EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 5A

Definition: If $A\vec{x} = \lambda\vec{x}$, then $\lambda \in \mathbb{R}$ is called an eigenvalue of A. $\vec{x}$ belongs to the eigenspace of A corresponding to eigenvalue λ. All vectors $\vec{x}$ in the eigenspace are called eigenvectors corresponding to the eigenvalue λ.

1. Mechanical Eigenvalues and Eigenvectors

In each part, find the eigenvalues of the matrix **M** and the associated eigenvectors. If the inverse of **M** exists, find it.

(a) $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(b) $\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

(c) **(PRACTICE)** $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

(d) **(PRACTICE)** $\mathbf{M} = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$

2. Steady State Reservoir Levels

We have 3 reservoirs: A, B and C. The pumps system between the reservoirs is depicted in Figure 1.

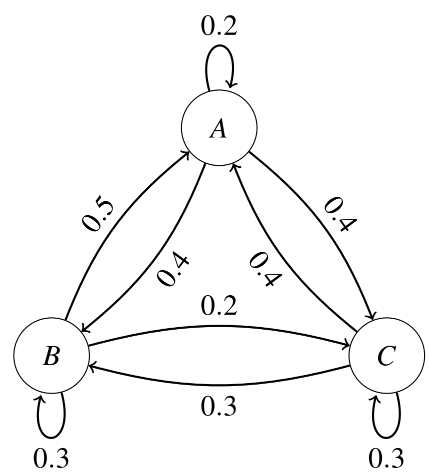

Figure 1: Reservoir pumps system.

(a) Write out the transition matrix **T** representing the pumps system.

(b) You are told that $\lambda_1 = 1$, $\lambda_2 = \frac{-\sqrt{2}-1}{10}$, $\lambda_3 = \frac{\sqrt{2}-1}{10}$ are the eigenvalues of **T**. Find a steady state vector $\vec{x}$, i.e. a vector such that $T\vec{x} = \vec{x}$.

3. Proofs

(a) Let $\mathbf{A}$ be an invertible matrix. Show that if λ is an eigenvalue of $\mathbf{A}$, then $\frac{1}{\lambda}$ is an eigenvalue of $\mathbf{A}^{-1}$.

