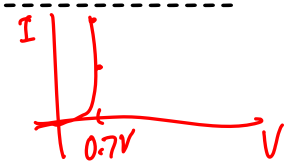
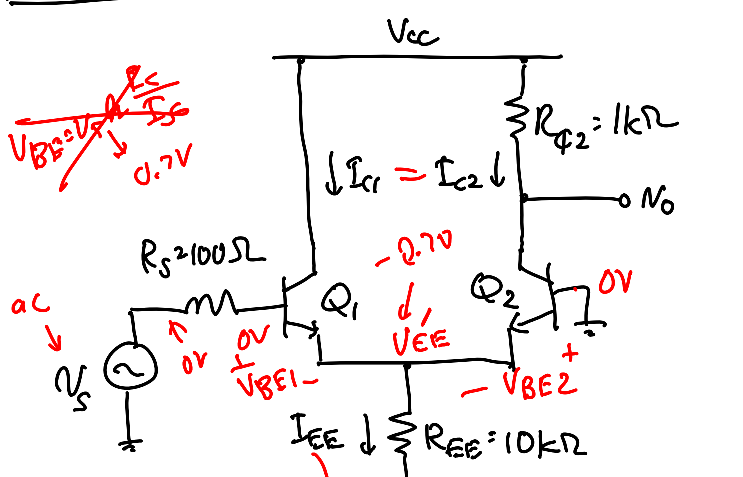
Lecture 5: MOS Inspection Analysis

- **Announcements:**
- Lab#1 is online
  - Labs start next week → go to your lab section
- **Lecture Topics:**
  - Multi-Tx Amplifier Examples
  - MOS Inspection Analysis
- ------------------------------------
- **Last Time:**
- Multi-TX inspection analysis
- Continue with this



$\boxed{\text{Inspection Analysis of a Multi-Transistor Ckt.}}$



Assume Q_1 & Q_2 identical.
Want R_i, R_o, gain.

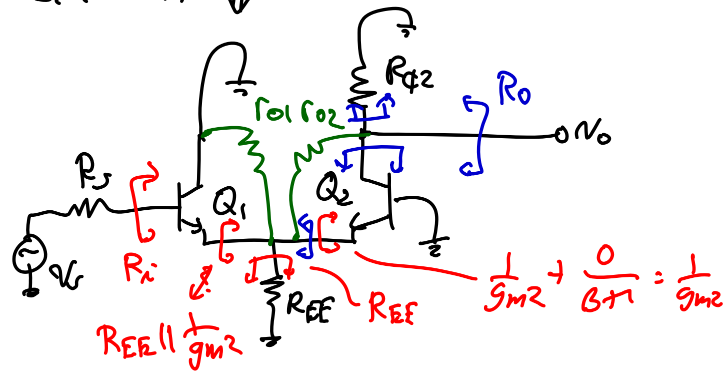
$I_{EE} = \frac{0.7 - V_{EE}}{R_{EE}}$

$V_{BE1} = V_{BE2} \rightarrow I_{C1} = I_{C2} = \frac{I_{EE}}{2}$

First, get DC operating pt.

$I_{C1} = I_{C2} = \frac{I_{EE}}{2} \rightarrow r_{\pi 1} = r_{\pi 2} = r_\pi$
$r_{o1} = r_{o2} = r_o$
$g_{m1} = g_{m2} = g_m$

S.S. ac ckt.



$R_i = r_\pi + (\beta+1)R_E$

$= r_{\pi 1} + (\beta+1)\left(\cancel{R_{EE}} \| \frac{1}{g_{m2}}\right)$ large $\| r_{o1} \| r_{o2}$ for completeness! (but usually they are big, so can be ignored!)

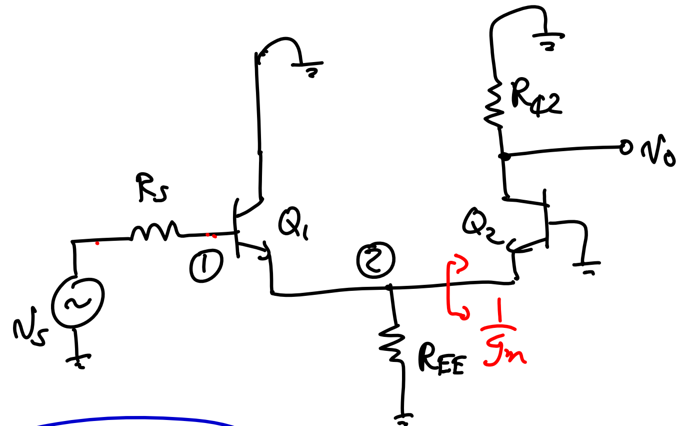
$= r_\pi + \frac{\beta+1}{g_{m2}}$

$= r_{\pi 1} + r_{\pi 2} = 2 r_\pi$

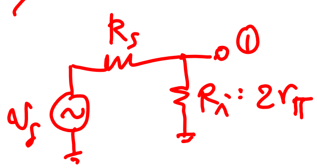
$R_o = R_{C2} \| r_{o2}\left(1 + \frac{g_{m2}\left(\frac{1}{g_{m1}}\right)}{1+0}\right) = R_{C2} \| 2r_{o2} \approx R_C$

$R_{EE} \| \left(\frac{1}{g_{m1}} + \frac{R_S}{\beta+1}\right) \| r_{o1} \| r_{o2}$ assume small

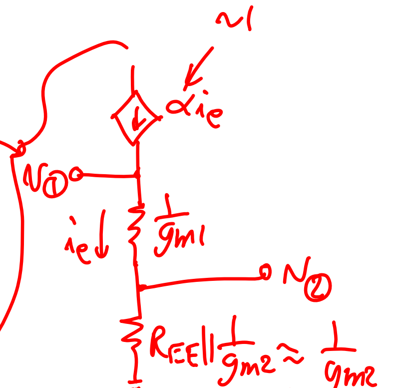
Assume r_{o1} & r_{o2} are very large.



$$\frac{v_o}{v_s} = \frac{v_①}{v_s}\cdot\frac{v_②}{v_①}\cdot\frac{v_o}{v_②}$$

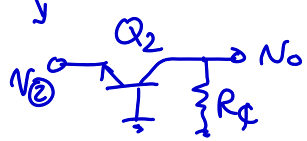


$$\frac{v_①}{v_s} = \frac{2r_\pi}{R_s+2r_\pi}$$



$R_{EE} \gg \frac{1}{g_{m2}}$

$$\frac{v_②}{v_①} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}}+\frac{1}{g_{m2}}} = \frac{1}{2}$$

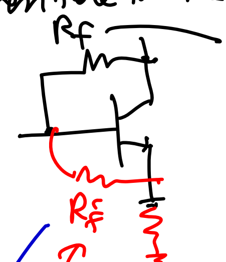


$$\frac{v_o}{v_②} = -\underbrace{G_{m2}}_{(-g_{m2})}R_C = g_{m2}R_C$$

$$G_{m2} = -\left(\underbrace{\frac{r_{\pi2}}{r_{\pi2}+\cancel{R_{B2}}}}_{1}\right)g_{m2} = -g_{m2}, \quad R_{B2} = 0$$

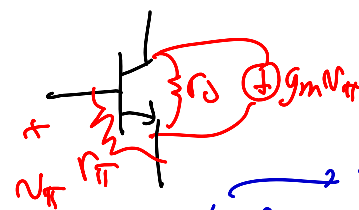
$$\boxed{\frac{v_o}{v_s} = \left(\frac{2r_\pi}{R_s+2r_\pi}\right)\frac{1}{2}g_{m2}R_C}$$

→ Reminder: Inspection analysis might not work when there is feedback:



feedback → not really inspectable (until the last week)

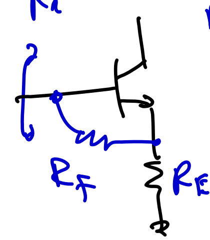
feedback → actually inspectable if recognize certain things



→ you can use it!

$$R_i = r_\pi' + (\beta'+1)R_E$$

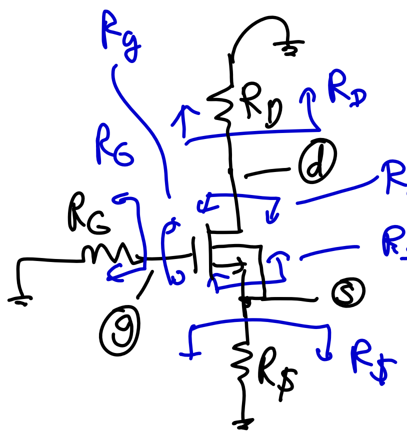
$$r_\pi \to r_\pi' = (r_\pi \| R_F)$$



MOS Xsistor Ckts.

⇒ for now, ignore Body effect (i.e., ignore g_{mb})

↳ use the same inspection formulas as bipolar, but use $\beta \to \infty$, $r_\pi = \frac{\beta}{g_m} \to \infty$



⇒ referring to the bipolar "Inspection Formula Sheet":

Bipolar		MOS
$R_b = \left(\frac{1}{g_m} + R_E\right)(\beta+1)$	$\xrightarrow{\beta\to\infty}$	$R_g = \infty$
$R_e = \frac{1}{g_m} + \frac{R_B}{\beta+1}$	$\xrightarrow{\beta\to\infty}$	$R_s = \frac{1}{g_m}$
$R_c = r_o\left[1 + \frac{g_m R_E}{1 + R_B/\underbrace{r_\pi}_{\infty}}\right]$	$\xrightarrow{\beta\to\infty}$	$R_d = r_o(1 + g_m R_S)$

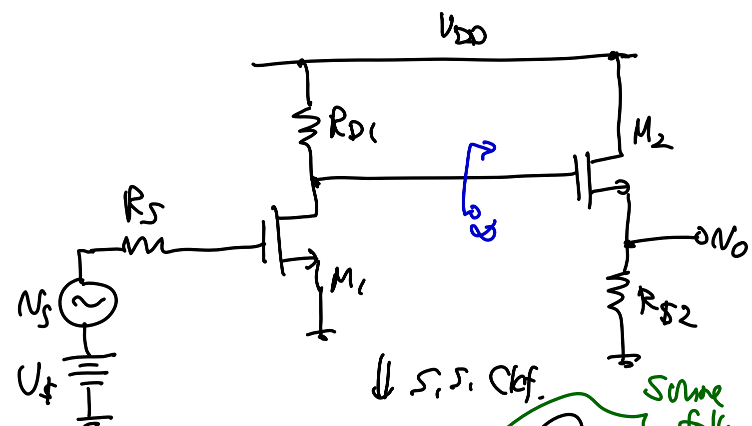
$$\frac{v_d}{v_g} = -G_m R_{d}, \quad G_m = \frac{g_m}{1 + g_m R_S}$$

$$\frac{v_d}{v_s} = -G_m R_{d}, \quad G_m = -g_m$$

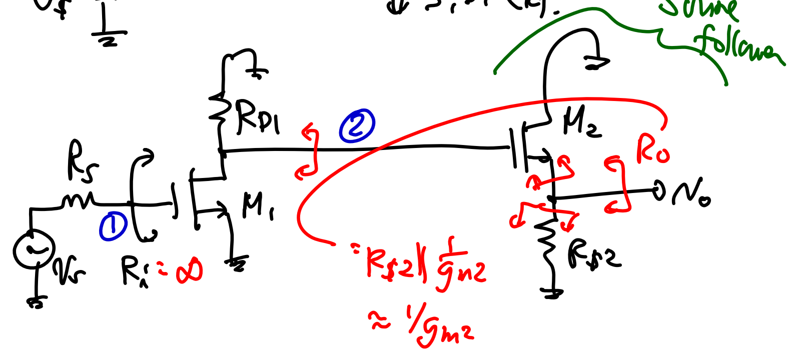
$$\frac{v_s}{v_g} = \frac{g_m R_S}{1 + g_m R_S} = \frac{R_S}{\frac{1}{g_m} + R_S}$$

MOS Inspection Analysis

Ex. Common-Source Common-Drain Cascode



⇓ s.s. Ckt.



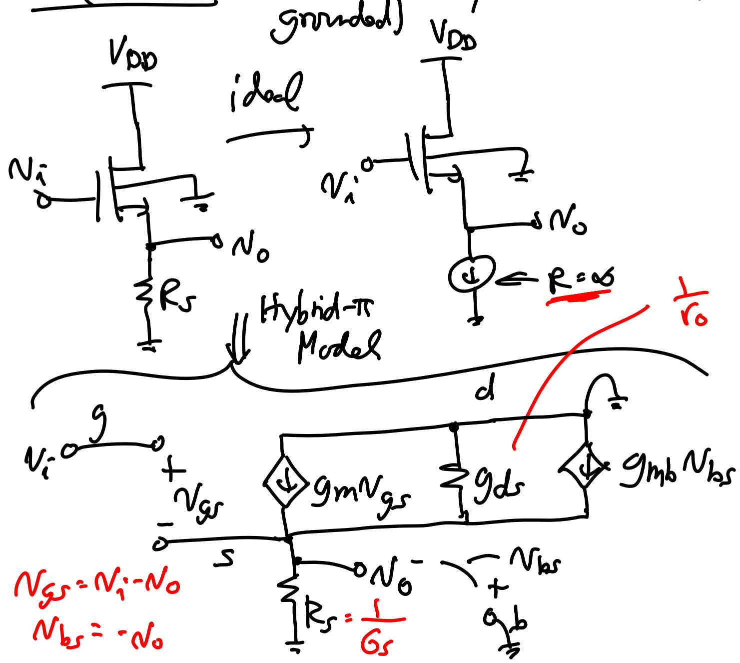
$$\frac{v_o}{v_s} = \frac{v_1}{v_s}\cdot\frac{v_2}{v_1}\cdot\frac{v_o}{v_2}$$

$$= (1)(-g_{m1}R_{D1})\left(\frac{R_{S2}}{\frac{1}{g_{m2}}+R_{S2}}\right) = \frac{v_o}{v_s}$$

Problem: Simulate via SPICE → the gain will be 80–90% of what is calculated using the problem is w/ g_{mb} in the source follower

this is the difference between bipolar & MOS hybrid-π models!

Source Follower: (w/ substrate grounded)



$$g_m(v_i - v_o) = v_o(g_{ds} + G_s + g_{mb})$$

$$\Rightarrow A_v = \frac{v_o}{v_i} = \frac{g_m}{g_m + g_{mb} + g_{ds} + G_s}$$

$R_s \to \infty \to G_s = 0$

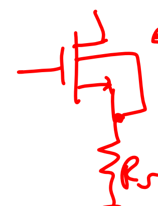
$g_{ds} \ll g_m + g_{mb}$

Body factor

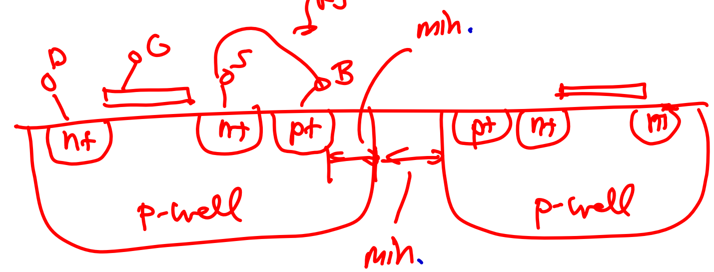
$$\boxed{A_v \simeq \frac{g_m}{g_m+g_{mb}} = \frac{1}{1+\eta}, \quad \eta = \frac{\gamma}{2\sqrt{V_{SB}+2\phi_f}}}$$

$\neq 1$

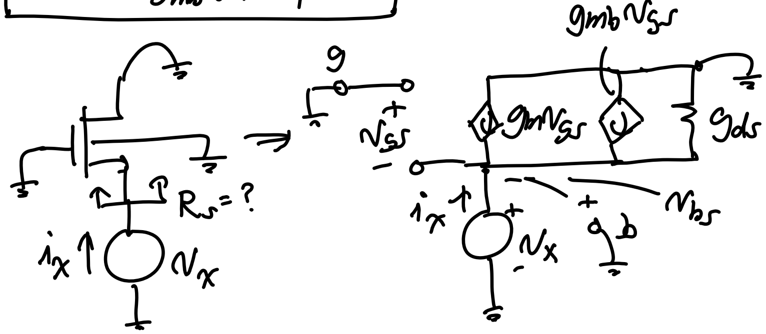
To make it '1', do this:



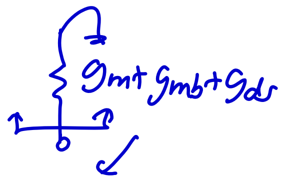
→ not always practical!



Effect of g_{mb} on Impedance



$$\begin{bmatrix} v_{gs} = -v_x = v_{bs} \\ v_{ds} = -v_x \end{bmatrix}$$



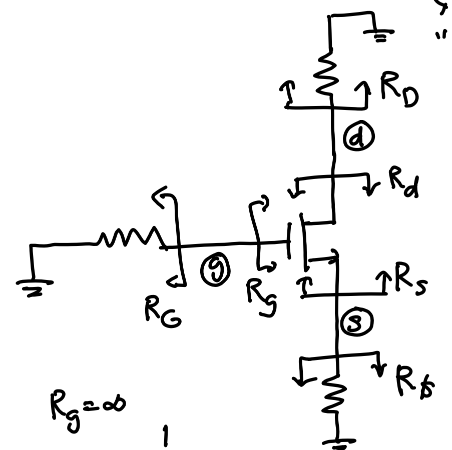
$$R_s = \frac{1}{g_m + g_{mb} + g_{ds}} = \frac{1}{g_m} \| \frac{1}{g_{mb}} \| r_o$$

$$\boxed{R_s \approx \frac{1}{g_m + g_{mb}}}$$

⇒ more extensive analysis shows that other inspection formulas change to accommodate a grounded body by replacing "g_m" in the numerators w/ "$g_m + g_{mb}$"

⇒ end up w/ the following: ↩ over

MOS Inspection Formulas w/ Substrate Grounded



→ only difference from "substrate tied to source" case is that g_m is replaced by $g_m + g_{mb}$ in some of the formulas → particularly ones where the source is involved!

$R_g = \infty$

$R_s = \dfrac{1}{g_m + g_{mb}}$

$R_d = r_o\left[1 + (g_m + g_{mb})R_S\right]$

$\dfrac{v_d}{v_g} = -G_m R_{d}$, $G_m = \dfrac{g_m}{1 + (g_m + g_{mb})R_S}$

$\dfrac{v_d}{v_s} = -G_m R_{d}$, $G_m = -(g_m + g_{mb})$

$\dfrac{v_s}{v_g} = \dfrac{g_m R_S}{1 + (g_m + g_{mb})R_S}$

Remark. When the substrate is tied to the source, $g_{mb} = 0$.