Lecture 18: Diffusion II

- Announcements:
- Midterms graded and passed back today
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- Lecture Topics:
  - Diffusion
    - Basic Process for Selective Doping
    - Diffusion Modeling
    - Predeposition Modeling
    - Drive-in Modeling
    - Successive Diffusions
    - Diffusion Coefficient
    - Junction Depth
    - Sheet Resistance
    - Irvin's Curves
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- Last Time:

Midterm Stats

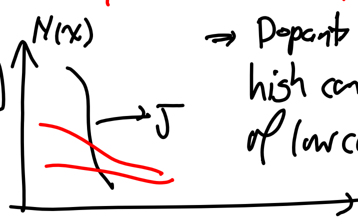
Top Score: 93

Average: 65

Median: 65

Std. Dev.: 17

Modeling



→ Dopants f/ points of high conc. move to points of low conc. w/ flux J

Question: What's $N(x,t)$?

↑ fcn of time + temperature

Fick's Law of Diffusion – (First Law)

$$J(x,t) = -D\frac{\partial N(x,t)}{\partial x} \qquad (1)$$

flux [#/cm²·s]

Diffusion Coefficient

Continuity Equation for Particle Flux –

General Form: $\dfrac{\partial N(x,t)}{\partial t} = -\vec{\nabla}\cdot\vec{J}$

rate of increase of conc. w/ time

negative of the divergence of the particle flux

⟹ we're interested in the one-dimensional form:

$$\frac{\partial N(x,t)}{\partial t} = -\frac{\partial J}{\partial x} \qquad (2)$$

$\left(\frac{\partial}{\partial x}(1) \text{ and substitute } (2) \text{ in } (1)\right)$

$$* \rightarrow \frac{\partial N(x,t)}{\partial t} = D\frac{\partial^2 N(x,t)}{\partial x^2}$$

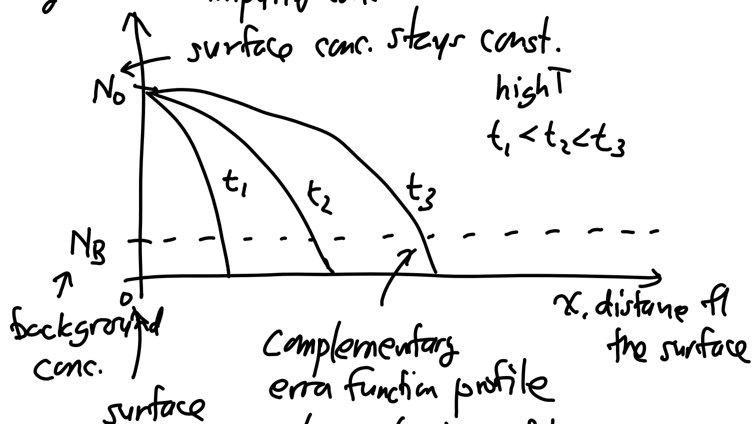
[Fick's 2nd Law of Diffusion in 1-D]

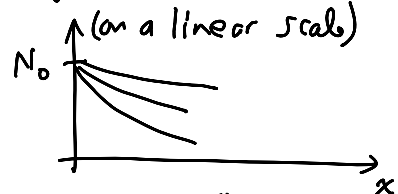
Solutions: → dependent upon boundary conditions

↳ use variable separation or Laplace Xform techniques

Case I: Predeposition

↳ constant source diffusion → surface concentration stays the same during the diffusion





Boundary Conditions: (predeposition)

(i) $N(0,t) = N_0$ → by definition of

(ii) $N(\infty, t) = 0$

↳ solve *

Get: $N(x,t) = N_0 \left[1 - \frac{1}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2} dy \right]$

$$\boxed{N(x,t) = N_0 \, \mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)}$$

complementary error fcn (read table or a graph)

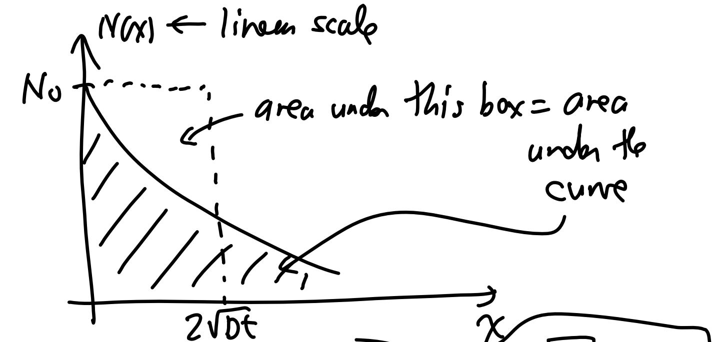
↳ on TI-89 (download flash app)

$D \triangleq$ diffusion const.

T dependence is here

Dose, $Q \triangleq$ total # of impurity atoms per unit area in the Si

= area under the curve

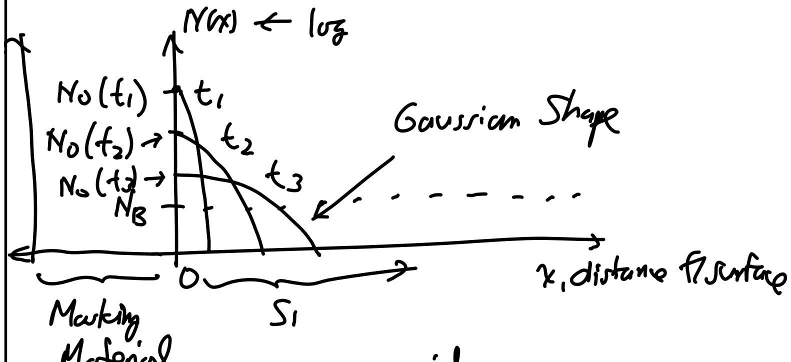


$Q = \int_0^\infty N(x,t)\,dx \Rightarrow \boxed{Q(t) = N_0 \frac{2\sqrt{Dt}}{\sqrt{\pi}} \ \mathrm{cm}^{-2}}$

$\boxed{2\sqrt{Dt} \triangleq \text{characteristic diffusion length}}$

temperature time

Case 2: Drive-in → limited source diffusion, i.e., constant dose Q

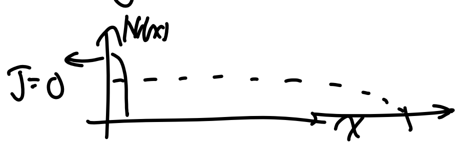


Boundary Condition

(i) $N(\infty, t) = 0$

(ii) $\left.\frac{\partial N(x,t)}{\partial x}\right|_{x=0} = 0$

This equivalent to saying that there's no flux going out of the Si



Constant Dose:

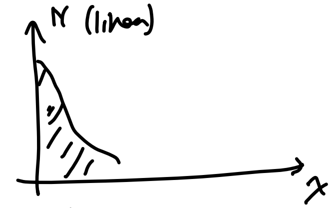
$$\int_0^\infty N(x,t)\,dx = Q = \text{const.}$$

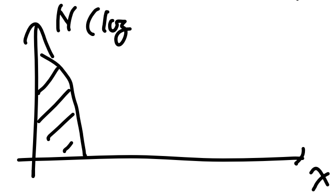
(iii) Usually make delta fcn. approx: $N(x,0) = Q\,\delta(x)$

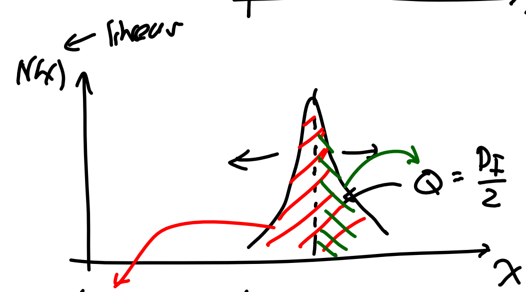
Get one-sided Gaussian Distribution:

$$N(x,t) = \frac{Q}{\sqrt{\pi D t}} \exp\left[-\left(\frac{x}{2\sqrt{Dt}}\right)^2\right]$$

Correspond to a "half-Gaussian":







$D_I \triangleq$ implanted dose

Two-Step Diffusion

Predeposition or I/I → erfc profile / Gaussian

↓

Drive-In → Gaussian

Which profile do we end up with?

→ Governed by the relative (Dt) product.

$(Dt)_{predep} >> (Dt)_{drive\text{-}in} \Rightarrow$ impurity profile is complementary error function

$(Dt)_{drive} >> (Dt)_{predep} \Rightarrow$ " " is Gaussian

↗ usually the case

Successive Diffusions

For actual process → junction formation is only one of many high temp. steps.

① Selective Doping

(a) implant → effective $(Dt)_1 = \frac{\Delta R_p^2}{2}$ (it's Gaussian)

(b) drive-in/activation ↳ $D_2 t_2$

② Other high temp. steps: $D_3 t_3, D_4 t_4, D_5 t_5,$

$$\boxed{(Dt)_{tot} = \sum_i D_i t_i}$$