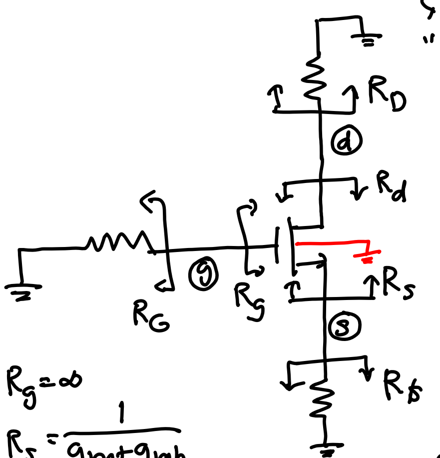
Lecture 6: Frequency Response Inspection Analysis

- Announcements:
- HW#2 due tomorrow at 8 a.m.
- HW#3 now online
- Lecture Topics:
  - Amplifier Bode plot
  - Open Circuit Time Constant (OCTC) Analysis
  - Frequency Response Inspection Analysis
- ----------------------------------
- Last Time:

MOS Inspection Formulas w/ Substrate Grounded

or as well as tied to V_{DD}



↳ only difference from "substrate tied to source" case is that g_m is replaced by g_m+g_{mb} in some of the formulas

particularly ones where the source is involved!

$R_g = \infty$

$R_s = \frac{1}{g_m+g_{mb}}$

$R_d = r_o\left[1+(g_m+g_{mb})R_S\right]$

$$\frac{v_s}{v_g} = \frac{g_m R_S}{1+(g_m+g_{mb})R_S}$$

$$\frac{v_d}{v_g} = -G_m R_{(d)} \quad , \quad G_m = \frac{g_m}{1+(g_m+g_{mb})R_S}$$

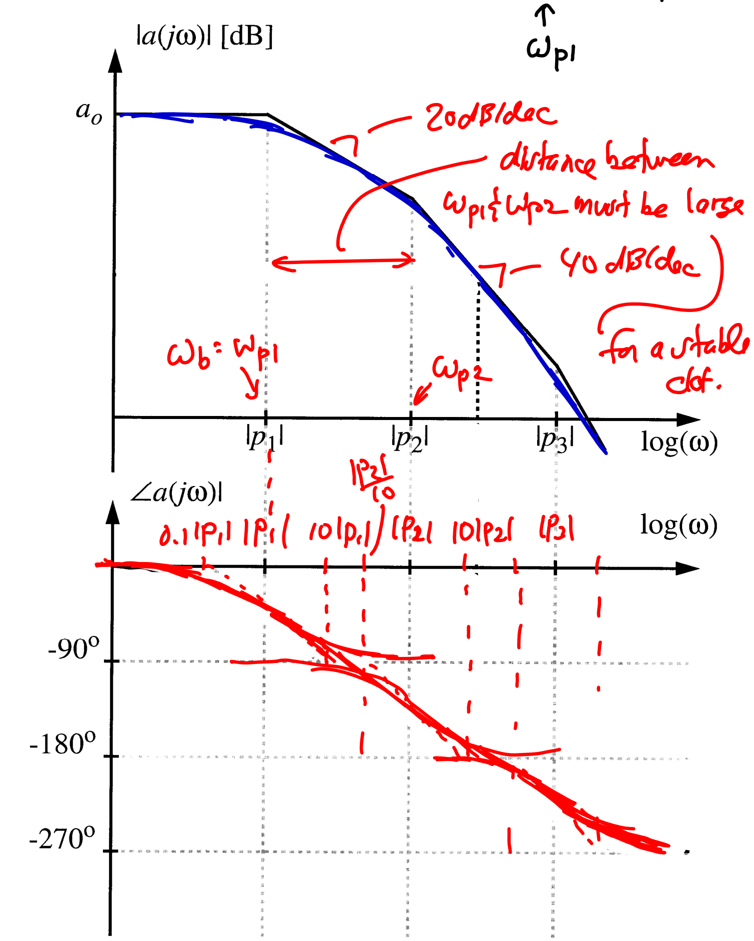
$$\frac{v_d}{v_s} = -G_m R_{(d)} \quad , \quad G_m = -(g_m+g_{mb})$$

Remark. When the substrate is tied to the source, $g_{mb} = 0$.

Poles & Zeros → Bode Plots

$$A(s) = \frac{v_o}{v_i}(s) = \frac{a_o}{\left(1+\frac{s}{\omega_b}\right)\left(1+\frac{s}{\omega_{p2}}\right)(1+\cdots)}$$

↑ ω_{p1}

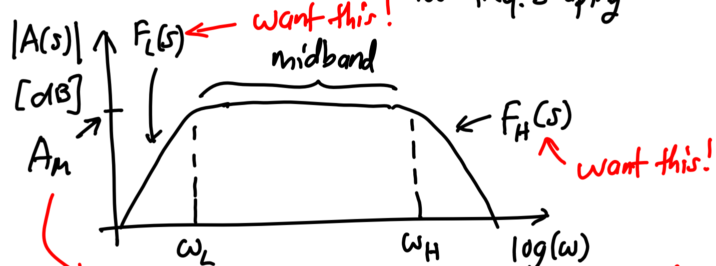


Freq. Response

Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s) \qquad s = j\omega$$

midband gain (A_M); low freq. shaping ($F_L(s)$); high frequency shaping ($F_H(s)$)



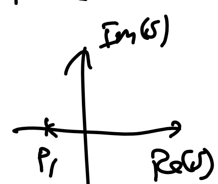
we've already found this using small-signal analysis

High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$



from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo}$$

where C_j are capacitors in the H.F. ckt., i.e., small ones

$R_{jo} \hat{=}$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$

(ii) $F_H(s) \cong \frac{1}{1 + \frac{s}{\omega_H}}$

(iii) $b_1 \cong \frac{1}{\omega_{p1}} \rightarrow \boxed{\omega_H = \omega_{p1} \cong \frac{1}{b_1} = \frac{1}{\sum_j \tau_{jo}} = \frac{1}{\sum_j C_j R_{jo}}}$

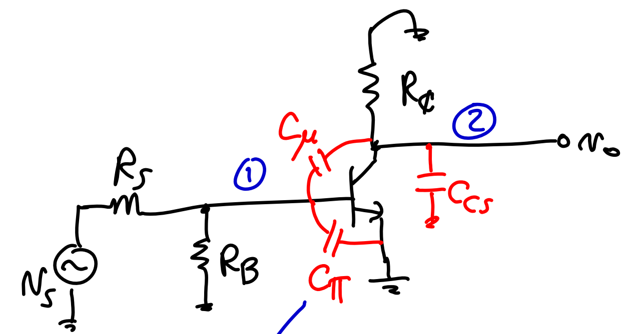
When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

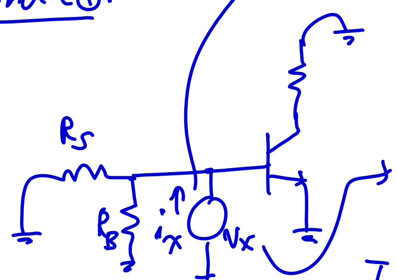
(just FYI)

- Now, go to inspection formula sheet and go over how to use the frequency response parts

Example H.F. Analysis (C.E. Ckt.)



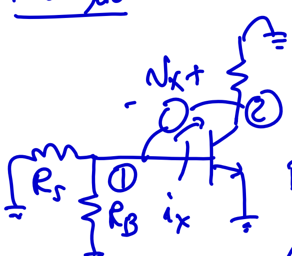
Find τ_1:



$R_1 = \frac{v_x}{i_x} = R_S \| R_B \| r_\pi$

$\tau_1 = C_\pi (R_S \| R_B \| r_\pi)$

Find $\tau_{\mu o}$:



$R_{\mu o} = \frac{v_x}{i_x} = R_1 + R_2 + g_m R_1 R_2$

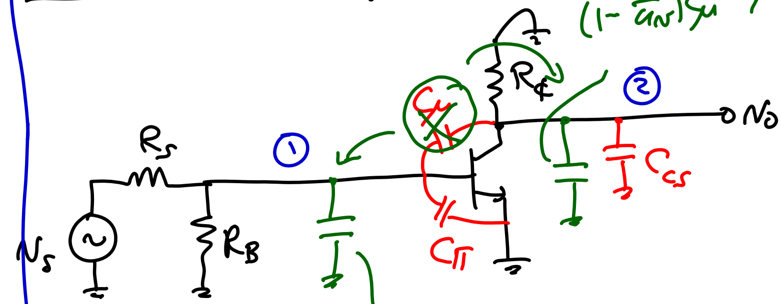
$R_{\mu o} = (R_S \| R_B \| r_\pi) + r_o \| R_C$
$+ g_m (R_S \| R_B \| r_\pi)(r_o \| R_C)$

$\tau_{\mu o} = C_\mu R_{\mu o}$

Find τ_2: $\tau_2 = C_{CS} R_C \leftarrow (r_o \| R_C)$

$$\omega_H = \frac{1}{\tau_1 + \tau_2 + \tau_{\mu o}}$$

Now, use Miller's Theorem:



$C_M = (1 - a_v) C_\mu = (1 + g_m R_C) C_\mu$

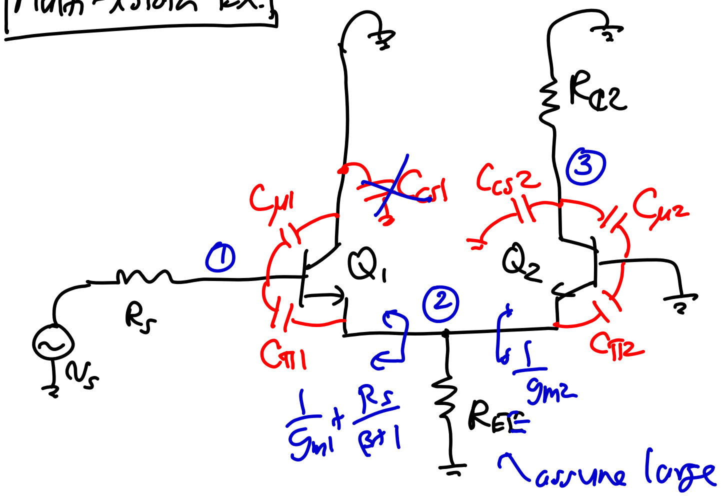
$\tau_1 = (R_S \| R_B \| r_\pi)(C_\pi + C_\mu (1 + g_m R_C))$

$\tau_2 = R_C (C_\mu + C_{CS})$

$$\omega_H = \frac{1}{\tau_1 + \tau_2}$$

Same as

Multi-Xsistor Ex.



$$\tau_{1} : (R_s \| 2r_\pi) C_{\mu 1}$$

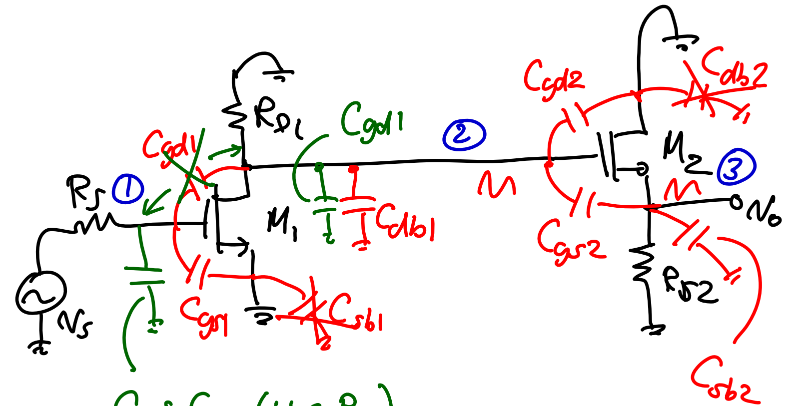
$$\tau_{\pi 1} = C_{\pi}\left(r_\pi \,\Big\|\, \frac{R_s + \frac{1}{g_{m2}}}{1 + g_{m1}\left(\frac{1}{g_{m2}}\right)}\right)$$

$$\tau_{2} : C_{\pi 2}\left(\left(\frac{1}{g_{m1}} + \frac{R_s}{\beta+1}\right) \Big\| \frac{1}{g_{m2}} \Big\| R_{EE}\right)$$

$$\tau_{3} = (C_{cs2} + C_{\mu 2}) R_C$$

$$\omega_H = \frac{1}{\tau_{1} + \tau_{2} + \tau_{3} + \tau_{\pi 1}}$$

MOS Two-Stage Amplifier



$$C_M = C_{gd1}(1 + g_{m1} R_{D1})$$

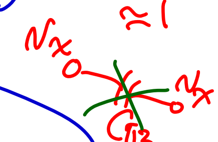
$$\tau_{1} = [C_{gs1} + C_{gd1}(1 + g_{m1} R_{D1})] R_s$$

$$\tau_{2} = [C_{gd1} + C_{db1} + C_{gd2}](r_{o1} \| R_{D1})$$

$$\tau_{3} = C_{sb2}\left(\frac{1}{g_{m2} + g_{mb2}} \Big\| R_{S2}\right)$$

because no signal across $C_{\pi 2}$ w/ 2nd stage gain ≈ 1

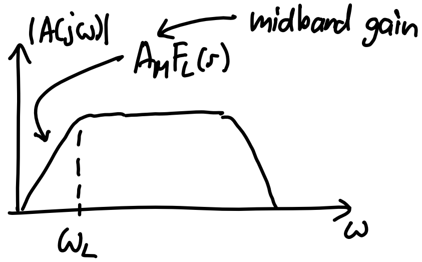
$$\cancel{\tau_{gs2} = C_{gs2}\left(\frac{R_{D1} + R_{S2}}{1 + (g_{m2} + g_{mb2}) R_{S2}}\right)}$$



$$\omega_H = \frac{1}{\tau_{1} + \tau_{2} + \tau_{3} + \tau_{gs2}}$$

Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall:



In general, for the low freq. response:

$$F_L(s) = \frac{s^{n_L} + d_1 s^{(n_L-1)} + \cdots}{s^{n_L} + e_1 s^{(n_L-1)} + \cdots}, \quad n_L = \#\text{poles} = \#\text{zeros}$$

We can express the coefficient e_1 by:

$$e_1 = \omega_{p1} + \omega_{p2} + \cdots + \omega_{pn_L}$$

Similar analysis to that used for OCTC...

For the case of a dominant pole:
↳ i.e., the highest freq. pole

$$F_L(s) \cong \frac{s}{s+\omega_L} = \frac{s}{s+e_1} \rightarrow e_1 \cong \omega_{p1} = \omega_L$$

$$\therefore \boxed{\omega_L \cong e_1 = \sum_j \omega_{pj} = \sum_j \frac{1}{C_j R_{js}} = \sum_j \frac{1}{\tau_{js}}}$$

where $C_j \triangleq$ various large (>10nF) capacitors in the ckt. (e.g., the bypass caps.)

$R_{js} \triangleq$ driving point resistance seen between the terminals of C_j determined with:

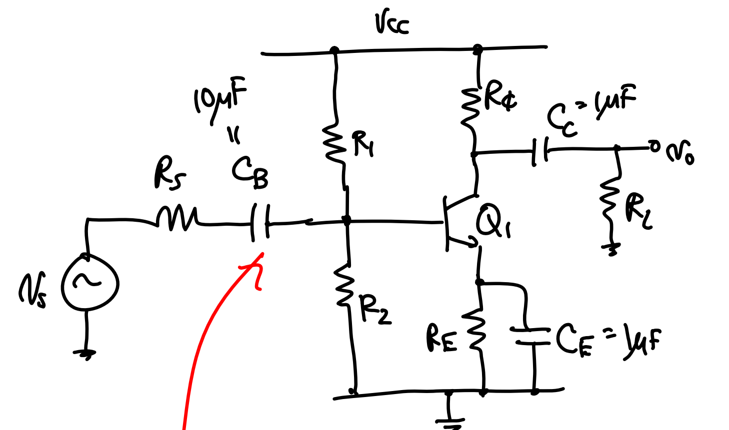
For reading, can go to Sedra & Smith

1. all large capacitors short-circuited, except C_j, which is replaced by the test voltage source for R determination
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. open all H.F. capacitors (i.e., small caps in the pF range, or <1nF)

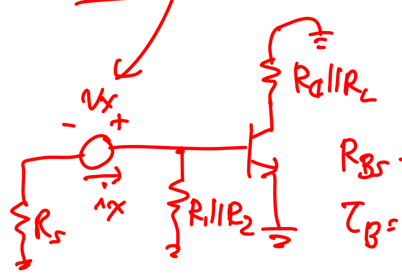
Again, for the case where there are no dominant poles, a reasonable approximation is:

$$\omega_L \cong \sqrt{\omega_{p1}^2 + \omega_{p2}^2 - 2\omega_{z1}^2 - 2\omega_{z2}^2}$$

Ex: Determine the L.F. response of the C.E. Amplifier



(a) τ due to C_B: short ckt. C_C & C_E



$R_{BS} = \frac{v_x}{i_x}$ = resistance in series w/ v_x

$R_{BS} = R_S + R_1 \| R_2 \| r_\pi$

$\tau_B = R_{BS} C_B = (R_S + R_1 \| R_2 \| r_\pi) C_B$

$$\omega_{pB} = \frac{1}{\tau_B} = \frac{1}{(R_s + R_1 \| R_2 \| r_\pi) C_B}$$