

Ideal Voltage Amplifier

→ ideal when  $\frac{V_o}{V_s} = A_{nr}$ ; i.e., when source and load R's do not influence the gain of the amplifier.

For this to occur, the voltage division at the input & output must be eliminated.  
This happens when:

$R_i = \infty$  } These resistance values define an  
 $R_o = 0$  } ideal voltage amplifier.

We'll look at other amplifier types later.

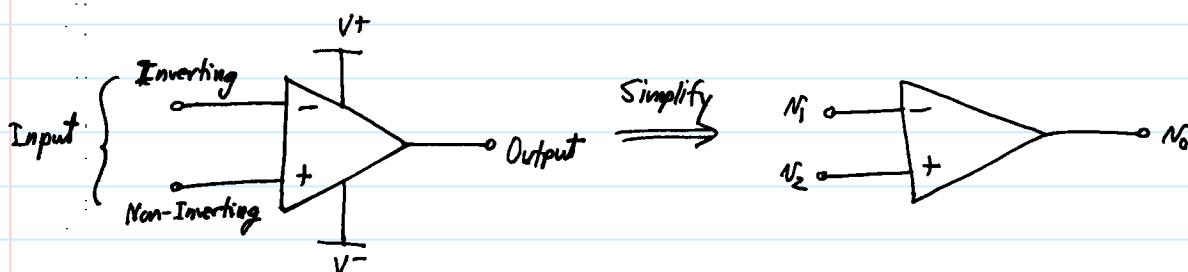
This, then, naturally leads us to:

Ideal Operational Amplifiers (Op Amps)

The work horse of analog electronics → combinations of op amps w/ feedback components allow the implementation of analog computers, sampled-data systems, analog filters, A/D Converters, DAC's, instrumentation amplifiers

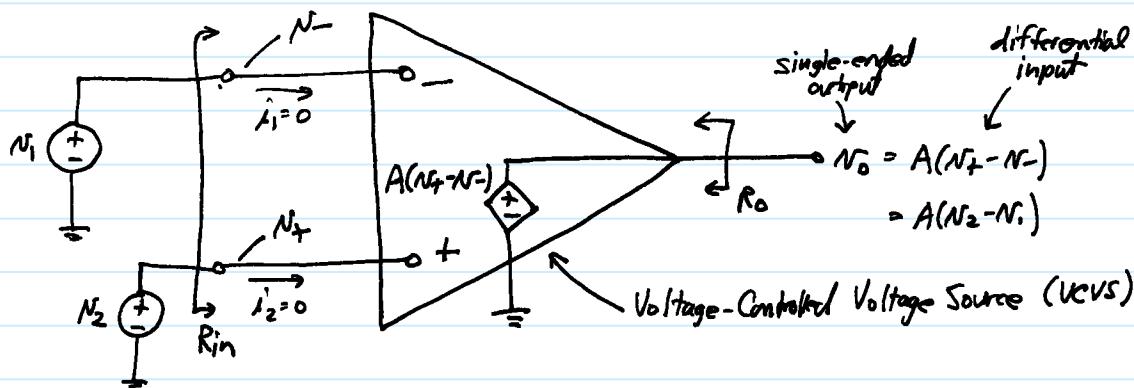
In general,

have a minimum of 5 terminals:



Perhaps the best way to define an op amp is thru its equivalent ckt:

Equivalent Ckt. of an Ideal Op Amp:



Properties of Ideal Op Amps:

$$\textcircled{1} \quad R_{in} = \infty \quad \xrightarrow{\text{leads to}} \quad \textcircled{4} \quad i_+ = i_- = 0$$

$$\textcircled{2} \quad R_o = 0$$

$$\textcircled{3} \quad A = \infty \quad \xrightarrow{\text{leads to}} \quad \textcircled{5} \quad V_+ = V_- \text{, assuming } N_o = \text{finite}$$

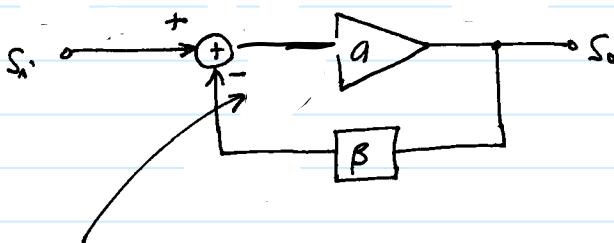
↳ Why? Because for  $\infty(V_+ - V_-) \Rightarrow V_o = \text{finite}$

$$\therefore \underbrace{V_+ - V_-}_{\frac{N_o}{\infty}} = 0 \rightarrow V_+ = V_- \Rightarrow \begin{array}{l} \text{virtual short ckt.} \\ (\text{virtual ground}) \end{array}$$

Big assumption! ( $N_o = \text{finite}$ )

How can we assume this?  $\Rightarrow$  only when there is an appropriate negative feedback path!

### Negative Feedback



where  $S$  could be a current, voltage, displacement, etc., ...

Negative feedback acts to oppose or subtract from input.

$$\left. \begin{array}{l} S_o = aS_i \\ S_o = S_i - \beta S_o \end{array} \right\} \Rightarrow S_o = a(S_i - \beta S_o) \quad \text{overall transfer function} \\ S_o(1 + a\beta) = aS_i \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1 + a\beta}}$$

$$[a \rightarrow \infty] \Rightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} > \text{finite!}$$

$$\therefore S_o = \frac{1}{\beta} S_i = \text{finite} \checkmark$$

(when there is neg. FB around the amplifier)

### In Summary:

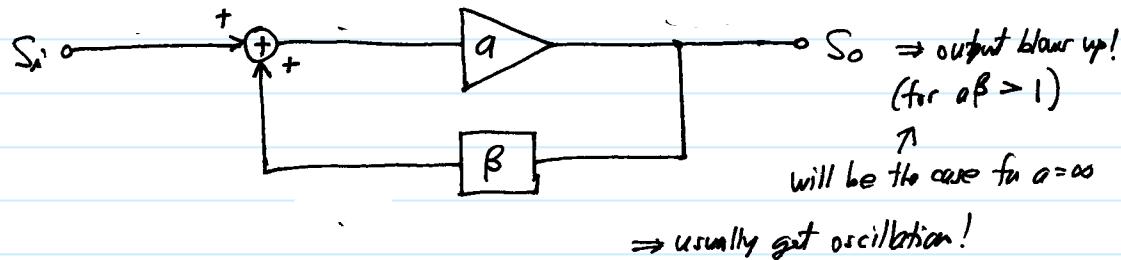
① Neg. FB can insure  $S_o = \text{finite}$  even with  $a = \infty$ .  
Overall

② Gain dependent (or overall T.F.) dependent only on external components. (e.g.,  $\beta$ )

③ Overall (closed-loop) gain  $\frac{S_o}{S_i}$  is independent of amplifier gain  $a$ .

↖ very important!  $\Rightarrow$  as you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain.  
i.e., if you're shooting for  $a = 50,000$ , you might get 47,000 or 52,000 instead.

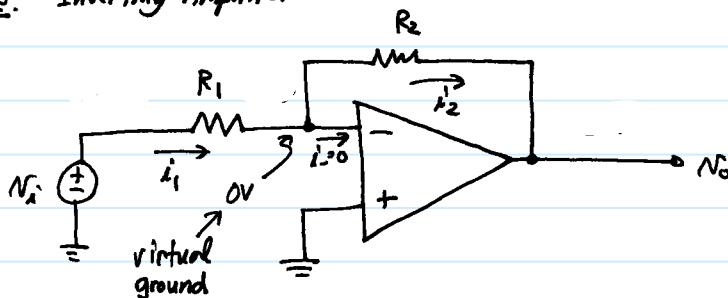
Contrast w/ **Positive Feedback**



Thus, for a bounded, controllable function, need negative FB around an op-amp.

### Op Amp Ckt's.

#### Example. Inverting Amplifier

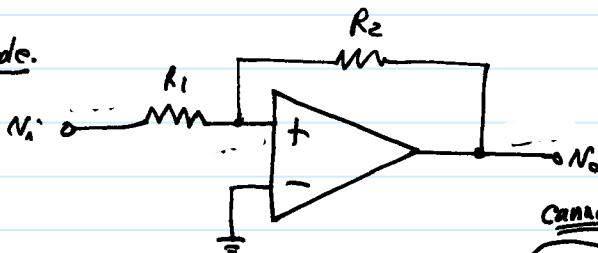


- ① Verify that there is negative FB. ✓
- ②  $\therefore V_{out} = \text{finite} \rightarrow V_+ = V_- \rightarrow$  node attached to (-) terminal is virtual ground
- ③  $i_- = 0 \therefore i_1 = i_2$

$$\left. \begin{aligned} i_1 &= \frac{V_{in} - 0}{R_1} = \frac{V_{in}}{R_1} = i_2 \\ V_{out} &= 0 - i_2 R_2 = -i_2 R_2 \end{aligned} \right\} \Rightarrow V_{out} = -\left(\frac{V_{in}}{R_1}\right) R_2 = -\frac{R_2}{R_1} V_{in} \therefore \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

Note: Gain dependent only on  $R_1$  &  $R_2$  (external components), not on the op-amp gain.

#### Example.



- ① Verify that there is neg. FB X

$V_{out} = L^+$  or  $L^-$  depending on initial condns.

Cannot analyze using ideal op-amp method!

$\therefore V_{out} \neq \text{finite}, V_+ \neq V_-$   
 $\Rightarrow$  this ckt. will "rail out"

$V_+ = (+) \rightarrow L^+$   
 $V_+ = (-) \rightarrow L^-$ .

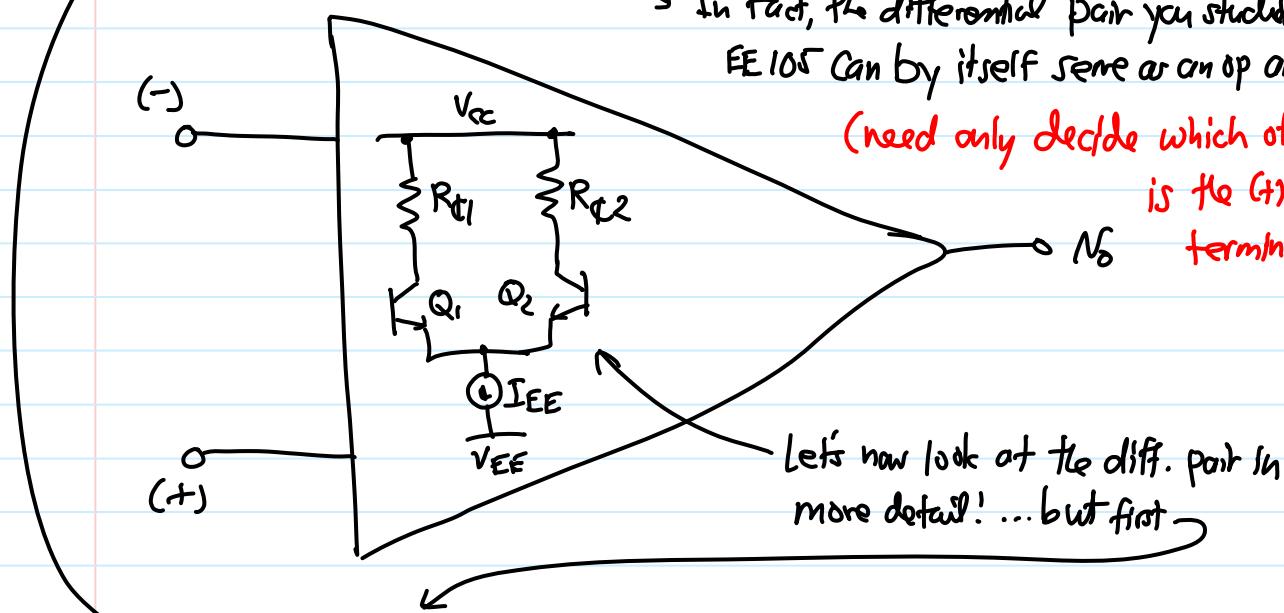
How does one make an op amp? (It turns out, you already know!)

$\Rightarrow$  Basic Needed Attributes:

- ① Gain (voltage gain).
- ② Two inputs, (+) and (-).
- ③ One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE 105 can by itself serve as an op amp!

(need only decide which of the inputs is the (+) and (-) terminals)

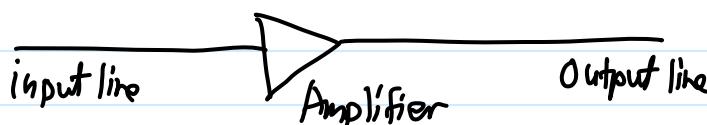


Let's now look at the diff. pair in more detail! ... but first

Why have 2 inputs?

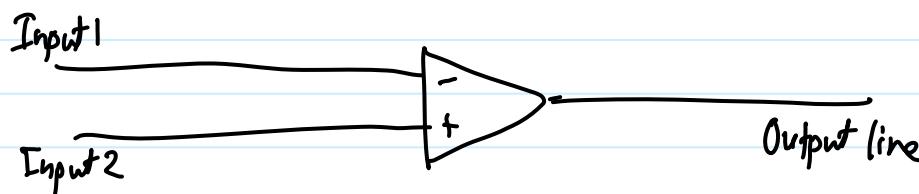
- ① To get a virtual short for op amp dc's.
- ② To suppress common-mode noise:

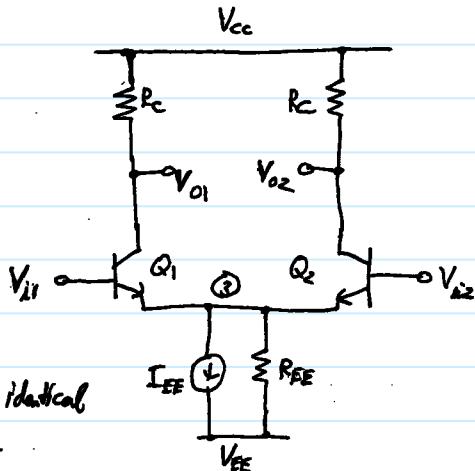
← nearby noisy line



Can avoid this w/ a differential input:

← nearby noisy line



**Differential Pair (Emitter-Coupled Pair)** $V_{CC}$ 

Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

$$\text{Definition: } V_{id} = V_{i1} - V_{i2} \quad (\text{differential input})$$

$$V_{icm} = \frac{V_{i1} + V_{i2}}{2} \quad (\text{common-mode input})$$

Assumptions:  
Q<sub>1</sub> & Q<sub>2</sub> identical  
 $R_{c1} = R_{c2}$

$$I_{EE} \downarrow \quad R_{EE}$$

$$\Rightarrow \begin{cases} V_{i1} = V_{icm} + \frac{V_{id}}{2} \\ V_{i2} = V_{icm} - \frac{V_{id}}{2} \end{cases}$$

$$\text{Differential Gain} = A_d = \frac{V_{o1} - V_{o2}}{V_{id}} = \frac{V_{od}}{V_{id}} \quad (\text{want this to be large for this differential amplifier})$$

$$\text{Common-Mode Gain} = A_{cm} = \frac{V_{o1}}{V_{icm}} \approx \frac{V_{o2}}{V_{icm}} \quad (\text{want this to be small so that the amp rejects common-mode signals})$$

$$\text{Common-Mode Rejection Ratio} = CMRR = \frac{A_{dm}}{A_{cm}} \quad (\text{should be very high to favor the differential mode and reject the common-mode})$$

$\Rightarrow$  we also want a high Common-Mode Input Range to reject DC input offsets

$\Rightarrow$  Note: No need for bypass capacitors (large) to the inputs or outputs  $\rightarrow$  can just use direct coupling!

**Biasing & Large Signal Common-Mode Behavior**

Case:  $R_{EE} = \infty \rightarrow$  ideal current source biasing  $\rightarrow I_{E1} = I_{E2} = \frac{I_{EE}}{2} \rightarrow V_{o1} = V_{o2} \Rightarrow V_{od} = 0$

If  $V_{icm} \uparrow \rightarrow V_{(3)} \uparrow$ , but current draw from I<sub>EE</sub> stays constant  $\therefore I_{c1} \neq I_{c2}$  stay constant  $\rightarrow$  bias pt.

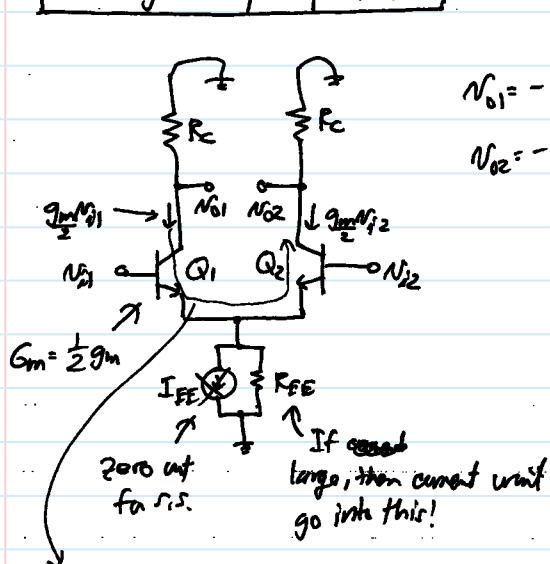
$$g_m = \frac{1}{2} \frac{I_{EE}}{V_T} \quad \text{doesn't change}$$

Case:  $R_{EE}$  finite  $\rightarrow V_{(3)} = V_{i1} - V_{BE(\text{on})}$

If  $V_{icm} \uparrow \rightarrow V_{(3)} \uparrow \rightarrow I_{E1} > I_{E2} \uparrow$  (current draw =  $I_{EE} + \frac{V_{od}}{R_{EE}}$ )

$\Rightarrow$  in general,  $R_{EE}$  will be large, so this component won't be large, and the bias pt. won't change much

## Small-Signal Analysis of Diff. Pair



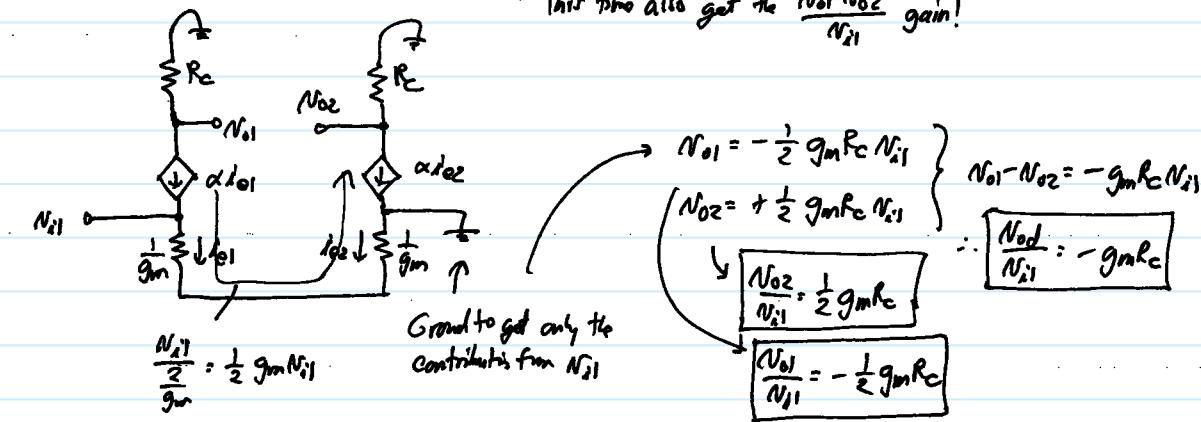
$$\begin{aligned} N_{01} &= -\frac{1}{2} g_m N_{11} R_c \\ N_{02} &= +\frac{1}{2} g_m N_{12} R_c \end{aligned} \quad \left. \begin{array}{l} N_{01} = -\frac{1}{2} g_m R_c (N_{11} - N_{12}) \\ N_{02} = +\frac{1}{2} g_m R_c (N_{11} - N_{12}) \end{array} \right\}$$

$$\therefore N_{0d} = N_{01} - N_{02} = -g_m R_c (N_{11} - N_{12})$$

$$\therefore \frac{N_{0d}}{N_{1d}} = A_{dm} = -g_m R_c$$

→ Easiest to see this happening using the T-model! (for those who must see the model chh.)

→ This time also get the  $\frac{N_{01}-N_{02}}{N_{11}}$  gain!



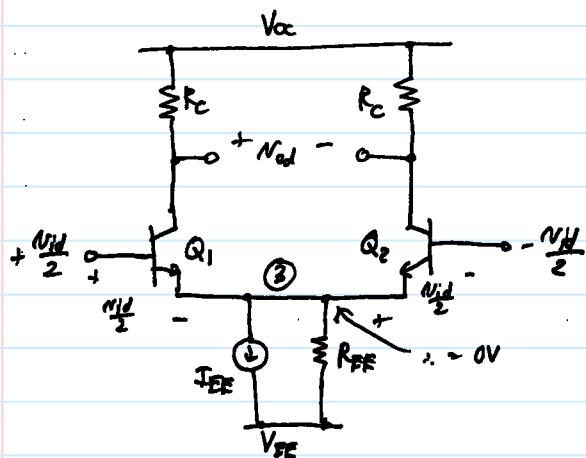
$$\begin{aligned} N_{01} &= -\frac{1}{2} g_m R_c N_{11} \\ N_{02} &= +\frac{1}{2} g_m R_c N_{11} \end{aligned} \quad \left. \begin{array}{l} N_{01} - N_{02} = -g_m R_c N_{11} \\ \frac{N_{02}}{N_{11}} = \frac{1}{2} g_m R_c \end{array} \right\}$$

$$\therefore \frac{N_{0d}}{N_{1d}} = -g_m R_c$$

$$\frac{N_{01}-N_{02}}{N_{11}} = -\frac{1}{2} g_m R_c$$

## Diff. Mode Analysis

Assume a diff. w/ only diff. input:



Total current thru  $I_{FEE}$  = const.

→  $V_E = \text{const.}$  as input changes

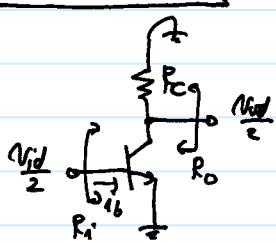
→ ③ acts as an incremental ground! →  $V_{③} = 0V$  (Always!)

∴ we can ground ③, and then, have

a Differential Half Ckt.

Note: Can really only make this for a purely symmetrical ckt!

Differential Half Ckt.



$$\text{By inspection: } \frac{N_{id}/2}{N_{id}} = \frac{N_{id}}{N_{id}} = A_{dm} = -g_m R_c$$

$$\frac{N_{id}/2}{i_b} = r_T \rightarrow R_{id} \frac{N_{id}}{i_b} = 2r_T = R_{id}$$

$$\frac{N_{id}/2}{i_b} = r_o || R_c \rightarrow R_{id} \frac{N_{id}}{i_b} = 2(r_o || R_c) \approx 2R_c = R_{id}$$

S.S. params. determined  
w/  $I_C = \frac{I_{FE}}{2}$

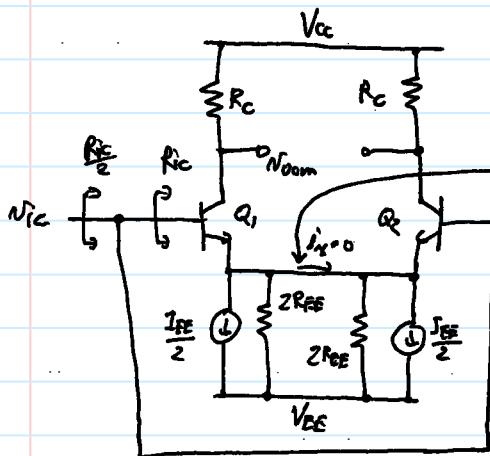
First define

$$r_T + \frac{1}{g_m(\beta+1)} - 2r_T \quad R_{id} = \frac{N_{id}}{i_b} = 2r_T$$

$$R_{id} = \frac{N_{id}}{i_b} = 2r_T$$

↳ this is actually general,  
so can inject that

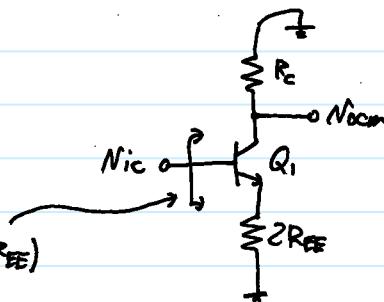
Common-Mode Analysis

Assume a pure CM input  $\rightarrow$  tie inputs together

By symmetry,  $i_X = 0 \Rightarrow$  thus, really have the equivalent of an open ckt. has

$\therefore \Rightarrow$  can split the ckt. into CM half-ckt's!

S.S. CM Half-Ckt.



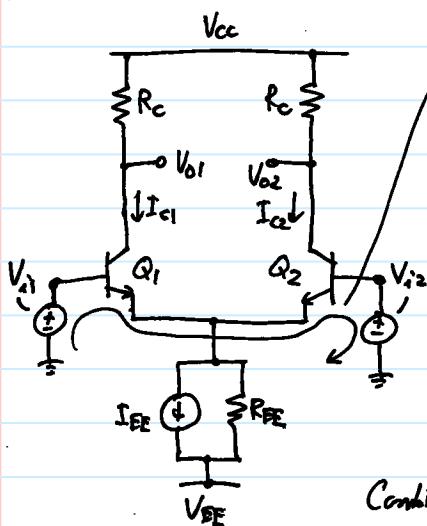
$$A_{cm} = \frac{N_{id}}{R_{ic}} = -\frac{g_m R_c}{1 + g_m(2R_EE)} \approx -\frac{R_c}{2R_EE}$$

Want small for large CMRR  $\therefore$  want $R_EE = \text{large!}$ 

$$\text{Common-Mode Rejection Ratio} = \text{CMRR} = \frac{A_{dm}}{A_{cm}} = \frac{-g_m R_c}{-\frac{g_m R_c}{1 + g_m(2R_EE)}} = \boxed{\text{CMRR} = 1 + 2g_m R_EE}$$

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the EOP.

## Large Signal ECP Performance

Find  $I_{c1} \neq I_{c2}$ :

$$\text{EVL: } V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$$

$$I_{c1} = I_{s1} \exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{c1}}{I_{s1}}\right), V_{be2} = V_T \ln\left(\frac{I_{c2}}{I_{s2}}\right)$$

$$V_{i1} = V_T \ln\left(\frac{I_{c1}}{I_{c2}} \frac{I_{c2}}{I_{s1}}\right) - V_{i2} = 0 \rightarrow \ln \frac{I_{c1}}{I_{c2}} = \frac{V_{i1} - V_{i2}}{V_T} = \frac{V_{id}}{V_T}$$

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{id}}{V_T}\right) \quad (1)$$

$$I_{EE} = I_{c1} + I_{c2} = \alpha (I_{c1} + I_{c2}) \quad (2)$$

Combine (1) &amp; (2) to get:

$$I_{c1} = \frac{\alpha I_{EE}}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)}, \quad I_{c2} = \frac{\alpha I_{EE}}{1 + \exp\left(\frac{V_{id}}{V_T}\right)} \quad (3)$$

Find  $V_{od}$ :

$$V_{o1} = V_{cc} - I_{c1} R_c$$

$$V_{o2} = V_{cc} - I_{c2} R_c$$

$$V_{od} = V_{o1} - V_{o2} = (I_{c2} - I_{c1}) R_c$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{1}{1 + \exp\left(\frac{V_{id}}{V_T}\right)} - \frac{1}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)} \right\}$$

$$\times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)}$$

$$\times \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right)}$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right) + \exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

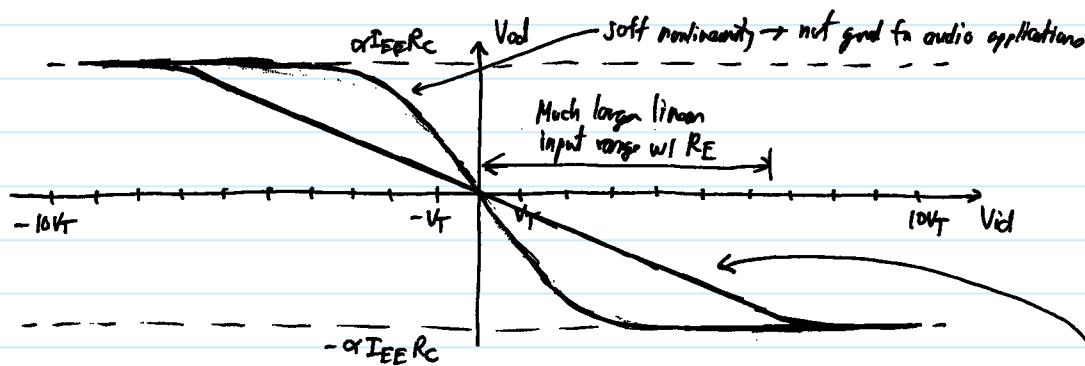
$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_c \frac{\sinh\left(\frac{V_{id}}{2V_T}\right)}{\cosh\left(\frac{V_{id}}{2V_T}\right)}$$

$$\begin{cases} \sinh u = \frac{1}{2}(e^u - e^{-u}) \\ \cosh u = \frac{1}{2}(e^u + e^{-u}) \end{cases} \quad u = -\frac{V_{id}}{2V_T}$$

$$\therefore V_{od} = \alpha_F I_{EE} R_c \tanh\left(-\frac{V_{id}}{2V_T}\right)$$

From our knowledge of the Taylor series for  
 $\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \dots$

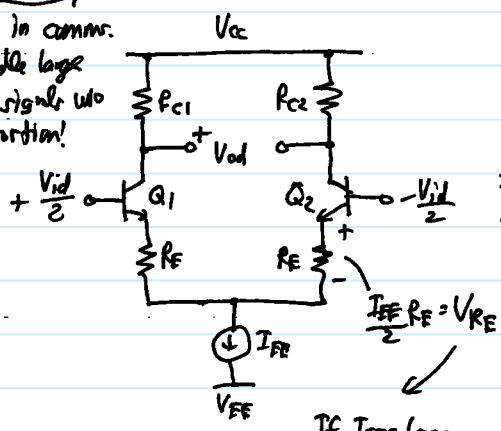
this is fairly linear for small  $V_{id}$ , but gets nonlinear  
 abruptly when  $V_{id}$  approaches a threshold value!



In the above curve, the  $\frac{V_{od}}{V_{id}}$  Xfer function is really only linear for  $Vid \ll V_t \rightarrow$  beyond  $V_t$ , start to enter the nonlinear realm of curve  $\rightarrow$  causes signal distortion: e.g., phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xfer in amplifiers)

Needed in common:  
to handle large  
input signals w/o  
distortion!



$$Adm = -\frac{gmr_c}{1 + gmr_e}$$

$\Rightarrow$  s.r. gain reduced, but the linear range is increased

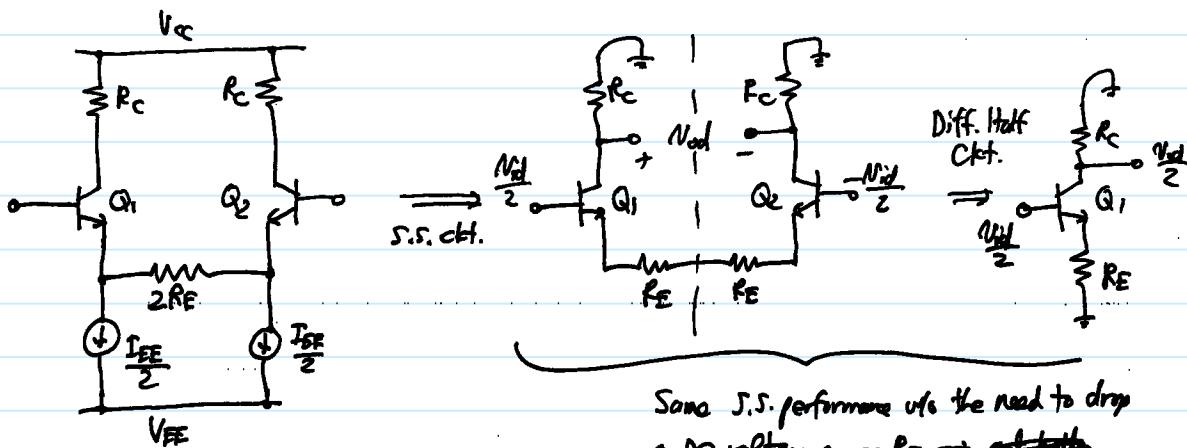
If  $I_{EE}$  is large,  
then this can  
force large  
supply voltages

$$\frac{N_{id}}{2} = N_{ad} + N_{re}$$

This can utilize

$N_{ad} \ll V_t$  if this absorbs some of the input voltage!

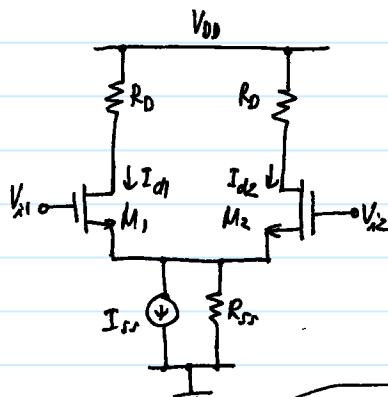
Alternative biasing technique if need larger DC current:-



Same S.S. performance w/o the need to drop  
a DC voltage across RE  $\rightarrow$  ~~good/better~~

Can use lower  $V_{cc}$  &  $V_{ee}$ .

## MOSFET Source-Coupled Pair



Assume:  $M_1, M_2$  are identical.

Find  $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$ .

$\Rightarrow$  approach: get  $V_{id} = f(\Delta I_d) \rightarrow$  then invert to get  $\Delta I_d = f(V_{id})$

$$I_{d1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{gs1} - V_t)^2 \Rightarrow V_{gs1} = V_t + \sqrt{\frac{2 I_{d1}}{k}}$$

$$\therefore V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2 I_{d1}}{k}} - \sqrt{\frac{2 I_{d2}}{k}}$$

Define.

$$\begin{cases} \Delta I_d = I_{d1} - I_{d2} \\ I_d = \frac{I_{d1} + I_{d2}}{2} \end{cases} \quad \begin{cases} I_{d1} = I_d + \frac{\Delta I_d}{2} \\ I_{d2} = I_d - \frac{\Delta I_d}{2} \end{cases}$$

$$V_{id} = \sqrt{\frac{2(I_d + \frac{\Delta I_d}{2})}{k}} - \sqrt{\frac{2(I_d - \frac{\Delta I_d}{2})}{k}} \Rightarrow \frac{k}{2} V_{id}^2 = I_d + \frac{\Delta I_d}{2} - 2\sqrt{I_d^2 - (\frac{\Delta I_d}{2})^2} + I_d - \frac{\Delta I_d}{2}$$

$$\frac{k}{2} V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - (\frac{\Delta I_d}{2})^2}$$

$\Rightarrow$  now rearrange to get  $\Delta I_d$  (algebra)

$$\text{Solve for } \Delta I_d: \quad \Delta I_d = \frac{k}{2} V_{id} \left( \frac{2I_{ss}}{k/2} - V_{id}^2 \right)^{\frac{1}{2}} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{id} \sqrt{\left( \frac{2I_{ss}}{\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)} \right) - V_{id}^2} = \Delta I_d$$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

$$|V_{id}| \leq \sqrt{\frac{2I_{ss}}{k}} = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox} \left( \frac{W}{L} \right)}} = \sqrt{2} (V_{GS} - V_t)$$

if true then input devices are both saturated

Thus, to extend the linear input range:

$$\textcircled{1} \quad I_{ss} \uparrow \rightarrow (V_{GS} - V_t) \uparrow$$

$$\textcircled{2} \quad W/L$$

$$\textcircled{3} \quad L \uparrow$$

To derive this:  $+V_{id} \xrightarrow{+V_{GS1}} M_1 \xrightarrow{-V_{GS2}} M_2 \xrightarrow{-V_{id}} -V_{id}$

$$V_{GS1} - \frac{V_{id}}{2} \quad V_{GS2} - \frac{V_{id}}{2}$$

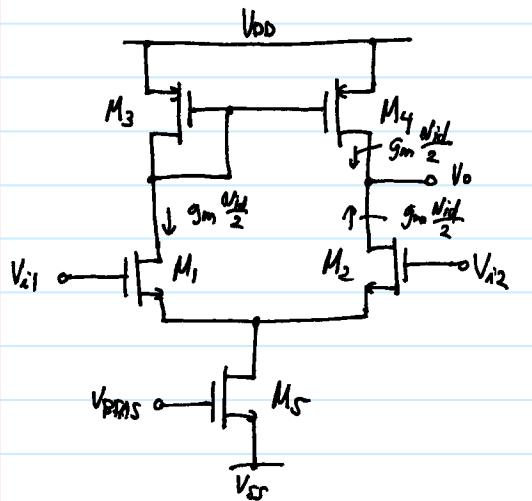
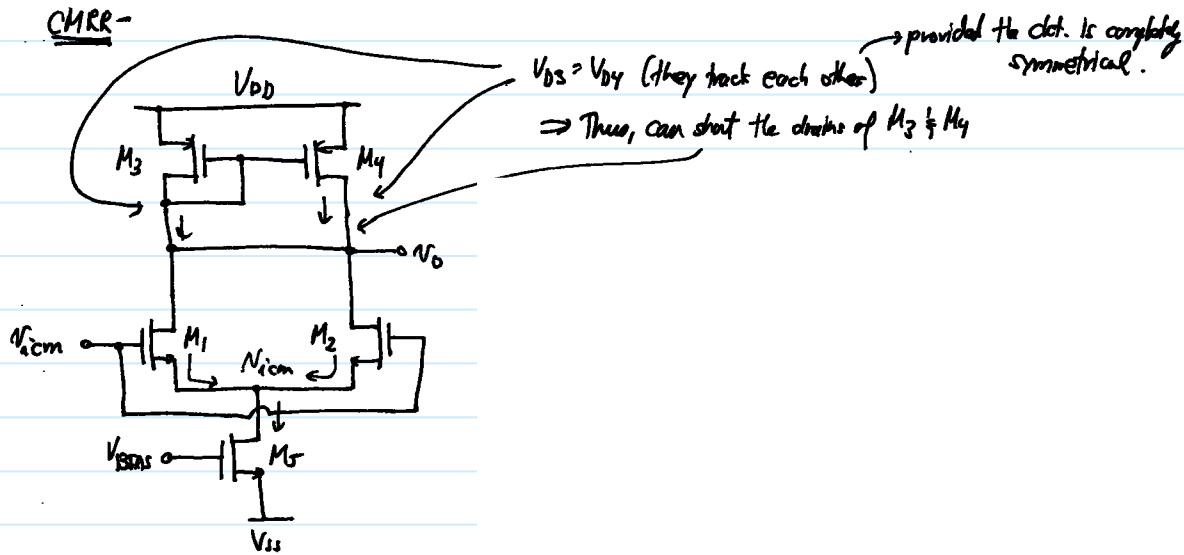
When  $\frac{V_{id}}{2} \geq V_{GS2} - V_t = \Delta V$  then  $M_2$  will cut-off

$\therefore V_{id} \leq 2(V_{GS2} - V_t) \rightarrow$  to maintain saturation

$$V_{GS2} - V_t = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox} \left( \frac{W}{L} \right)}} = \sqrt{\frac{2(I_d - \frac{\Delta I_d}{2})}{\mu_n C_{ox} \left( \frac{W}{L} \right)}} = \frac{V_{id}}{2}$$

then plug in  $\Delta I_d$  & solve for  $V_{id}$

## MOS Differential Stage w/ Current Mirror Load

CMRR -Common-Mode Input Range - Range of input voltages in which all devices remain in saturation.Low End - must keep  $M_5$  saturated

High End - keep  $M_1, M_2$  saturated

