
EECS 16A Designing Information Devices and Systems I Discussion 5B
 Fall 2019

1. Eigenvalues and Special Matrices – Visualization

An eigenvector \vec{v} belonging to a square matrix \mathbf{A} is a nonzero vector that satisfies

$$\mathbf{A}\vec{v} = \lambda\vec{v}$$

where λ is a scalar known as the **eigenvalue** corresponding to eigenvector \vec{v} .

The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

- (a) Does the identity matrix in \mathbb{R}^n have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

- (b) Does a diagonal matrix $\begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}$ in \mathbb{R}^n have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

- (c) Does a rotation matrix in \mathbb{R}^2 have any eigenvalues $\lambda \in \mathbb{R}$?

- (d) Does a reflection matrix, where the reflection is around any line passing through the origin, have any eigenvalues $\lambda \in \mathbb{R}$?

- (e) If a matrix \mathbf{M} has an eigenvalue $\lambda = 0$, what does this say about its null space? What does this say about the solutions of the system of linear equations $\mathbf{M}\vec{x} = \vec{b}$?

- (f) Does the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

2. Mechanical Determinants

- (a) Compute the determinant of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.
- (b) Compute the determinant of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

3. Steady and Unsteady States

- (a) You're given the matrix \mathbf{M} (below) which describes some physical system (could describe either people or water):

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

Find the eigenspaces associated with the following eigenvalues:

- i. $\text{span}(\vec{v}_1)$, associated with $\lambda_1 = 1$
 - ii. $\text{span}(\vec{v}_2)$, associated with $\lambda_2 = 2$
 - iii. $\text{span}(\vec{v}_3)$, associated with $\lambda_3 = \frac{1}{2}$
- (b) Define $\vec{x} = \alpha\vec{v}_1 + \beta\vec{v}_2 + \gamma\vec{v}_3$. The values α, β , and γ are the coordinates for the basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. For each of the cases in the table, determine if

$$\lim_{n \rightarrow \infty} \mathbf{M}^n \vec{x}$$

converges. If it does, what does it converge to?

α	β	γ	Converges?	$\lim_{n \rightarrow \infty} \mathbf{M}^n \vec{x}$
0	0	$\neq 0$		
0	$\neq 0$	0		
0	$\neq 0$	$\neq 0$		
$\neq 0$	0	0		
$\neq 0$	0	$\neq 0$		
$\neq 0$	$\neq 0$	0		
$\neq 0$	$\neq 0$	$\neq 0$		