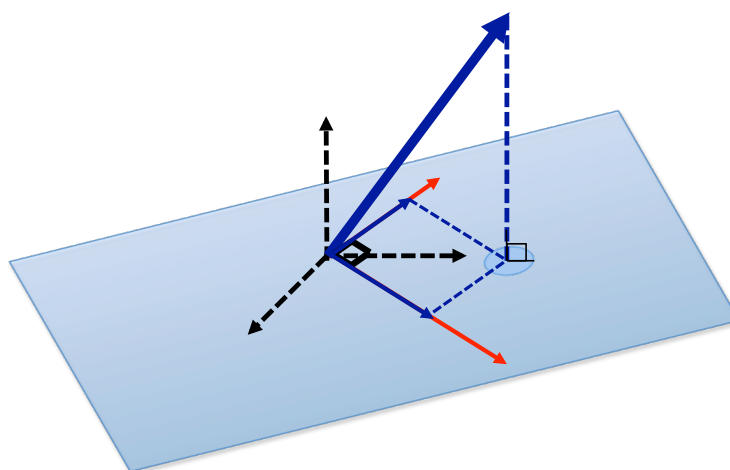


- (b) We now consider the special case of least squares where the columns of \mathbf{A} are orthogonal (illustrated in the figure below). Given that $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \vec{b}$ and $A\hat{\mathbf{x}} = \text{proj}_{\mathbf{A}}(\vec{b}) = \hat{x}_1 \vec{a}_1 + \hat{x}_2 \vec{a}_2$, show that

$$\text{proj}_{\vec{a}_1}(\vec{b}) = \hat{x}_1 \vec{a}_1$$

$$\text{proj}_{\vec{a}_2}(\vec{b}) = \hat{x}_2 \vec{a}_2$$



(c) Compute the least squares solution to

$$\min_{\vec{x}} \left\| \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right\|^2.$$

3. Polynomial Fitting

Let's try an example. Say we know that the output, y , is a quartic polynomial in x . This means that we know that y and x are related as follows:

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

We're also given the following observations:

x	y
0.0	24.0
0.5	6.61
1.0	0.0
1.5	-0.95
2.0	0.07
2.5	0.73
3.0	-0.12
3.5	-0.83
4.0	-0.04
4.5	6.42

- What are the unknowns in this question? What are we trying to solve for?
- Can you write an equation corresponding to the first observation (x_0, y_0) , in terms of a_0, a_1, a_2, a_3 , and a_4 ? What does this equation look like? Is it linear in the unknowns?
- Now, write a system of equations in terms of a_0, a_1, a_2, a_3 , and a_4 using *all of the observations*.
- Finally, solve for a_0, a_1, a_2, a_3 , and a_4 using IPython. You have now found the quartic polynomial that best fits the data!