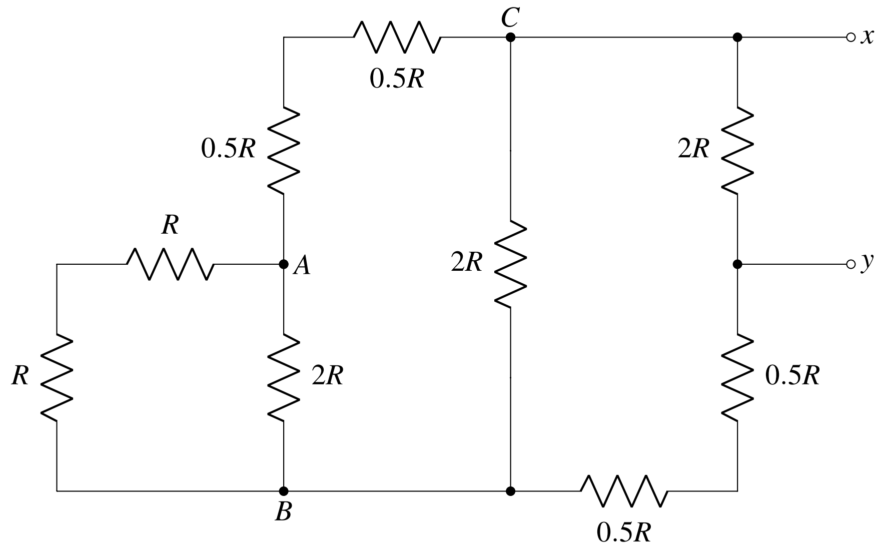
EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 8A

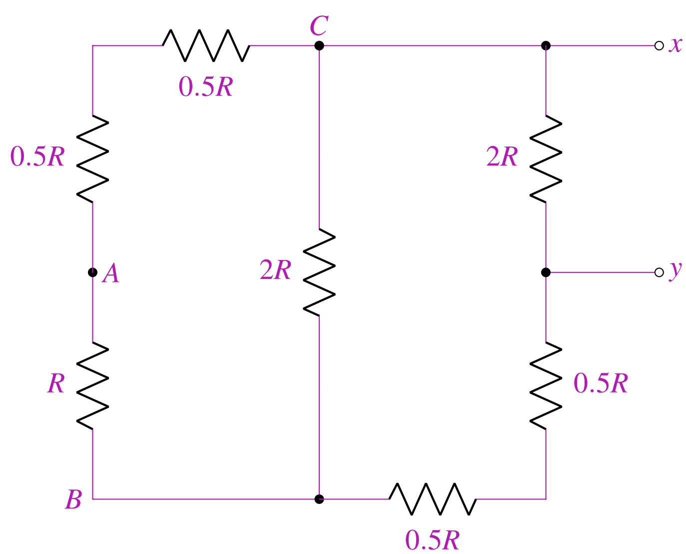
1. Equivalence

For the circuit shown below, find the equivalent resistance looking in from points x and y.

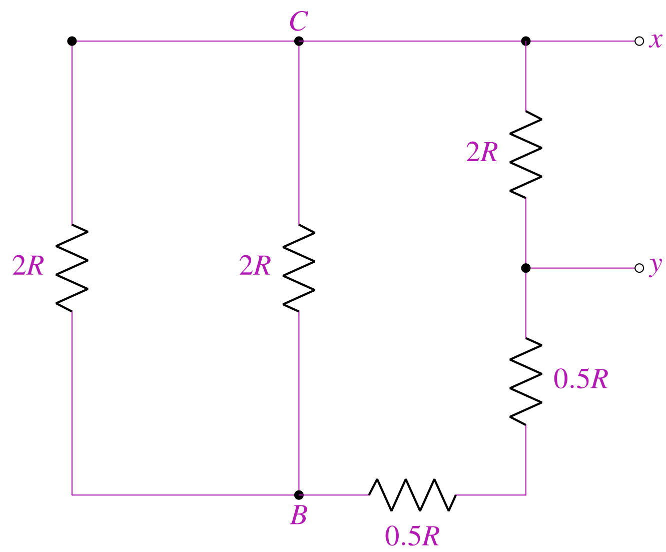


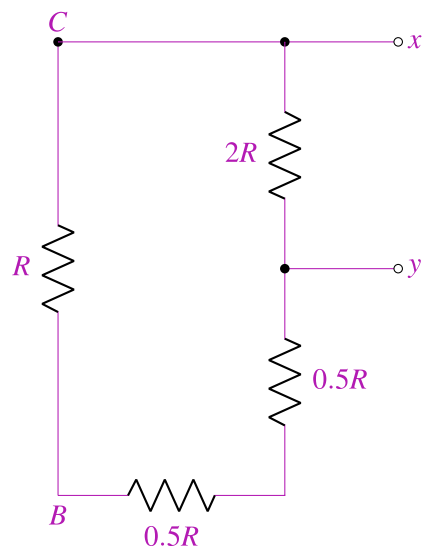
Answer:

We find the equivalent resistance for the resistors from left to right. First we find R_{AB}.

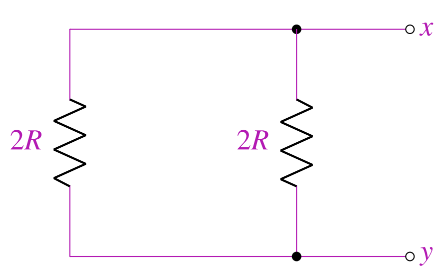


Then we can find R_{CB}.

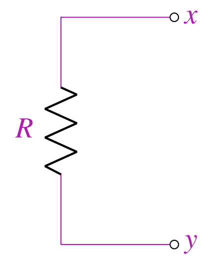




If we move from node C counter-clockwise to node y, the resistance seen is $R+0.5R+0.5R=2R$. Therefore we have,



Now we can find R_{xy}.



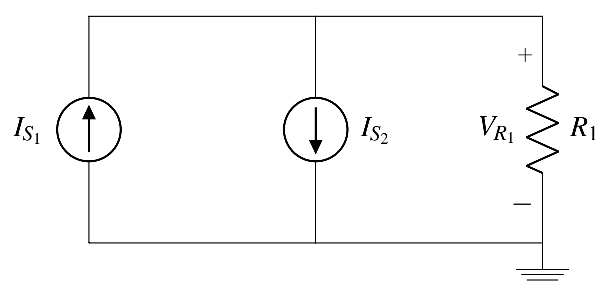
Therefore, the equivalent resistance is $R_{xy} = R$.

2. Superposition

For the following circuits:

i. Use the superposition theorem to solve for the voltages across the resistors.
ii. For parts (a) and (b) only, find the power dissipated/generated by all components. Is power conserved?

(a)



Answer:

i. While we could apply the algorithm we have learned in class, let's see if there's a way to find the answer quicker than before. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{S_1} - I_{S_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

We could also solve this using superposition. Turning on I_{S_1} gives $V_{R_1} = I_{S_1}R_1$. Turning on I_{S_2} gives $V_{R_1} = -I_{S_2}R_1$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

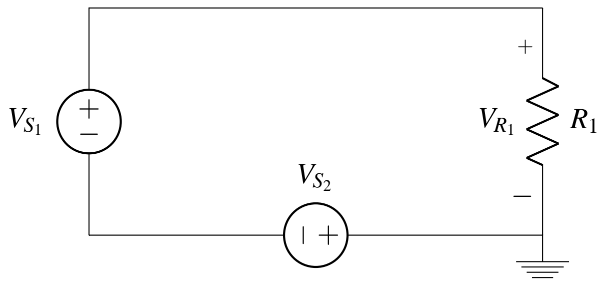
ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = (I_{S_1} - I_{S_2})^2 R_1$$
$$P_{I_{S_1}} = -I_{S_1}V_{R_1} = -(I_{S_1} - I_{S_2})I_{S_1}R_1$$
$$P_{I_{S_2}} = I_{S_2}V_{R_1} = (I_{S_1} - I_{S_2})I_{S_2}R_1$$

$$P_{R_1} + P_{I_{S_1}} + P_{I_{S_2}} = (I_{S_1} - I_{S_2})^2 R_1 - (I_{S_1} - I_{S_2})I_{S_1}R_1 + (I_{S_1} - I_{S_2})I_{S_2}R_1 = 0$$

Power is conserved.

(b)



Answer:

i. Once again, we could apply the circuit analysis algorithm or find the answer directly. Notice the circuit only has one loop, so we can use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

We could also solve with superposition. Turning on V_{S_1} gives $V_{R_1} = V_{S_1}$. Turning on V_{S_2} gives $V_{R_1} = -V_{S_2}$. The overall voltage is then the sum.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = \frac{(V_{S_1} - V_{S_2})^2}{R_1}$$

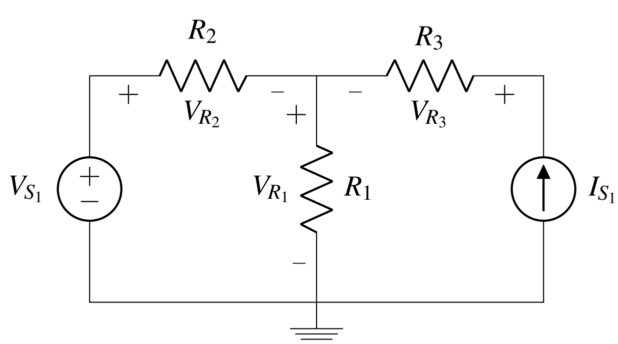
$$P_{V_{S_1}} = -I_{R_1} V_{S_1} = -\frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{V_{S_2}} = I_{R_1} V_{S_2} = \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{R_1} + P_{V_{S_1}} + P_{V_{S_2}} = \frac{(V_{S_1} - V_{S_2})^2}{R_1} - \frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1} + \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1} = 0$$

Power is conserved.

(c)



Answer: Turning on only V_{S_1}, we have the following voltages across the resistors:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1}$$

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_{S_1}$$
$$V_{R_3} = 0$$

Then turning only I_{S_1}, we have the following voltages:

$$V_{R_1} = \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_2} = -\frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_3} = I_{S_1} R_3$$

Using superposition we can sum up the contributions from both V_{S_1} and I_{S_1} to get:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1} + \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_2} = V_{S_1} - V_{R_1} = \frac{R_2}{R_1 + R_2} V_{S_1} - \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_3} = I_{S_1} R_3$$

3. Resist the Touch

In this question, we will be re-examining the 2-dimensional resistive touchscreen previously discussed in both lecture and lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length L and width W and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position $(1,1)$.

The top layer has N vertical strips denoted by $x_1, x_2, ..., x_N$. These vertical strips all have cross sectional area A, and resistivity ρ_x.

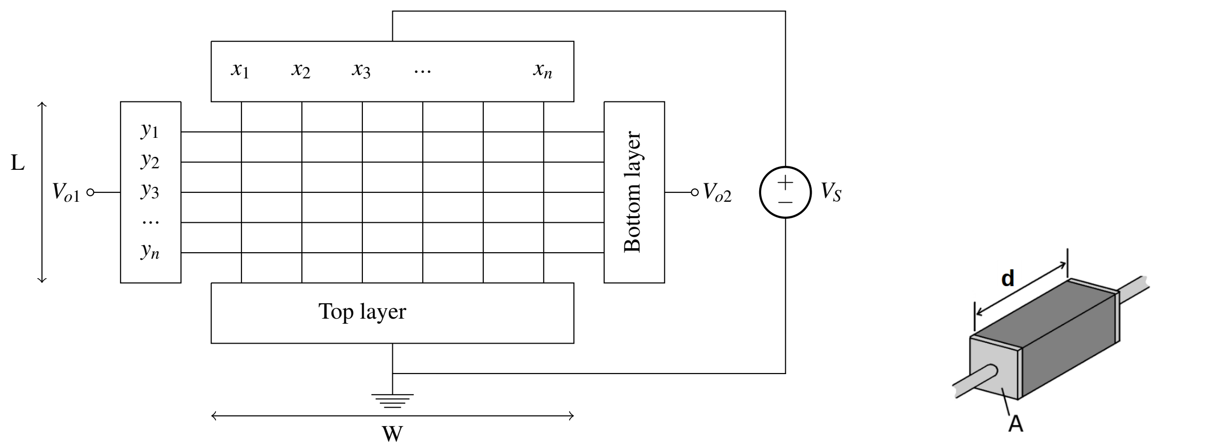
The bottom layer has N horizontal strips denoted by $y_1, y_2, ..., y_N$. These horizontal strips all have cross sectional area A as well, and resistivity ρ_y.

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).

(a) (3 points) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance R_x for the vertical strips and R_y for the horizontal strips, as a function of the screen dimensions W and L, the respective resistivities, and the cross-sectional area A.

Answer: The equation for resistance is $R = \frac{\rho l}{A}$
Therefore, $R_x = \frac{\rho_x L}{A}$.
For the bottom, $R_y = \frac{\rho_y W}{A}$.

(b) (5 points) Consider a 2×2 example for the touchscreen circuit.
Given that $V_s = 3\,\text{V}$, $R_x = 2000\,\Omega$, and $R_y = 2000\,\Omega$, draw the equivalent circuit for when the point $(2,2)$ is pressed and solve for the voltage at terminal V_{O2} with respect to ground.
Answer:
Since all of the resistive strips are equally spaced, the resistor above point $(2,2)$ on strip x_2 becomes $\frac{2}{3}R_x$ and the resistor below point $(2,2)$ on strip x_2 becomes $\frac{1}{3}R_x$.
The bottom layer resistors, although they must be drawn in the equivalent circuit, do not affect the voltage at V_{O2} as they are open circuits.



(a) 2-D Resistive Touch Screen

(b) 3D Model of a Single Resistive Strip

Figure 1:

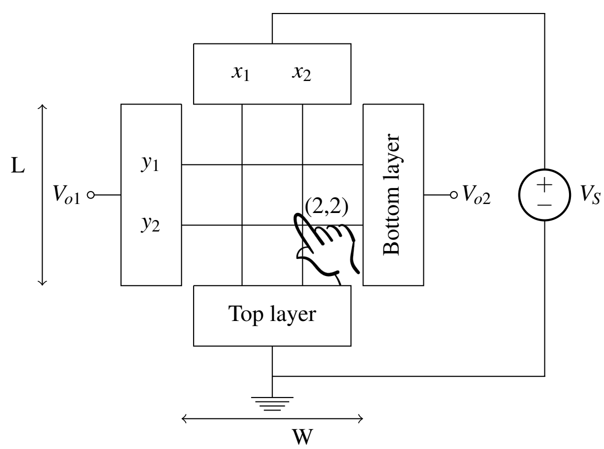
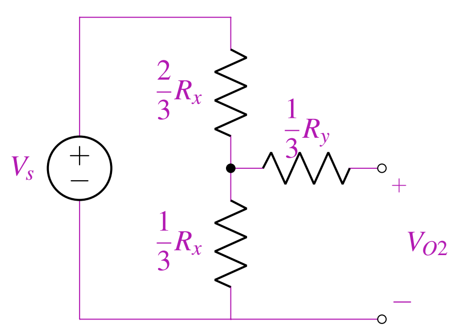


Figure 2: 2×2 Case of the Resistive Touchscreen



Observing that the resistive strips form a voltage divider, we can determine V_{O2} using the voltage divider equation.

Therefore, $V_{O2} = V_{(2,2)} = V_s \frac{\frac{1}{3}R_x}{\frac{1}{3}R_x + \frac{2}{3}R_x} = \frac{1}{3}V_s = 1\text{V}$.

(c) (8 points) Suppose a touch occurs at coordinates (i, j) in Figure 1(a). Find an expression for V_{O2} as a function of V_s, N, i, and j. The upper left corner is the coordinate $(1,1)$ and the upper right coordinate is $(N,1)$.

Answer:

$$\begin{aligned} V_{O2} &= \frac{\frac{N+1-j}{N+1}R_x}{R_x}Vs \\ &= \frac{N+1-j}{N+1}V_s \end{aligned}$$

