EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 4B

1. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that output the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of **A**? What is its dimension?
ii. What is the null space of **A**? What is its dimension?
iii. Are the column spaces of the row reduced matrix **A** and the original matrix **A** the same?
iv. Do the columns of **A** form a basis for $\mathbb{R}^2$? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

2. Constructing a Basis

Let's consider a subspace of $\mathbb{R}^3$ called V which has the following property: for every vector in V, the first entry is equal to two times the sum of the second and third entries. That is, if $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \in V$, $a_1 = 2(a_2 + a_3)$.

Find a basis for V. What is the dimension of V? Is this basis unique?

3. Identifying a Subspace: Proof

Is the set

$$V = \left\{ \vec{v} \;\middle|\; \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \right\}$$

a subspace of $\mathbb{R}^3$? Why/why not?

4. (PRACTICE) Exploring Dimension, Linear Independence, and Basis

In this problem, we are going to talk about the connections between several concepts we have learned about in linear algebra – linear independence, dimension of a vector space/subspace, and basis.

Let's consider the vector space $\mathbb{R}^k$ and a set of n vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ in $\mathbb{R}^k$.

(a) For the first part of the problem, let $k > n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^k$? Why/why not? What conditions would we need?

(b) Let $k = n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^k$? Why/why not? What conditions would we need?

(c) Now, let $k < n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^k$? What vector space could they form a basis for?

Hint: Think about whether the vectors can be linearly independent.