EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 3A

1. **Matrix Multiplication**

   Consider the following matrices:

   $\mathbf{A_1} = \begin{bmatrix} 1 & 4 \end{bmatrix}$ $\mathbf{B_1} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix}$ $\mathbf{D} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix}$ $\mathbf{E} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$

   $\mathbf{F} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$

   For each matrix multiplication problem, if the product exists, find the product by hand. Otherwise, explain why the product does not exist.

   (a) $\mathbf{A_1 B_1}$

   (b) $\mathbf{AB}$

   (c) $\mathbf{BA}$

   (d) $\mathbf{AC}$

   (e) $\mathbf{DC}$

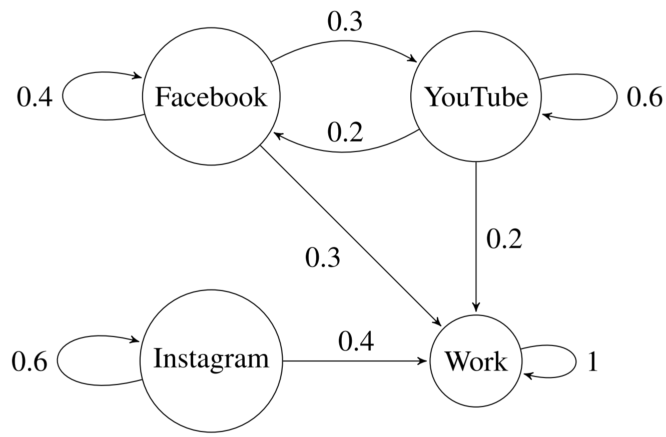
   (f) $\mathbf{CD}$ (Write down the dimensions of the product if it exists. For practice, you can compute the product on your own)

   (g) $\mathbf{EF}$ (Practice on your own)

   (h) $\mathbf{FE}$ (Practice on your own)

2. **Social Media**

   As a tech-savvy Berkeley student, the distractions of social media are always calling you away from productive stuff like homework for your classes. You're curious—are you the only one who spends hours switching between Facebook or YouTube? How do other students manage to get stuff done and balance pursuing Insta-fame? You conduct an experiment, collect some data, and notice Berkeley students tend to follow a pattern of behavior similar to the figure below. So, for example, if 100 students are on Facebook, in the next timestep, 30 of them will click on a link and move to YouTube.



(a) Derive the corresponding transition matrix.

(b) There are 1500 of you in the class. Suppose on a given Friday evening (the day when HW is due), there are 700 EECS16A students on Facebook, 450 on YouTube, 200 on Instagram, and 150 actually doing work. In the next timestep, how many people will be doing each activity? In other words, after you apply the matrix once to reach the next timestep, what is the state vector?

(c) Compute the sum of each column in the state transition matrix. What is the interpretation of this?

(d) You want to predict how many students will be on each website n timesteps in the future. How would you formulate that mathematically? Without working it out, can you predict roughly how many students will be in each state 100 timesteps in the future?

(e) **Challenging Practice Problem:** Suppose, instead of having 'Work' as an explicit state, we assume that any student not on Facebook/Youtube/Instagram is working. Work is like the "void," and if a student is "leaked" from any of the other states, we assume s/he has gone to work and will never come back. How would you reformulate this problem? Redraw the figure and rewrite the appropriate transition matrix. What are the major differences between this problem and the previous one?

3. Span Proofs

Given some set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$, show the following:

(a)

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = \text{span}\{\alpha\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}, \text{ where } \alpha \text{ is a non-zero scalar}$$

In other words, we can scale our spanning vectors and not change their span.

(b) **(Practice)**

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = \text{span}\{\vec{v}_2, \vec{v}_1, \ldots, \vec{v}_n\}$$

In other words, we can swap the order of our spanning vectors and not change their span.