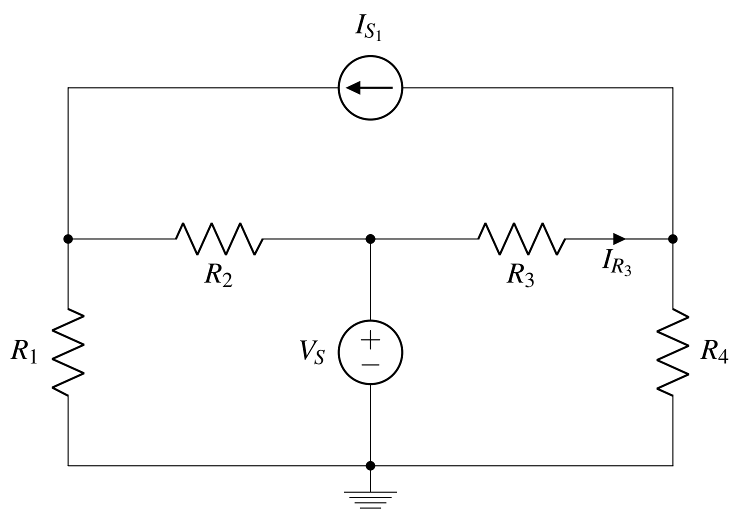
EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 7B

Fundamental Power Equation: The power dissipated by a circuit element is equal to $P = IV$, where V is the voltage across its terminals and I the current flowing through it.
Note: If this quantity is negative it means that the element in question is supplying power to the circuit instead of dissipating it.

1. Circuit Analysis

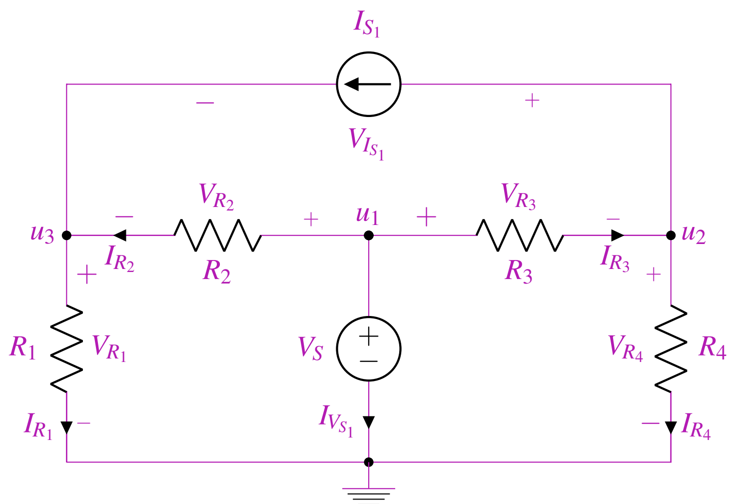
(a) Use nodal analysis to solve for all node voltages.

(b) Find current I_{R_3} flowing through resistor R_3.



Answer:

(a) Let us follow the Since Step 1 (selecting a ground node) has already been done for us, we will go through steps 2-4 in the following circuit diagram. As a reminder the steps are the following:
Step 2: Label all nodes with voltage set by voltage sources (denoted below as u_1),
Step 3: Label remaining nodes (denoted below as u_2, u_3),
Step 4: Label element voltages and currents (V_{R_1}, I_{R_1}, etc.)



Note: You can pick any direction you want for the currents but once you have done so you the voltages you choose have to abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5: Write KCL equations for all nodes with unknown voltages (namely u_2, u_3):

$$-I_{R_3} + I_{S_1} + I_{R_4} = 0$$
$$-I_{R_2} - I_{S_1} + I_{R_1} = 0$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$u_3 - 0 = I_{R_1} R_1$$
$$V_s - u_3 = I_{R_2} R_2$$
$$u_2 - 0 = I_{R_4} R_4$$
$$V_s - u_2 = I_{R_3} R_3$$

Where we used the fact that $u_1 = V_s$

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

$$-\frac{V_s - u_2}{R_3} + I_{S_1} + \frac{u_2}{R_4} = 0$$
$$-\frac{V_s - u_3}{R_2} - I_{S_1} + \frac{u_3}{R_1} = 0$$

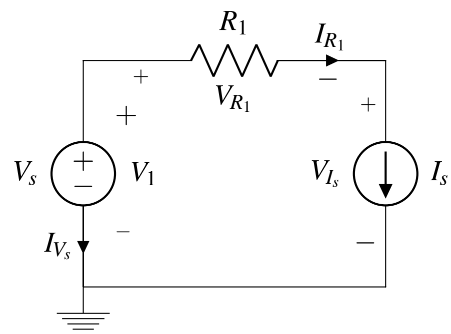
Notice that we can now directly solve for u_2 and u_3 and then substitute into the equations of Step 6 to find the currents through all resistors. Finally, KCL in node V_1 will give us the current through the voltage source, V_s.

(b)

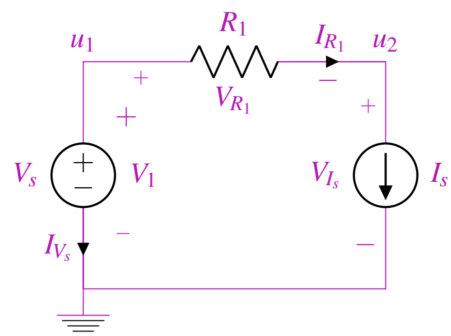
$$I_{R_3} = \frac{V_s - u_2}{R_3}$$

2. Passive Sign Convention and Power

Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}, I_s = 0.5\,\text{A}$ and $R_1 = 5\,\Omega$.



Answer: We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equations for nodes u_1, u_2 in this circuit are:

$$I_{V_s} + I_{R_1} = 0$$

$$-I_{R_1} + I_s = 0$$

The Element equations for the voltage source and the resistor are:

$$u_1 - 0 = V_1 = V_s$$

$$V_{R_1} = u_1 - u_2 = I_{R_1} R_1$$

Finally, the KVL equation around the one loop formed by the circuit is:

$$V_1 - V_{R_1} - V_{I_s} = 0$$

Solving the above equations with $V_s = 5\,\text{V}, I_s = 0.5\,\text{A}$ and $R_1 = 5\,\Omega$:

$$I_{R_1} = 0.5\,\text{A}$$
$$I_{V_s} = -0.5\,\text{A}$$
$$V_{R_1} = 2.5\,\text{V}$$
$$V_{I_s} = 2.5\,\text{V}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = IV = I_{R_1}V_{R_1} = 0.5\,\text{A} \cdot 2.5\,\text{V} = 1.25\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = IV = I_{V_s}V_1 = -0.5\,\text{A} \cdot 5\,\text{V} = -2.5\,\text{W}$$

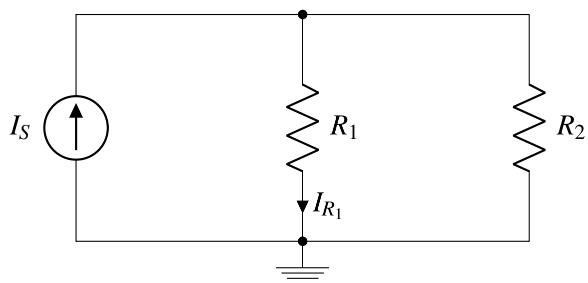
Finally, we can solve for the power dissipated across the current source:

$$P_{I_s} = IV = I_sV_{I_S} = 0.5\,\text{A} \cdot 2.5\,\text{V} = 1.25\,\text{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is actually ***supplying*** power to the circuit.
Note: In this case the current source is also dissipating power but it could be also supplying if the numbers were picked differently. For example, if $I_s = 2$A the same equations would give $P_{R_1} = 20$W, $P_{I_s} = -10$W, $P_{I_s} = -10$W. Also, numbers could have been selected such that the voltage source dissipated and the current source supplied power to the circuit.

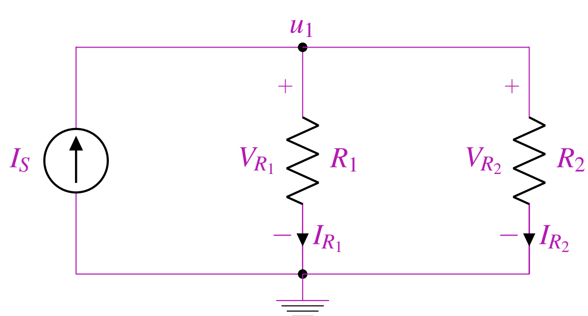
3. A simple circuit (aka current divider)

For the following circuit calculate the current I_{R_1}, through resistor R_1 in the direction indicated in the diagram.



Answer:

Since I_{R_1} is now specified by the problem, we only need to pick the direction of I_{R_2}, and label the node and element voltages. We do so in the following diagram:



We can now write a KCL equation on node u_1:

$$-I_S + I_{R_1} + I_{R_2} = 0$$

From Ohm's law in the two resistors we get that:

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - 0}{R_1}$$
$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_1 - 0}{R_2}$$

Now plugging back into the KCL equation we have:

$$u_1 = I_S \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = I_S \frac{R_1 R_2}{R_1 + R_2}$$

So we can now estimate the required current as:

$$I_{R_1} = \frac{u_1}{R_1} = I_S \frac{R_2}{R_1 + R_2}$$

