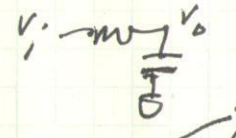
solving linear ~~~~ ODEs

poles move in feedback

stability & phase margin

compensation



$RC\frac{dV_o}{dt} + V_o = V_i$

if $V_i = \sin(\omega t)\, V_I$

assume $V_o = A\sin\omega t + B\cos\omega t = V_O \sin(\omega t + \phi)$

$RCA\omega\cos(\omega t) + RCB(-\omega)\sin\omega t + A\sin\omega t + B\cos\omega t = V_I \sin\omega t$

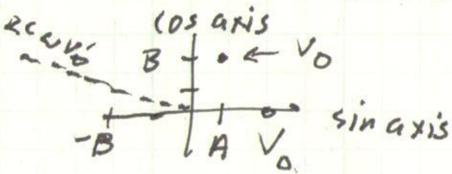
$(A - \omega RCB)\sin(\omega t) + \underbrace{(B + RCA\omega)}_{=0}\cos(\omega t) = V_I \sin(\omega t)$

always end up w/ $\{\ \}\sin + \{\ \}\cos = \{\ \}\sin + \{\ \}\cos$

even if you add more derivatives

so ~~write~~ think of $\sin\omega t$ and $\cos\omega t$ as basis functions

plot their coefficients



notice that

$\frac{d}{dt}\sin = \omega\cos$

$\frac{d}{dt}\cos = -\omega\sin$ | $\frac{d^2}{dt^2}\sin = -\omega^2\sin$

$\frac{d^3}{dt^3}\sin = -\omega^3\cos$

$\frac{d^4}{dt^4}\sin = \omega^4\sin$

$\boxed{\text{derivatives scale by } \omega \text{ and rotate left } 90^\circ}$

Use real & imaginary axis and derivative is just multiplication by $j\omega$

so $RC\frac{dV_o}{dt} + V_o = V_I$ becomes

$(RCj\omega + 1)V_o = V_I$

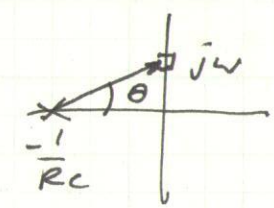
$\frac{V_o}{V_I} = \frac{1}{RCj\omega + 1} = \frac{1/RC}{j\omega + 1/RC} = \frac{1/RC}{j\omega - P}$

$P = \frac{-1}{RC}$

$|H(j\omega)| = \left|\frac{V_o}{V_I}\right| = \frac{1/RC}{|j\omega - P|}$

$\angle H(j\omega) = \angle\frac{1}{RC} - \angle(j\omega - P)$

$= 0 - \theta$



2 poles co-located $A = \frac{A_0}{(1 + s/\omega_p)^2}$ $A_{CL} = \frac{A}{1 + Af}$

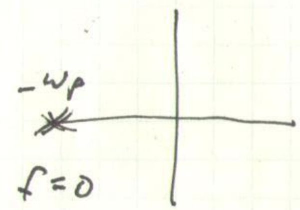
$A_{CL} = \frac{A_0}{(1 + s/\omega_p)^2 + A_0 f} = \frac{A_0}{1 + \frac{2s}{\omega_p} + \frac{s^2}{\omega_p^2} + A_0 f}$

$= \frac{A_0 \omega_p^2}{s^2 + 2\omega_p s + A_0 f \omega_p^2 + 1}$

poles are located @ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\omega_p \pm \frac{\sqrt{(2\omega_p)^2 - A_0 f \omega_p^2 - 1}}{2}$

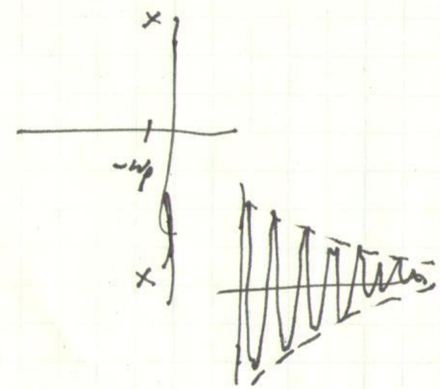
$P_{1,2} = -\omega_p \pm \omega_p \sqrt{\cancel{4} - \frac{A_0 f - 1}{4}}$

if $f = 1$ $\quad -\omega_p \pm j\omega_p \frac{\sqrt{A_0}}{2}$



$f = 0$

Open loop



3 poles co-located

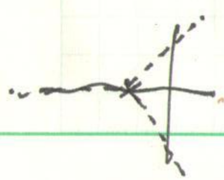
$A = \frac{A_0}{(1 + s/\omega_p)^3} = \frac{\omega_p^3 A_0}{(s + \omega_p)^3}$

$A_{CL} = \frac{\omega_p^3 A_0}{(s + \omega_p)^3 + \omega_p^3 A_0 f}$

poles: $(s + \omega_p)^3 = -\omega_p^3 A_0 f$

$s - (-\omega_p) = \sqrt[3]{-\omega_p^3 A_0 f} = \omega_p \sqrt[3]{A_0 f}\, e^{j\theta_i}$

$\theta_i = 180, \pm 60$

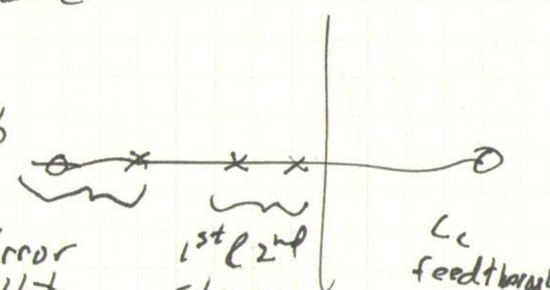


turns out that we have

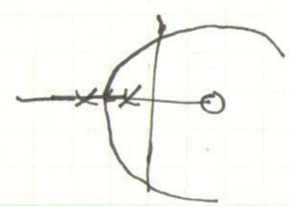
2 poles

pole/zero doublet

RHP zero



with 2 poles & zero



Root locus

stability and phase margin

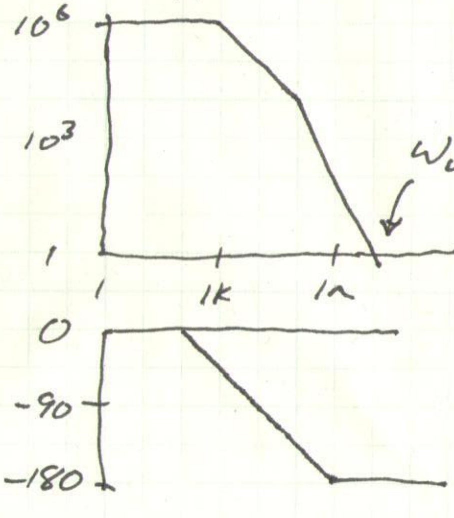
if $Af = -1$ at some freq,

the system oscillates $\frac{A}{1+Af}$

same is true if Af is more negative

for stability, we need $|Af| < 1$ when $\angle Af = \pm\pi$
± 180

$A_o = 10^6$ $\omega_{p1} = 10^3$ $\omega_{p2} = 10^5$



unstable w/ $f = 1$

fine if $f < 10^{-4}$

phase margin : $\angle Af(j\omega_u) - 180°$

How far away from oscillation is the system?

PM = 45° minimum requirement for most systems

less and you see a lot of ringing.

PM = 60° or 70° very stable

single pole amp : PM = 90

what if we move ω_{p1} much lower (lab 3)

$\omega_{p1} = 10$

stable w/ $f = 1$

but PM = 22°