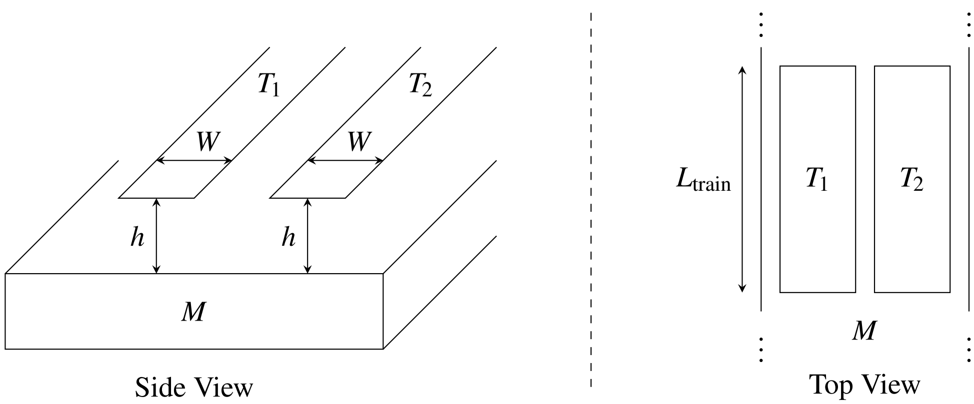
EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 9A

1. **Maglev Train Height Control System**

One of the fastest forms of land transportation are trains that actually travel slightly elevated from ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how the maglev trains use capacitors to keep them elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you e.g. get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, let's imagine that all along the bottom of the train, we put two parallel strips of metal (T_1, T_2), and that on the ground below the train (perhaps as part of the track), we have one solid piece of metal (M).



Assuming that the entire train is at a uniform height above the track and ignoring any fringing fields (i.e., all capacitors are purely parallel plate), as a function of L_{train} (the length of the train), W (the width of T_1/T_2), and h (the height of the train off of the track), what is the capacitance between T_1 and M? How about the capacitance between T_2 and M?

Answer:

The distance between the plates (T_1 & M or T_2 & M) is h. The area of plate for the parallel plate capacitor is $A = WL_{\text{train}}$. Using the formula for capacitance of a parallel plate capacitor, we get:

$$C = \frac{\varepsilon A}{d}$$

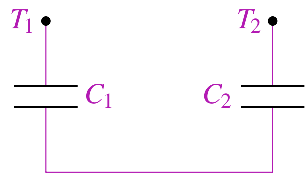
$$C_1 = \frac{\varepsilon W L_{\text{train}}}{h} \text{ (Capacitance between T}_1\text{ and M)}$$

$$C_2 = \frac{\varepsilon W L_{\text{train}}}{h} \text{ (Capacitance between T}_2\text{ and M)}$$

(b) Any circuit on the train can only make direct contact at T_1 and T_2. To detect the height of the train, it would only be able to measure the effective capacitance between T_1 and T_2. Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other.

Answer:

The capacitors C_1 and C_2 are in series. To realize this, let's consider the train circuit that is in contact with T_1 and T_2. If there is current entering plate T_1, the same current has to exit plate T_2. Thus, the circuit can be modeled as follows:



(c) Using the same parameters as in part (a), provide an expression for the capacitance between T_1 and T_2.

Answer:

Since the two capacitors are in series, the effective capacitance between T_1 and T_2 is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

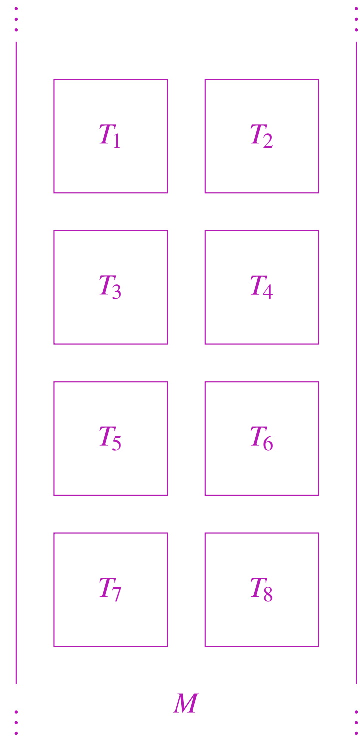
Thus, we get

$$\frac{1}{C_{\text{eq}}} = \frac{h}{\varepsilon W L_{\text{train}}} + \frac{h}{\varepsilon W L_{\text{train}}}$$

$$C_{\text{eq}} = \frac{\varepsilon W L_{\text{train}}}{2h}$$

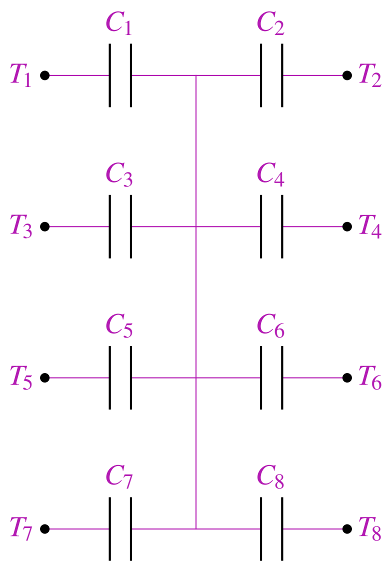
(d) So far we've assumed that the height of the train off of the track is uniform along its entire length, but in practice, this may not be the case. Suggest and sketch a modification to the basic sensor design (i.e., the two strips of metal T_1/T_2 along the entire bottom of the train) that would allow you to measure the height at the train at 4 different locations.

Answer:



Top View

One important thing to note about this circuit is that it works only if extra care is taken during the capacitance measurement circuit. The equivalent model for this is:

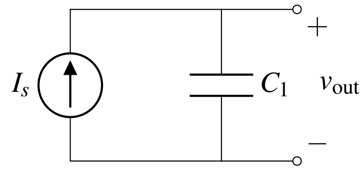


Therefore, the circuit needs separate switches on each T, so that you can measure the capacitance between only two terminals (like T_1 & T_2) and so that the effect of other capacitors is nullified.

2. Current Sources And Capacitors

For the circuits given below, give an expression for $v_{out}(t)$ in terms of I_s, C_1, C_2, C_3, and t. Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is 0 V.

(a)

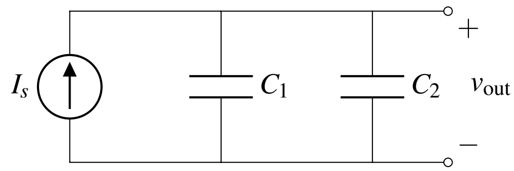


Answer:

$$I_s = C_1 \frac{dv_{out}(t)}{dt}$$
$$v_{out}(t) = \int \frac{I_s}{C_1} dt = \frac{I_s t}{C_1} + v_{out}(0)$$

Since the capacitor is initially uncharged, $v_{out}(0) = 0$, so $v_{out}(t) = \frac{I_s t}{C_1}$.

(b)



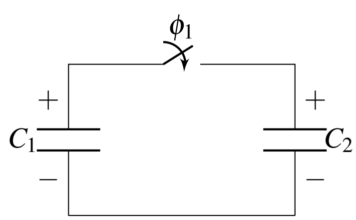
Answer:

We can combine the two capacitors into an equivalent capacitor with capacitance $C_1 + C_2$. Again, $v_{out}(0) = 0$ because all capacitors are initially uncharged.

$$I_s = (C_1 + C_2)\frac{dv_{out}(t)}{dt}$$
$$v_{out}(t) = \frac{I_s t}{C_1 + C_2} + v_{out}(0) = \frac{I_s t}{C_1 + C_2}$$

3. Capacitors and Charge Conservation

(a) Consider the circuit below with $C_1 = C_2 = 1\,\mu F$ and an open switch. Suppose that C_1 is initially charged to $+1\,V$ and that C_2 is charged to $+2\,V$. How much charge is on C_1 and C_2?



Answer:

$$q_1 = C_1 V_1 = 1\,\mu C$$
$$q_2 = 2\,\mu C$$

(b) Now the switch is closed (i.e. the capacitors are connected together.) What are the voltages across and the charges on C_1 and C_2?

Answer:

Charge is always conserved.

Let $Q_{C_1,1}, Q_{C_2,1}$ be the charges on the capacitors after the switch is closed. There was $3\,\mu\text{C}$ of total charge on the top two plates of the capacitors initially, so we must have

$$Q_{C_1,1} + Q_{C_2,1} = 3\,\mu\text{C}$$

Further, the voltages on the capacitors must be the same, so:

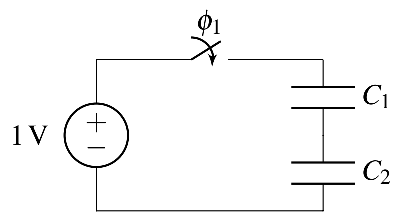
$$\frac{Q_{C_1,1}}{C_1} = \frac{Q_{C_2,1}}{C_2}$$

Solving this system gives:

$$Q_{C_1,1} = Q_{C_2,1} = 1.5\,\mu\text{C}$$

Comparing to the previous part, charge has moved from C_2 to C_1. This yields a voltage of $1.5\,\text{V}$.

(c) Consider the following circuit with $C_1 = 1\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$. Suppose that both capacitors are initially uncharged ($0\,\text{V}$).



What are the voltages across each capacitor after the switch is closed? What are the charges on each capacitor?

Answer:

Solution 1: Use equivalent capacitance:

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

The series has an equivalent capacitance $C_{eq} = \frac{3}{4}\mu\text{F}$, so $Q_{eq} = C_{eq}V = \frac{3}{4}\mu\text{C}$. Note that this means $+\frac{3}{4}\mu\text{C}$ is on the top plate of C_1 (and hence the top plate of C_2, etc.).

The voltage across C_1 is $\frac{Q_{eq}}{C_1} = \frac{3}{4}\text{V}$. The voltage across C_2 is $\frac{Q_{eq}}{C_2} = \frac{1}{4}\text{V}$.

Solution 2: Let an unknown $+q$ charge be on the top plate of C_1. Then by charge conservation, $-q$ charge is on the bottom plate of C_2. And since conductors have no $\vec{E}$ field, $-q$ is on the bottom plate of C_1, and $+q$ on the top of C_2.

Now, by KVL, the voltage across the series is $1\,\text{V}$:

$$\frac{q}{C_1} + \frac{q}{C_2} = 1\,\text{V}$$

Therefore:

$$q = \frac{1\,\text{V}}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Notice that we have derived the formula for equivalent capacitance of capacitors in series.