

1. Mechanical Inverses

In each part, determine whether the inverse of \mathbf{A} exists. If it exists, find it.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

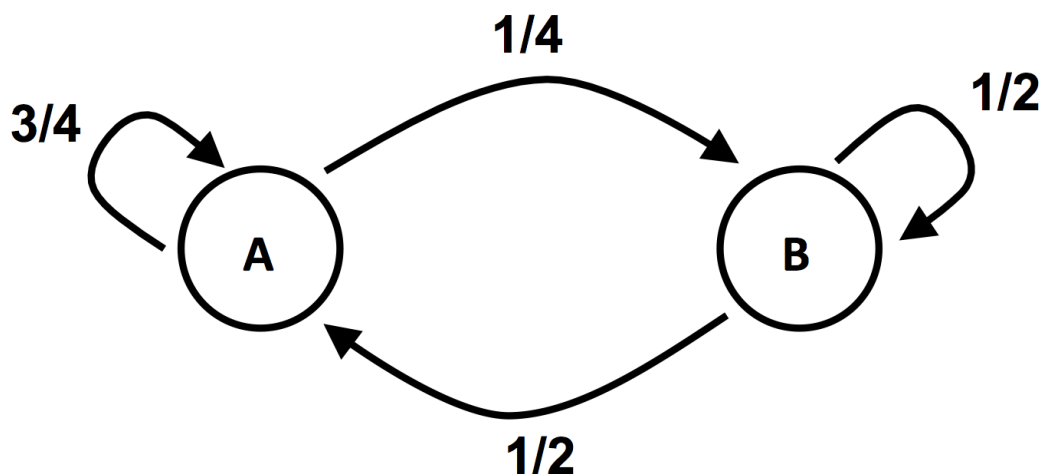
(d) (PRACTICE)

$$\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 0 & 4 \end{bmatrix}$$

2. Transition Matrix

(a) Suppose there exists some network of pumps as shown in the diagram below. Let $\vec{x}(n) = \begin{bmatrix} x_A(n) \\ x_B(n) \end{bmatrix}$ where $x_A(n)$ and $x_B(n)$ are the states at timestep n .

Find the state transition matrix S , such that $\vec{x}(n+1) = S\vec{x}(n)$.



- (b) Let us now find the matrix S^{-1} such that we can recover $\vec{x}(n-1)$ from $\vec{x}(n)$. Specifically, solve for S^{-1} such that $\vec{x}(n-1) = S^{-1}\vec{x}(n)$.
- (c) Now draw the state transition diagram that corresponds to the S^{-1} that you just found.
- (d) Redraw the diagram from the first part of the problem, but now with the directions of the arrows reversed. Let us call the state transmission matrix of this "reversed" state transition diagram T . Does $T = S^{-1}$?