EECS 16A Designing Information Devices and Systems I
Fall 2018 Discussion 4A

1. **Identifying a Basis**

   Does each of these sets of vectors describe a basis for $\mathbb{R}^3$? What about for some subspace of $\mathbb{R}^3$?

$$V_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\} \qquad V_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \qquad V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

2. **Constructing a Basis**

   Let's consider a subspace of $\mathbb{R}^3$ called V which has the following property: for every vector in V, the first entry is equal to two times the sum of the second and third entries. That is, if $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \in V$, $a_1 = 2(a_2 + a_3)$.

   Find a basis for V. What is the dimension of V?

3. **Exploring Dimension, Linear Independence, and Basis**

   In this problem, we are going to talk about the connections between several concepts we have learned about in linear algebra – linear independence, dimension of a vector space/subspace, and basis.

   Let's consider the vector space $\mathbb{R}^m$ and a set of n vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ in $\mathbb{R}^m$.

   (a) For the first part of the problem, let $m > n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^m$? Why/why not? What conditions would we need?

   (b) Let $m = n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^m$? Why/why not? What conditions would we need?

   (c) Now, let $m < n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^m$? What vector space could they form a basis for?

   Hint: Think about whether the vectors can be linearly independent.

4. **Exploring Column Spaces and Null Spaces**

   - The **column space** is the possible outputs of a transformation/function/linear operation. It is also the **span** of the column vectors of the matrix.
   - The **null space** is the set of input vectors that output the zero vector.

   For the following matrices, answer the following questions:

   i. What is the column space of $\mathbf{A}$? What is its dimension?

   ii. What is the null space of $\mathbf{A}$? What is its dimension?

   iii. Are the column spaces of the row reduced matrix $\mathbf{A}$ and the original matrix $\mathbf{A}$ the same?

   iv. Do the columns of $\mathbf{A}$ form a basis for $\mathbb{R}^2$ (or $\mathbb{R}^3$ for part (b))? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

5. Inverse Proof

Prove that a matrix $\mathbf{A}$ is invertible if and only if its columns are linearly independent.