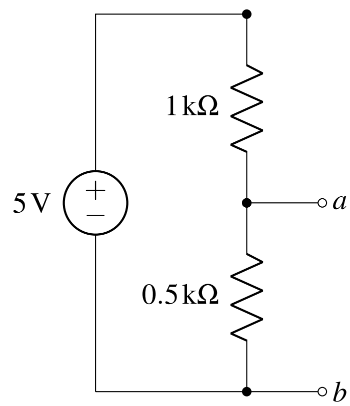
EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 8B

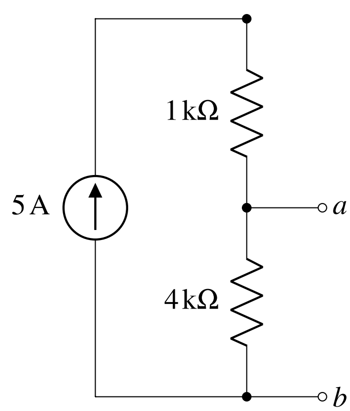
1. Equivalence

Find the Thévenin and Norton equivalents across terminals *a* and *b* for the circuits given below.

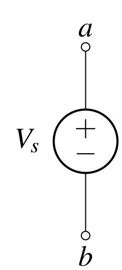
(a)



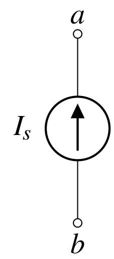
(b)



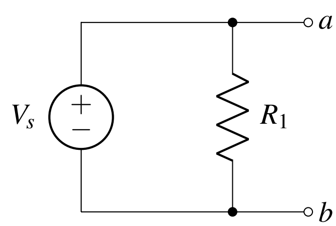
(c)



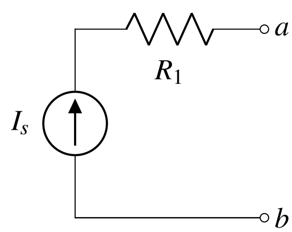
(d)



(e) (Practice)

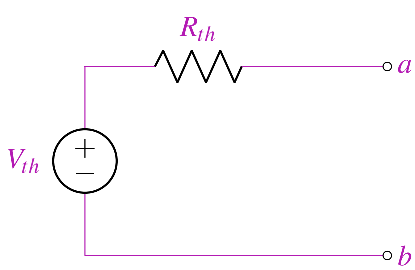


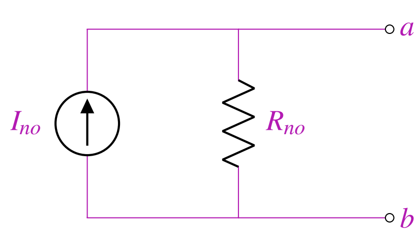
(f) (Practice)



Answer:

The general Thévenin and Norton equivalents are shown below:





(a)

$$V_{th} = 1.67\,\text{V}, I_{no} = 5\,\text{mA}, R_{th} = R_{no} = 333\,\Omega$$

(b)

$$V_{th} = 20\,000\,\text{V}, I_{no} = 5\,\text{A}, R_{th} = R_{no} = 4000\,\Omega$$

(c) A Norton equivalent of a voltage source is not necessary, since a voltage source is a basic element. The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$.

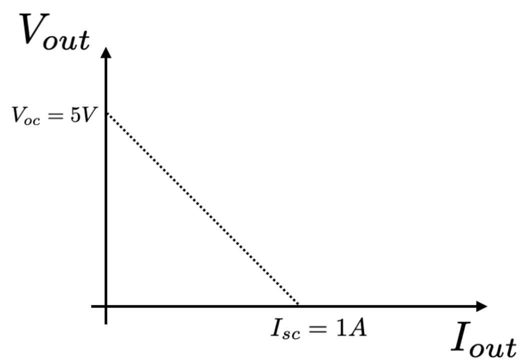
(d) A Thévenin equivalent of a voltage source is not necessary because a current source is a basic element and cannot be represented as a voltage source. The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$.

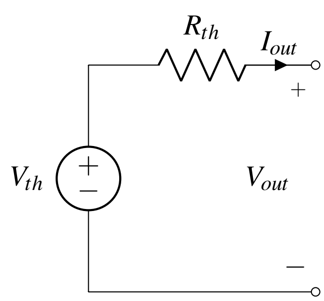
(e) The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$. Notice that adding a parallel resistor does not change the Thévenin equivalent. As before, since the circuit is effectively a voltage source, a Norton equivalent is not required.

(f) The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$. Notice that adding a series resistor does not change the Norton equivalent. With a similar argument as before, the Thévenin equivalent for the source is not required, as it is a current source.

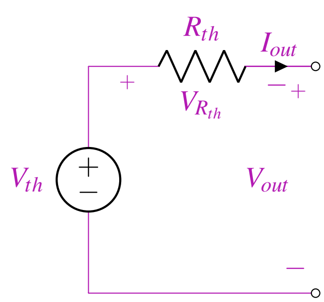
2. Thevenin equivalence

(a) You are given the following $I_{out} - V_{out}$ characteristic of the Thevenin model of a circuit. Find the Thevenin voltage and the Thevenin resistance.

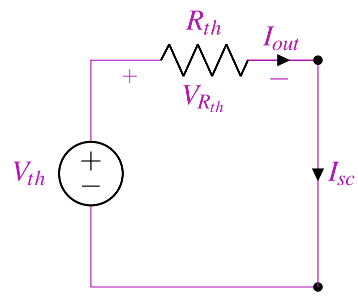




Answer: The Thevenin voltage corresponds to the open circuit voltage of the $I_{out} - V_{out}$ characteristic and is equal to 5V. By KVL, $-V_{th} + V_{R_{th}} + V_{out} = 0$. We also can see that $I_{out} = I_{R_{th}}$.

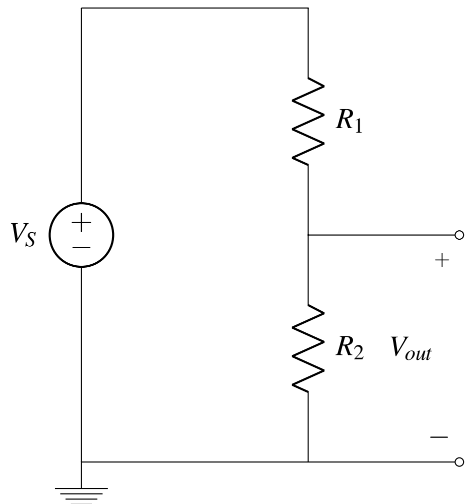


When the output is shorted, $I_{out} = I_{sc}$, and $V_{out} = 0V$, which implies that $V_{R_{th}} = V_{th}$, and $I_{R_{th}} = I_{sc}$.



Thus by Ohm's law, the Thevenin resistance is equal to $R_{th} = \frac{V_{R_{th}}}{I_{R_{th}}} = \frac{V_{th}}{I_{sc}}$.
Therefore $R_{th} = 5\Omega$.

(b) You are given a voltage divider as shown below. Find R_1 and R_2 such that the Thevenin equivalent model is the same as that of (a). You are given that $V_S = 10V$.



Answer:
From part (a), the Thevenin voltage corresponds to the open circuit voltage of the $I_{out} - V_{out}$ characteristic and is equal to 5V.
The Thevenin voltage V_{th} of the voltage divider is equal to:

$$V_{th} = V_s \frac{R_2}{R_1 + R_2}$$
$$= 10V \frac{R_2}{R_1 + R_2} = 5V$$
$$10V R_2 = 5V R_1 + 5V R_2$$
$$5V R_2 = 5V R_1$$

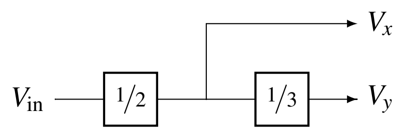
Therefore $R_1 = R_2$.
The short circuit current I_{sc} should be equal to $1A$. If we short the output of the voltage divider circuit we have:

$$I_{sc} = \frac{V_s}{R_1} = \frac{10}{R_1} = 1A$$

Therefore $R_1 = 10\Omega$ and $R_2 = 10\Omega$.

3. Modular Circuits

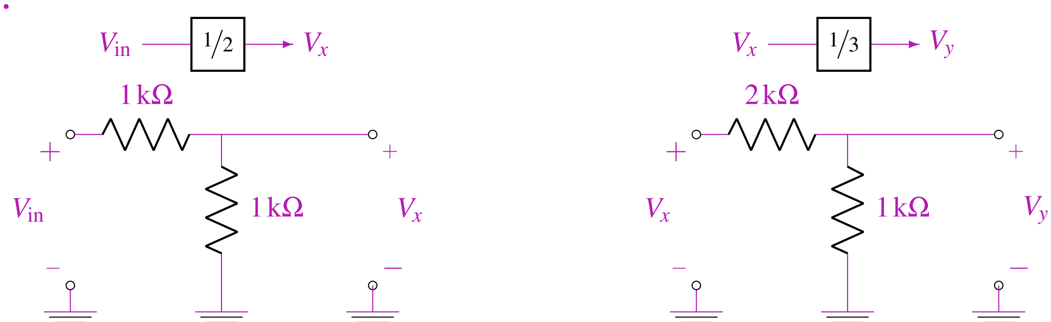
In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations. (Note that the so-called analog signal processing – where these kinds of mathematical operations are performed on continuously-valued voltages by analog circuits – is extremely common in real-world applications; without this capability, essentially none of our radios or sensors would actually work.) Specifically, let's assume that we want to implement the block diagram shown below:



In other words, we want to implement a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{\text{in}}$ and $V_y = \frac{1}{3}V_x$.

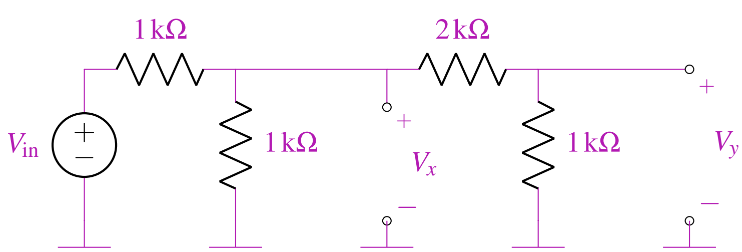
(a) Design two voltage divider circuits that each independently would implement the two multiplications shown in the block diagram above (i.e., multiply by 1/2 and multiply by 1/3). Note that you do not need to include the input voltage sources in your design – you can simply define the input to each block as being at the appropriate potential (e.g., V_{in} or V_x).

Answer:



(b) Assuming that V_{in} is created by an ideal voltage source, implement the original block diagram as a circuit by directly replacing each block with the designs you came up with in part (a).

Answer:



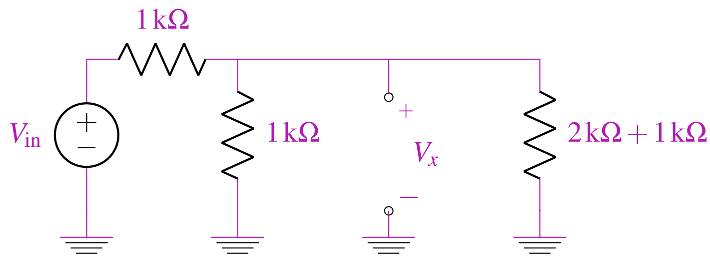
(c) For the circuit from part (b), find V_x and V_y. Do you get the desired relationship between V_y and V_x? How about between V_x and V_{in}? Be sure to explain why or why not each block retains its desired functionality.

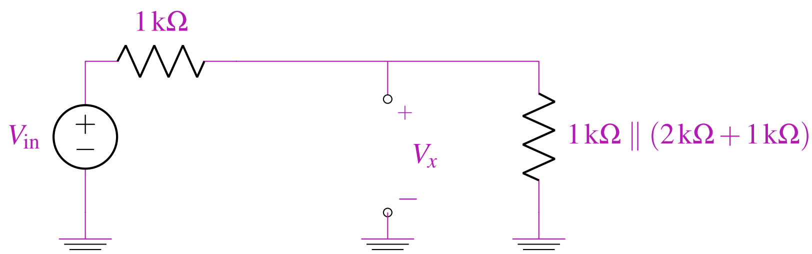
Answer:

The relationship between V_y and V_x will be correct. We can apply the voltage divider equation to the two rightmost resistors to see this:

$$V_y = \frac{1\,\text{k}\Omega}{1\,\text{k}\Omega + 2\,\text{k}\Omega} V_x = \frac{1}{3} V_x$$

The relationship between V_x and V_{in} will not be correct. To see this, we can redraw the circuit applying resistance series and parallel rules.



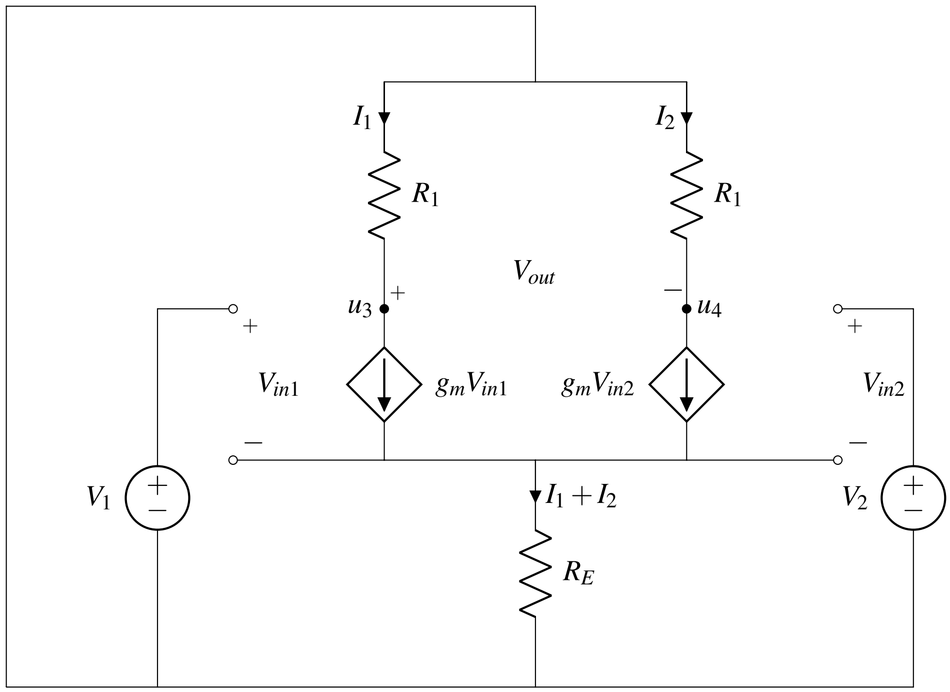


Now we can apply the voltage divider equation to see that:

$$V_x = \frac{1\,\text{k}\Omega \parallel (2\,\text{k}\Omega + 1\,\text{k}\Omega)}{1\,\text{k}\Omega + 1\,\text{k}\Omega \parallel (2\,\text{k}\Omega + 1\,\text{k}\Omega)} V_{in} = \frac{3}{7} V_{in}$$

which was not the desired relationship between V_x and V_{in}.

4. Superposition (Optional)



(a) Calculate V_{out} with only V_1 on. Repeat this with only V_2 on.

Answer:

First we calculate V_{out} when V_1 is on. We will call this $V_{out,1}$.

$$\begin{aligned} V_{in1} &= V_1 - (I_1 + I_2)(R_E) \\ I_1 = g_m V_{in1} &= g_m(V_1 - (I_1 + I_2)(R_E)) \\ u_3 = 0 - R_1 I_1 &= -R_1 g_m (V_1 - (I_1 + I_2)(R_E)) \\ V_{in2} &= -(I_1 + I_2)(R_E) \\ I_2 = g_m V_{in2} &= g_m(-(I_1 + I_2)(R_E)) \\ u_4 = 0 - R_1 I_2 &= -R_1 g_m(-(I_1 + I_2)(R_E)) \\ V_{out,1} = u_3 - u_4 &= -g_m R_1 (V_1) \end{aligned}$$

Similarly, we can find V_{out} when V_2 is on. Let us call this $V_{out,2}$. We see that the circuit in this situation is a mirror image of the one we just solved for. Therefore, we can conclude the expression for $u_3 - u_4$ in the new circuit is the same as $-(u_3 - u_4)$, where we substitute V_2 for V_1. We thus have:

$$V_{out,2} = g_m R_1 (V_2)$$

(b) Let's now turn on both V_1 and V_2. What is the output V_{out}? What does this circuit do to arbitrary input voltages?

Answer:

From superpostion, we can calculate $V_{out} = -g_m R_1(V_1 - V_2)$. For two arbitrary inputs, this circuit *amplifies* the *difference*. We will learn more about amplification in the coming weeks. This particular circuit actually forms one of the most essential blocks inside op-amps, a device we will explore in further detail in the coming weeks.