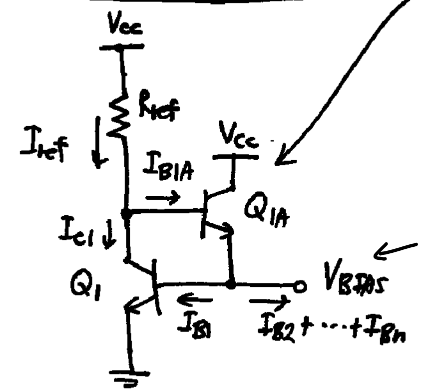
Lecture 9: Supply & Temperature Independent Biasing

- Announcements:
  - Lab#1's due next week in your lab section (on Friday, Feb. 29)
  - Lab#2 will be online this week
    - This is a hardware lab, so you will need to use the lab to make measurements
    - You are all being added to the access list for 353 Cory
  - Those taking 240A will soon get additional HW assignments to supplement the regular ones
- Lecture Topics:
  - Widlar Current Source
  - Supply & Temperature Independent Biasing
  - Output Swing (Headroom)
  - High Swing Current Sources
- ------------------------------------
- Last Time:
- Reviewed current sources using prepared lecture material
- Now finish this (with Widlar current sources)

over

To reduce the error term, use a

$\boxed{\text{Buffered } V_{BIAS}\text{-Generator}}$



Add a buffer Xsistor to attenuate base currents from Xsistor current sources.

This can now drive the base currents of many bipolar transistor current sources (i.e., active loads).

$$I_{ref} = I_{c1} + I_{B1A}$$

$$I_{B1A} = \frac{I_{B1}+I_{B2}+\cdots I_{Bn}}{\beta+1} = \frac{nI_{c1}}{\beta(\beta+1)}$$

[Assuming identical Xsistors]

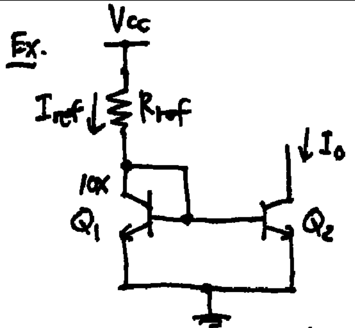
$$I_{ref} = I_{c1}\left(1+\frac{n}{\beta(\beta+1)}\right)$$

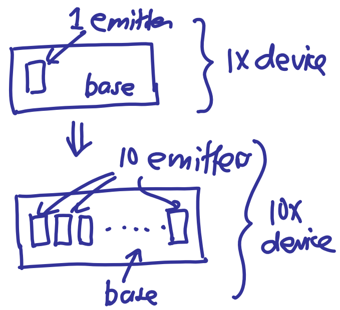
$$\rightarrow \boxed{I_o = I_{c2} = \frac{I_{ref}}{1+\frac{n}{\beta(\beta+1)}} \approx I_{ref}\left(1-\frac{n}{\beta^2}\right)}$$

Note: Now, $\boxed{I_{ref} = \frac{V_{cc}-2V_{BE(on)}}{R_{ref}}}$

Problem: For power savings reasons, oftentimes very small bias currents are needed, on the order of 5μA. This might force too large an R_{ref} in the above bipolar V_{BIAS} generator.

Ex.





i.e., larger emitter area: Layout like this to get more accurate ratio.

If Q_1 is 10X larger than Q_2.

$\therefore I_{S1} = 10 I_{S2} \rightarrow I_o \cong I_{ref}/10$

$\therefore I_o = \dfrac{(V_{CC} - V_{BE(on)})}{10 R_{ref}} \rightarrow R_{ref} = \dfrac{V_{CC} - V_{BE(on)}}{10 I_o}$

this helps to lower R_{ref}, but is it enough?

Ex. $I_o = 5\mu A$, $V_{CC} = 30V$

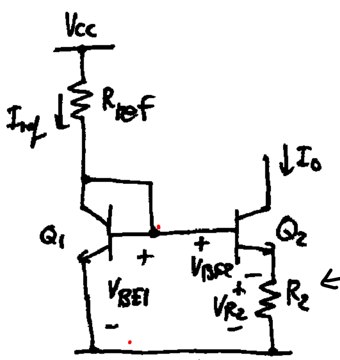
$R_{ref} \cong \dfrac{30}{5\mu} = \underline{600k\Omega}$ ← That's way too big!

(Yes, there's only one of them on the chip, but this takes up too much space!)

The Low Current Solution: $\boxed{\text{Widlar Current Source}}$

⇒ scale $I_{C2} = I_o$ by reducing V_{BE2} (relative to V_{BE1}):

Do this by emitter degenerating Q_2 via R_2



$$V_{BE1} = V_{BE2} + V_{R_2} = V_{BE2} + \frac{1}{\alpha} I_{C2} R_2 \approx V_{BE2} + I_{C2} R_2$$

$$I_{C2} R_2 = V_{BE1} - V_{BE2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}}$$

$$I_{C2} R_2 = V_T \ln \frac{I_{C1}}{I_{C2}}$$ [Assuming Q_1 & Q_2 are matched.]

$$\boxed{I_o R_2 = V_T \ln \frac{I_{ref}}{I_o}}$$

Rule of Thumb:

$V_{R_2} = I_{C2} R_2$	$I_{C2} = I_o$
18mV	$\frac{1}{2} I_{ref}$
42mV	$\frac{1}{5} I_{ref}$
60mV	$\frac{1}{10} I_{ref}$
120mV	$\frac{1}{100} I_{ref}$

5μA example again

Ex. scale by 100X using Widlar source

$V_{BE1} - V_{BE2} = 120mV \rightarrow R_2 = \dfrac{120mV}{5\mu A} = \underline{24k\Omega}$

$I_{ref} = 500\mu A \rightarrow R_{ref} = \dfrac{30V}{500\mu A} = \underline{60k\Omega}$

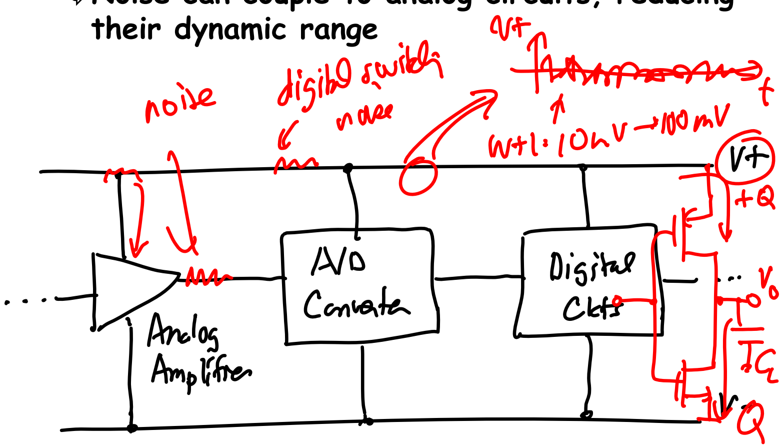
More reasonable than 600kΩ before.
If want smaller, scale by 1000X instead.

Another advantage of the Widlar: larger R_o ∴ a more ideal current source:

$R_o = r_{o2}(1 + g_{m2} R_2)$

Supply & Temperature Independent Biasing

- Why is it necessary?
- For battery-operated systems, battery voltages vary over time
  - Amplifier gains change
  - Power consumption changes
  - Frequency of oscillators changes
  - In summary: long-term stability degrades
  - Large uncertainty in biasing translates to overdesign that wastes power
- Same issues as above when temperature varies with time
- Short-term supply variations
  - In mixed signal circuits, i.e., both analog and digital together, digital switching generates noise on the supply lines
  - Noise can couple to analog circuits, reducing their dynamic range

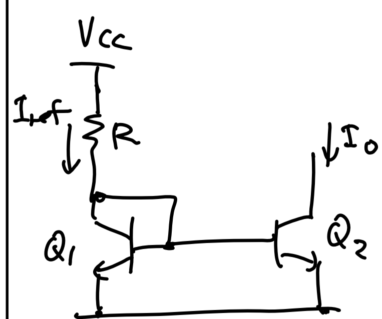


Definition. Sensitivity of Y to X

$$S_X^Y = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{X}{Y}\frac{\Delta Y}{\Delta X} = \frac{X}{Y}\frac{\partial Y}{\partial X}$$

For supply independence, we want $S_{V_{CC}}^{I_O} = 0$.

Simple Current Source



Neglecting base currents:

$$I_O = I_{ref} = \frac{V_{CC} - V_{BE(on)}}{R}$$

$$I_O \approx \frac{V_{CC}}{R} \quad [V_{CC} >> V_{BE(on)}]$$

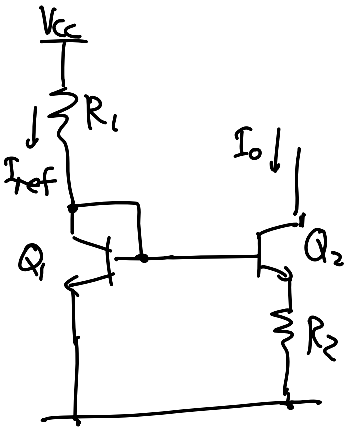
Thus:

$$S_R^{I_O} = \frac{R}{I_O}\frac{\partial I_O}{\partial R} = \frac{R^2}{V_{CC}}\left(-\frac{V_{CC}}{R^2}\right) \Rightarrow S_R^{I_O} = -1$$

$$S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{\partial I_O}{\partial V_{CC}} = R\left(\frac{1}{R}\right) \Rightarrow S_{V_{CC}}^{I_O} = 1$$

∴ a 10% change in V_{CC} leads to 10% change in I_O!
(terrible!)

$\boxed{\text{Widlar Current Source}}$ (Any better?)



$$\underbrace{V_T \ln \frac{I_{ref}}{I_o}}_{(\ln I_{ref} - \ln I_o)} = I_o R_2$$

↓ Differentiate w/r to Vcc:

$$V_T\left(\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}} - \frac{1}{I_o}\frac{\partial I_o}{\partial V_{cc}}\right) = R_2 \frac{\partial I_o}{\partial V_{cc}}$$

↓ math

$$\frac{\partial I_o}{\partial V_{cc}} = \frac{\frac{V_T}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}}}{\left(R_2 + \frac{V_T}{I_o}\right)}$$

$$\therefore S^{I_o}_{V_{cc}} = \frac{V_{cc}}{I_o}\frac{\partial I_o}{\partial V_{cc}} = \frac{V_T \underbrace{\left(\frac{V_{cc}}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}}\right)}_{S^{I_{ref}}_{V_{cc}}}}{I_o R_2 + V_T}$$

$$\Rightarrow \boxed{S^{I_o}_{V_{cc}} = \left(\frac{1}{1+\frac{I_o R_2}{V_T}}\right) S^{I_{ref}}_{V_{cc}}}$$

Since $I_{ref} = \frac{V_{cc} - V_{BE(on)}}{R_1} \approx \frac{V_{cc}}{R_1} \Rightarrow S^{I_{ref}}_{V_{cc}} \approx 1$

$$\therefore \boxed{S^{I_o}_{V_{cc}} = \frac{1}{1+\frac{I_o R_2}{V_T}}}$$

For $I_{ref} = 1mA$, $I_o = 10\mu A$, $R_2 = 11.9k\Omega$, then

10% Δ in V_{cc} → 1.3% Δ in I_o

(better than simple current source)

How can we do better? → Use another voltage reference:

✓ ① V_{BE} → base-emitter junction voltage

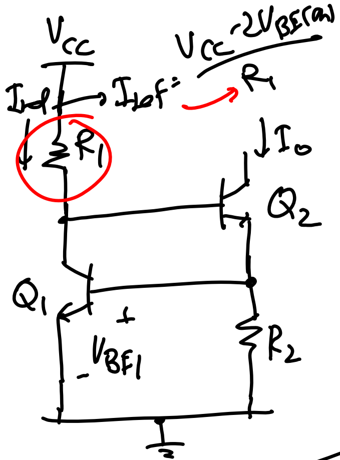
② V_Z → zener diode

③ V_t → threshold voltage (MOS)

✓ ④ $V_T = \frac{kT}{q}$ → thermal voltage

✓ ⑤ E_G → bandgap

V_{BE}-Referenced Biasing



$$I_{ref} = \frac{V_{CC} - 2V_{BE(on)}}{R_1}$$

$$I_o = \frac{V_{BE1}}{R_2} = \frac{V_T}{R_2}\ln\frac{I_{ref}}{I_{S1}}$$

$$\frac{\partial I_o}{\partial V_{CC}} = \frac{V_T}{R_2}\left(\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}} - \frac{1}{I_{S1}}\frac{\partial I_{S1}}{\partial V_{CC}}\right)$$

$$S_{V_{CC}}^{I_o} = \frac{V_{CC}}{I_o}\frac{\partial I_o}{\partial V_{CC}}$$

$$= \frac{V_{CC}}{\ln(I_{ref}/I_{S1})}\left(\underbrace{\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}}}_{=\frac{1}{V_{CC}}} - \frac{1}{I_{S1}}\frac{\partial I_{S1}}{\partial V_{CC}}\right)$$

$$S_{V_{CC}}^{I_{ref}} \approx 1 = \frac{V_{CC}}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}}$$

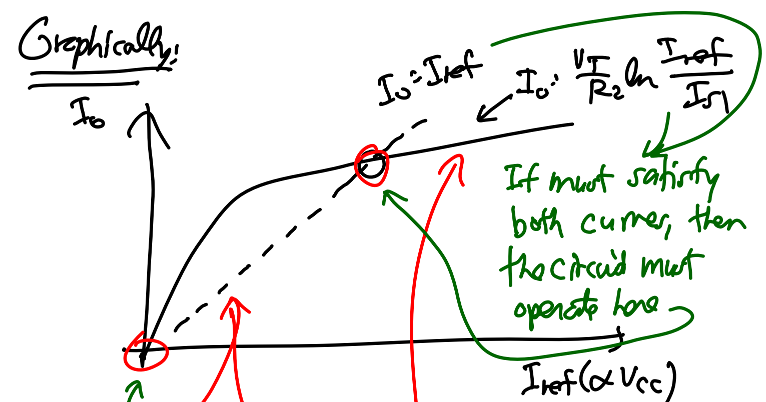
Problem: I_{ref} still dependent on V_{CC}

$$S_{V_{CC}}^{I_o} = \frac{1}{\ln(I_{ref}/I_{S1})}\left[1 - \underbrace{\frac{V_{CC}}{I_{S1}}\frac{\partial I_{S1}}{\partial V_{CC}}}_{=0}\right] = \frac{1}{\ln(I_{ref}/I_{S1})}$$

If we can eliminate this '1', then $S_{V_{CC}}^{I_o} = 0$ → need to eliminate I_{ref} dependence on V_{CC}

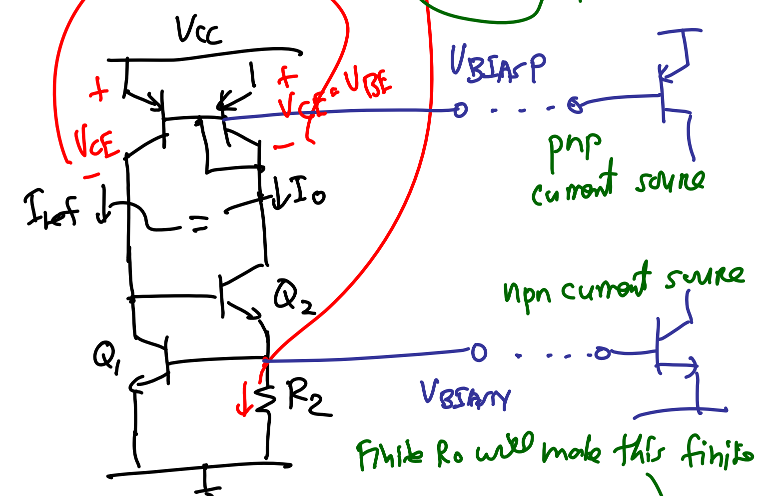
Solution: Derive I_{ref} independently of V_{CC}.
↳ use self-biasing

Graphically:



If must satisfy both curves, then the circuit must operate here

Need to bootstrap out of this point

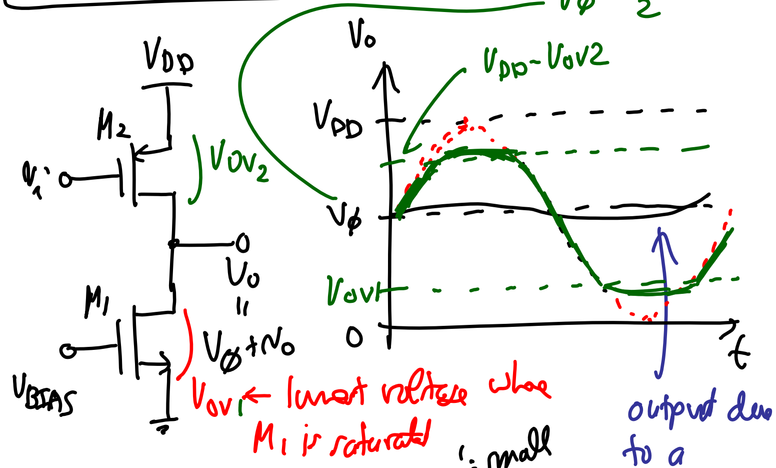


Finite Ro will make this finite.

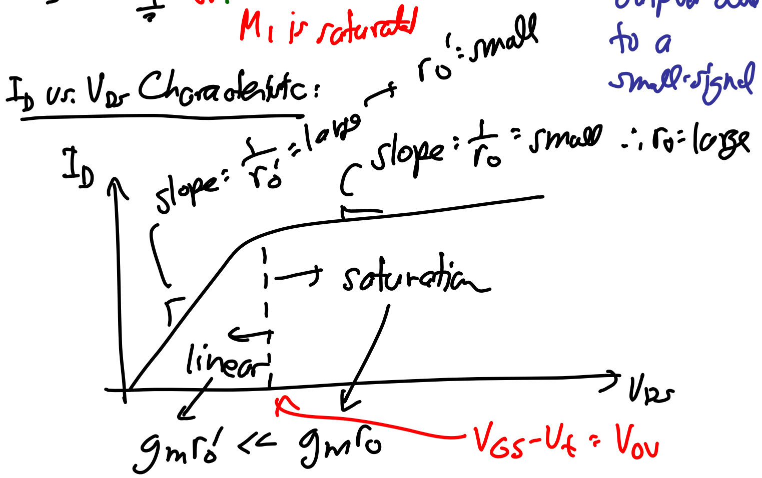
This is pretty darn sad! $S_{V_{CC}}^{I_{ref}} \to S_{V_{CC}}^{I_o} \cong 0$

- 240A folks: read Gray & Meyer
  - Sections 4.4.2 through 4.4.3
  - These cover supply and temperature independent biasing, including bandgap references
  - Can also read Razavi, Chpt. 11, on bandgap references

$\boxed{\text{Issue: Output Swing (Headroom)}}$



I_D vs. V_{DS} Characteristic:



Ex: Cell Phone