EECS 16A Designing Information Devices and Systems I
Fall 2019 Practice Handout

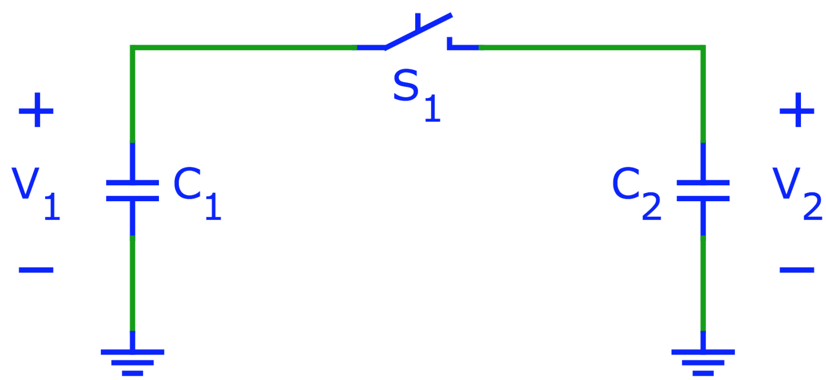
This handout contains a selection of problems that will serve as preparation for Midterm 2.

1. **Charge Sharing**

   In the circuit below, switch S_1 is initially open. Capacitor $C_1 = 10^{-3}$F is initially charged to $V_1 = 3$V and capacitor $C_2 = 3 \times 10^{-3}$F is initially charged to $V_2 = 2$V.

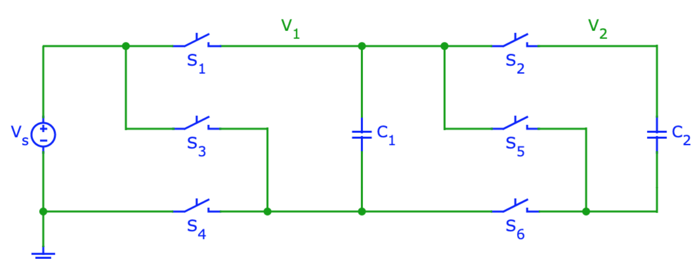
   Now S_1 is closed. Calculate the new value of V_2.

   Hint: Charge is conserved.



2. **Voltage Booster**

   We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply $V_S = 5V$? We can do this with capacitors!

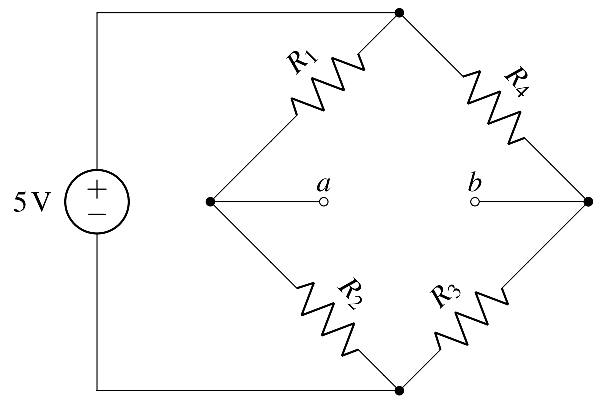


   (a) In the circuit above switches S1, S2, S4 and S6 are initially closed and switches S3 and S5 open. Calculate voltages V_1 and V_2.

   (b) Now, after the capacitors are charged, switches S1, S2, S4 and S6 are opened and switches S3 and S5 closed. Calculate the new voltages V_1 and V_2.

3. Wheatstone Bridge

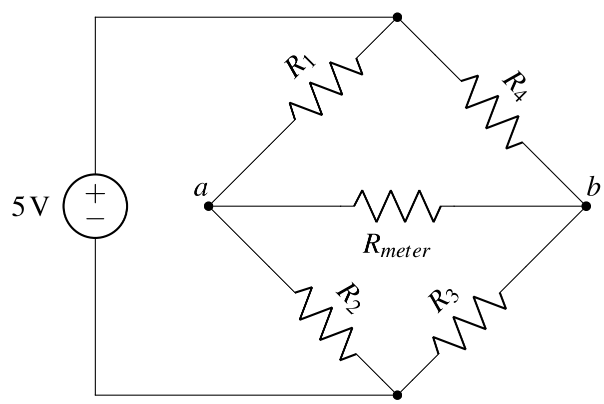
Thévenin equivalence is a powerful technique we can use to analyze the Wheatstone bridge circuit shown below. This circuit is used in many sensor applications where resistors $R_1 - R_4$ are varying with respect to some external actuation. For example, it can be used to build a strain gauge or a scale (remember Fruity Fred from HW 7). In that case the resistors $R_1 - R_4$ would vary with respect to a strain caused by a force, and the Wheatstone Bridge circuit would translate that variation into a voltage difference across the "bridge" terminals a and b. Assume that $R_1 = 2\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$, $R_4 = 4\,\text{k}\Omega$



(a) Calculate the voltage V_{ab} between the two terminals a and b.

(b) Next, draw the Thévenin equivalent of the Wheatstone bridge circuit.

(c) Now assume that you are trying to measure the voltage V_{ab} using a voltmeter, whose resistance is R_{meter}, so you end up with the circuit below.



Unfortunately, you voltmeter is far from ideal, so $R_{meter} = 4k\Omega$. Is the voltage V_{ab} you found in part (a) equal to the new voltage $V_{R_{meter}}$ across the voltmeter resistor? Why or why not? Calculate the current $I_{R_{meter}}$ through the voltmeter resistor and the voltage $V_{R_{meter}}$ across the meter resistor.

4. Digital to Analog Converter (DAC)

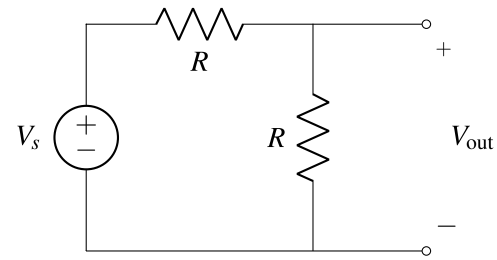
For some outputs, such as audio applications, we need to produce an analog output, or a continuous voltage from 0 to V_s. These analog voltages must be produced from digital voltages, that is sources, that can only be V_s or 0. A circuit that does this is known as a Digital to Analog Converter. It takes a binary representation of a number and turns it into an analog voltage.

The output of a DAC can be represented with the equation shown below:

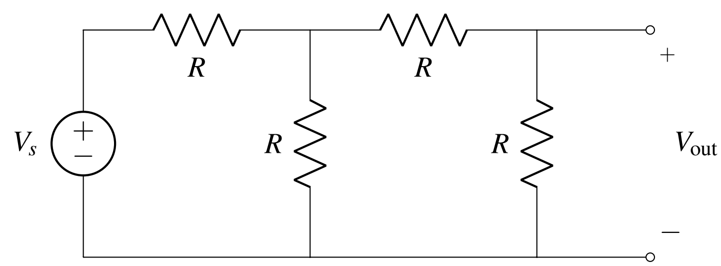
$$V_{\text{out}} = V_s \sum_{n=0}^{N} \frac{1}{2^n} \cdot b_n$$

where each binary digit b_n is multiplied by $\frac{1}{2^n}$.

(a) We know how to take an input voltage and divide it by 2:



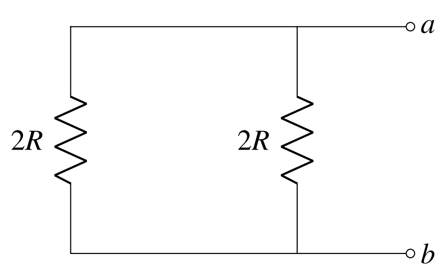
To divide by larger powers of two, we might hope to just "cascade" the above voltage divider. For example, consider:



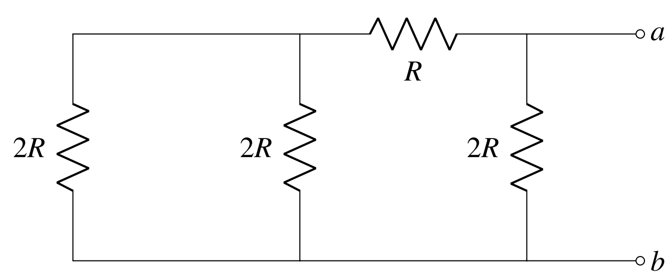
Calculate V_{out} in the above circut. Is $V_{\text{out}} = \frac{1}{4}V_s$?

(b) The R-$2R$ ladder, shown below, has a very nice property. For each of the circuits shown below, find the equivalent resistance looking in from points a and b. Do you see a pattern?

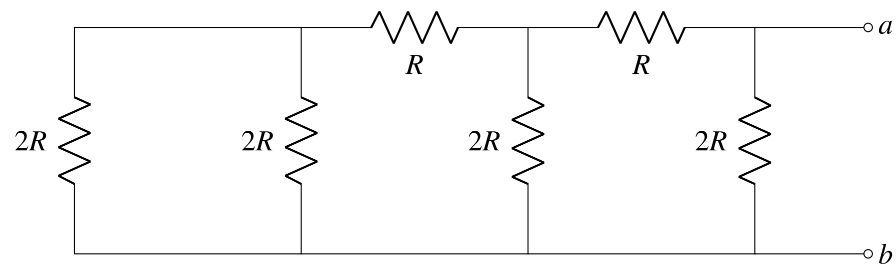
i.



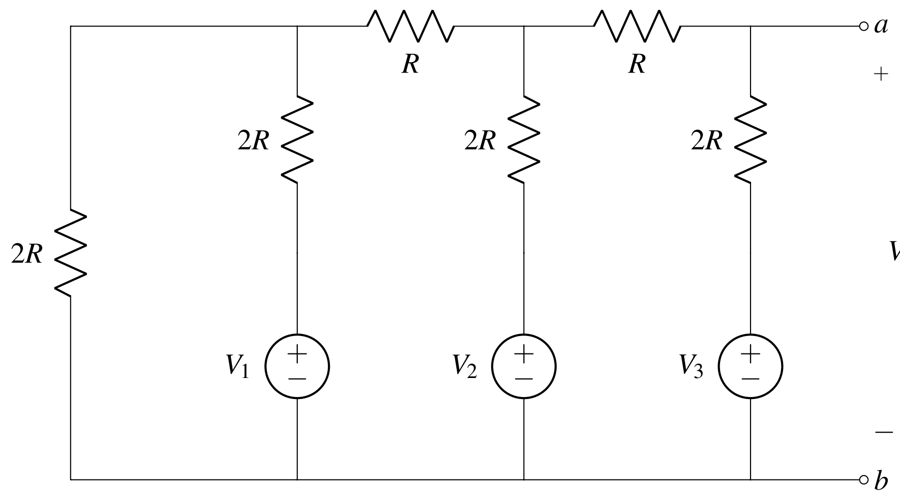
ii.



iii.



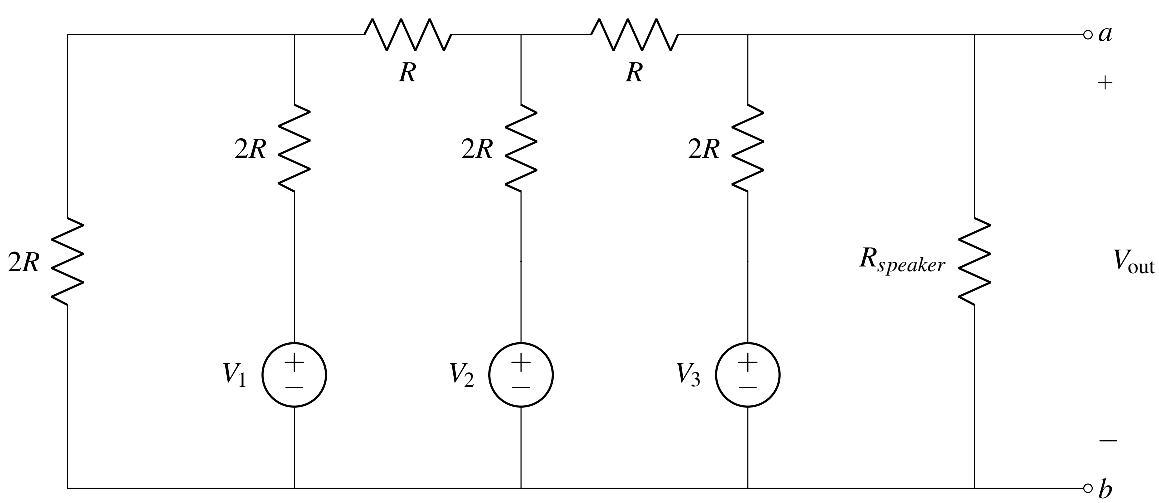
(c) The following circuit is an R-$2R$ DAC. To understand its functionality, use superposition to find V_{out} in terms of each V_k in the circuit.



(d) We've now designed a 3-bit R-$2R$ DAC. What is the output voltage V_{out} if $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$?

(e) Draw the Thévenin equivalent of the above circuit, looking in from the terminals a and b with $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$.

(f) Suppose that we now attach a speaker to the DAC with a resistance of $R_{speaker} = 50\Omega$ as shown in the figure below. Assume also, that the value of R is 50Ω as well. What is the voltage across the speaker and the power dissipated by the speaker? *Hint:* Use the Thevenin equivalent circuit to calculate the above.



(g) Repeat part (f) now assuming that the speaker resistance is 100Ω. The value of R remains 50Ω. How do the power and voltage values you found in the two parts compare? Why is the voltage across the speaker in both cases lower than V_{th}?

5. Challenge Problem: Average

The circuit in Figure 1 below operates in time steps k, as illustrated in Figure 2. Each step is of duration T. During each step, switch S_2 is opened, and then switch S_1 is closed immediately afterwards. Then, after time $T/2$, switch S_1 is opened just before switch S_2 is closed.

At the end of each time step the input $V_{in}(kT)$ changes and the process repeats.

Derive an expression for $V_2(kT)$ as a function of $V_{in}(kT)$. Use $C_1 = pC_0$ and $C_2 = (1-p)C_0$.

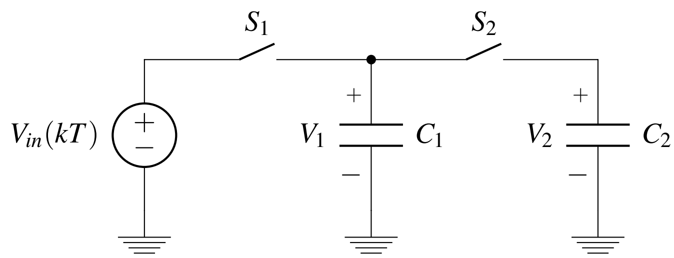


Figure 1: Averaging circuit

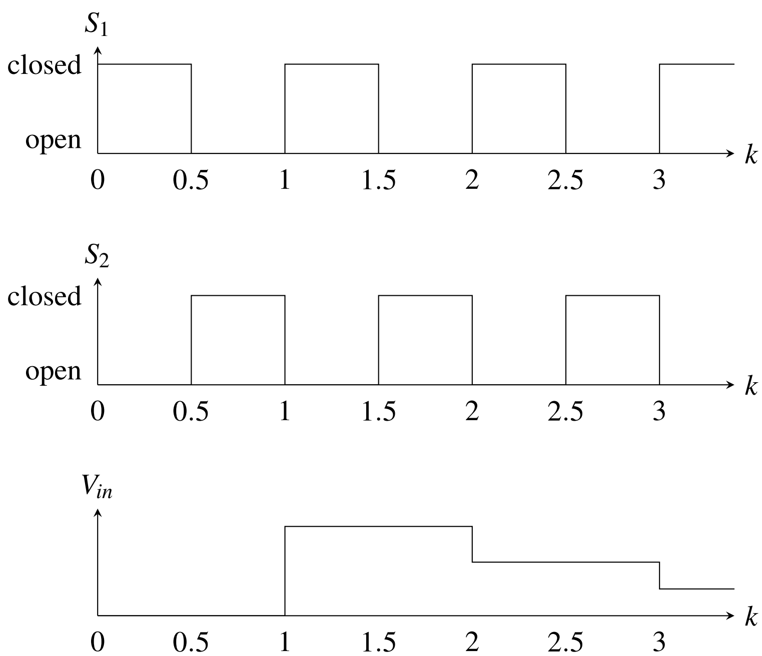


Figure 2: Timing diagram