

$$\sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}$$

$V_{t0} + \gamma(f(V_{SB}))$
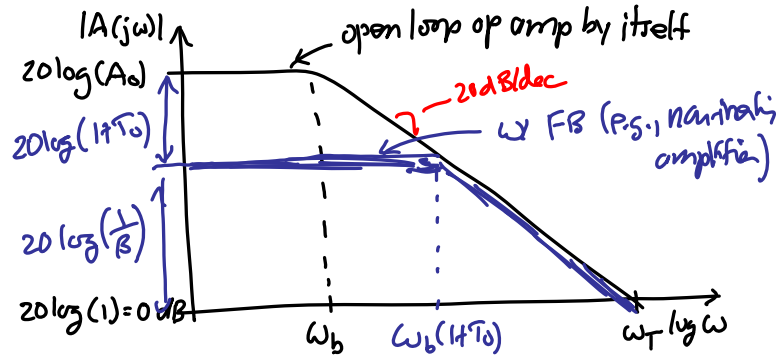
Easily possible for $\frac{\Delta R_D}{R_D} = (-)$; $\frac{\Delta(W/L)}{(W/L)} = (-)$

- Now, go through bipolar mismatch prepared material

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$.

In reality, the gain is given by: $A(s) = \dfrac{A_0}{1 + s/\omega_b}$

open loop op amp by itself

−20dB/dec

w FB (e.g., noninverting amplifier)

$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$

$\qquad\qquad (= 0dB)$

At ω_T:

$$|A(j\omega_T)| = 1 = \frac{A_0}{\sqrt{1 + \left(\frac{\omega_T}{\omega_b}\right)^2}}$$

$$[\omega_T \gg \omega_b] \Rightarrow \frac{A_0}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = A_0 \omega_b}$$
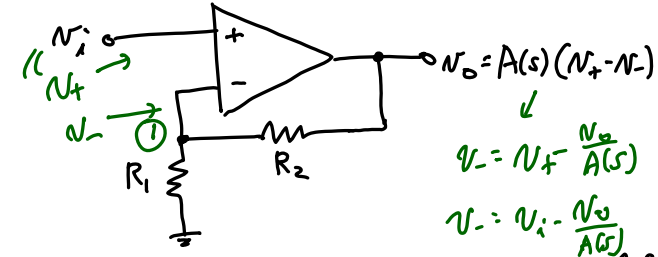
Gain-Bandwidth Product

for $\omega \gg \omega_b$:

$$A(s) \approx \frac{A_0}{\frac{s}{\omega_b}} = \frac{A_0 \omega_b}{s} = \frac{\omega_T}{s} = \frac{f_T}{f} \quad \left[\begin{array}{l}\text{Integrator w/ time}\\ \text{constant } \tau = \frac{1}{\omega_T}\end{array}\right]$$

The unity gain bandwidth f_T is usually specified on op amp data sheets. Knowing f_T, one can easily determine the op amp gain at a given frequency f.

Frequency Response of Closed loop Amplifiers

Example. Non-Inverting Amplifier

$v_o = A(s)(v_+ - v_-)$

$v_- = v_+ \dfrac{v_o}{A(s)}$

$v_- = v_i - \dfrac{v_o}{A(s)}$
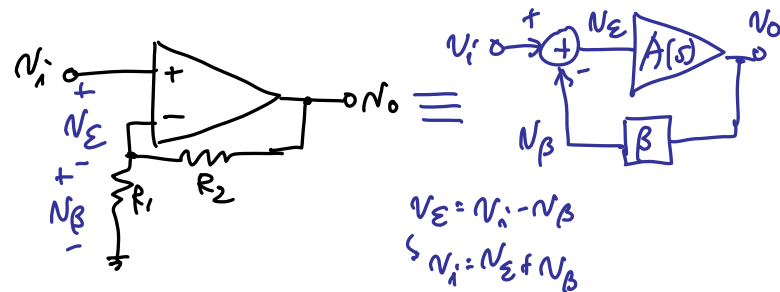
Find an expression for the gain as a function of frequency.

① Brute force derivation:

KCL ①: $\dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} \rightarrow \dfrac{v_o}{R_2} = v_- \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$\dfrac{v_o}{R_2} = \left(v_i - \dfrac{v_o}{A(s)}\right)\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) \rightarrow \dfrac{v_o}{v_i}(s) = \dfrac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)}\left(1 + \frac{R_2}{R_1}\right)}$

$\left[A(s) = \dfrac{A_0}{1 + \frac{s}{\omega_b}}\right] \Rightarrow \boxed{\dfrac{v_o}{v_i}(s) = \left(1 + \dfrac{R_2}{R_1}\right)\dfrac{1}{1 + \dfrac{s}{A_0\omega_b\left(\frac{R_1}{R_1 + R_2}\right)}}}$

② More insightful way to do this:

$v_\varepsilon = v_i - v_\beta$

$v_i = v_\varepsilon + v_\beta$

$\beta = \dfrac{N_\beta}{N_0} = \dfrac{R_1}{R_1 + R_2}$
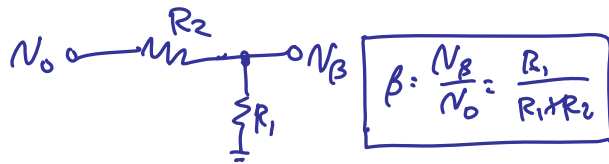
Recall from previous FB analysis:

$\dfrac{V_o}{V_i}(s) = \dfrac{A(s)}{1 + \beta A(s)}$

$\left[A(s) = \dfrac{A_0}{1 + \frac{s}{\omega_b}} \right] \longrightarrow \dfrac{N_o}{N_i}(s) = \dfrac{\frac{A_0}{1 + s/\omega_b}}{1 + \beta \left(\frac{A_0}{1 + s/\omega_b} \right)}$

if $\beta A_0 \gg 1 \longrightarrow \frac{1}{\beta}$

$\dfrac{N_o}{N_i}(s) = \dfrac{A_0}{1 + \beta A_0} \cdot \dfrac{1}{1 + \dfrac{s}{\omega_b(1 + \beta A_0)}}$

"closed loop dc gain" term

frequency shaping term

$T_0 = \beta A_0 \overset{\triangle}{=}$ "loop gain" at $\omega = 0$
(i.e., @ dc)

Plug in β:

$\dfrac{N_o}{N_i}(s) \cong \dfrac{1}{\beta} \cdot \dfrac{1}{1 + \dfrac{s}{\omega_b \beta A_0}} = \left[\left(1 + \dfrac{R_2}{R_1} \right) \dfrac{1}{1 + \dfrac{s}{\omega_b A_0 \left(\frac{R_1}{R_1 + R_2} \right)}} \right] = \dfrac{N_o}{N_i}(s)$

Observations:

① Closed loop DC gain $= \dfrac{A_0}{1 + \beta A_0} = \dfrac{A_0}{1 + T_0} \approx \dfrac{A_0}{T_0}$

i.e., the closed loop gain is reduced from the open loop gain by $1 + T_0 \longrightarrow$ show this on graph $\quad [T_0 \gg 1]$

② Alternatively, Closed loop DC gain $\approx \dfrac{A_0}{\beta A_0} = \dfrac{1}{\beta} \quad [T_0 \gg 1]$

③ ω_{-3dB} has increased from $\omega_b \longrightarrow \omega_b(1 + A_0 \beta) = \omega_b(1 + T_0)$

↳ To draw the Bode plot, just find the dc gain, draw a horizontal line across, then follow the open loop response after running into it!

④ Gain-BW Product $= \dfrac{A_0}{1 + \beta A_0} \omega_b(1 + \beta A_0) = A_0 \omega_b = \omega_T$

∴ the Gain-BW product remains the same for the open & closed loop FB cases!