This homework is due October 25, 2019, at 23:59.
Self-grades are due October 28, 2019, at 23:59.

Submission Format
Your homework submission should consist of one file.

- **hw8.pdf**: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. Power Analysis

   ![Circuit Diagram](image)

   (a) Find the power dissipated by each element in the circuit above. Remember to label voltages using passive sign convention.

   **Solution**: We label a ground node, and then solve for the currents $i_V, i_R$ and the voltages $V_R, V_I$.

   ![Solved Circuit Diagram](image)

   Solving the above circuit using nodal analysis, we get
   
   $$i_R = \frac{V_s}{R}$$
   $$i_V = I - \frac{V_s}{R}$$
   $$v_I = -V_s$$
   $$v_R = V_s$$
Using this we can calculate

\[ P_V = V_s i_V = IV_s - \frac{V_s^2}{R} \]
\[ P_I = I V_I = -IV_s \]
\[ P_R = i_r V_R = \frac{V_s^2}{R} \]

Note that \( P_V + P_I + P_R = 0 \), i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

(b) Use \( R = 5k\Omega \), \( V_s = 5V \), and \( I = 5mA \). Calculate \( P_V, P_I, \) and \( P_R \).

**Solution:**

\[ P_V = (0.005A)(5V) - \frac{(5V)^2}{5000\Omega} = 0.02W \]
\[ P_I = -(0.005A)(5V) = -0.025W \]
\[ P_R = \frac{(5V)^2}{5000\Omega} = 0.005W \]

Note that \( P_V + P_I + P_R = 0 \).

(c) Repeat part (b) but change the value \( I \) of the current source such that it dissipates 40mW. Calculate \( I, P_V, P_I, \) and \( P_R \).

**Solution:**

Remember that using passive sign convention, an element whose power is negative is supplying power, and an element whose power is positive is dissipating power. Therefore, we want \( P_I = 40mW \). We know that \( P_I = -IV_s \). Therefore, \( I = \frac{0.04W}{5V} = -0.008A \).

\[ P_V = (-0.008A)(5V) - \frac{(5V)^2}{5000\Omega} = -0.045W \]
\[ P_I = -(-0.008A)(5V) = 0.04W \]
\[ P_R = \frac{(5V)^2}{5000\Omega} = 0.005W \]

Note that \( P_V + P_I + P_R = 0 \).

2. **Maximum Power Transfer**

Smartphones use "bars" to indicate strength of the cellular signal. Few "bars" translate to slow or no connectivity.

But what do these "bars" actually stand for? Voltage, current? Well, not quite. Good radio - and a cellular modem is nothing but a particular kind of radio - reception depends on the power received from the transmitter.

In this assignment we design a receiver that maximizes the power received, and hence connection speed.

The figure below shows electronic circuit models for the antenna (a) and the receiver (b). The antenna consists of source \( V_s \), typically in the range of micro- or milli-Volts \( (10^{-6} \) and \( 10^{-3} \), respectively) depending on transmitter strength and resistance \( R_s \), usually 50\( \Omega \) or 75\( \Omega \), depending on the particular antenna design.

The radio receiver is represented by resistor \( R_L \) and chosen carefully by the designer to maximize the received power (i.e. the "number of bars").

It is important to understand that these are *models*. For example, the complete receiver circuit consists of many more elements. Likewise, the antenna consists of several appropriately shaped conductors. The resistor \( R_s \) is nowhere to be found, and neither is \( V_s \) physically present (i.e. you cannot connect a wire to it).
However, these simple models act like the devices they represent. In other words, the voltages and currents at their terminal are identical to the voltages and currents at the terminals of the actual circuits.

Models are very important in engineering design for their ability to abstract away details when they are not needed and are the key to successful design of complex systems.

We will discuss the use and properties of electronic circuit models further in class.

Use the following component values for your calculations: $V_s = 100 \mu V$, and $R_s = 50 \Omega$.

(a) Find the value of $R_L$ that maximizes the voltage $V_L$ across resistor $R_L$. Calculate the values of $V_L$, $I_L$, and the power $P_L$ delivered to (i.e. dissipated in) resistor $R_L$.

**Solution:**

Note that this circuit is a voltage divider, where the voltage across $R_L$ can be written as $V_L = V_S \left( \frac{R_L}{R_L + R_S} \right)$. Taking the derivative of $V_L$ with respect to $R_L$, we find $\frac{dV_L}{dR_L} = \frac{V_S R_S}{(R_S + R_L)^2}$. We note that as $R_L$ increases, $\frac{dV_L}{dR_L}$ approaches 0. Therefore, making $R_L$ as large as possible (ideally $R_L = \infty$) maximizes $V_L$.

If $R_L = \infty$ then $V_L = V_S = 100 \mu V$.

We can use the result $V_L = V_S \left( \frac{R_L}{R_S + R_L} \right)$ to find $I_L = \frac{V_L}{R_L} = \frac{V_S}{R_S + R_L}$. If $R_L = \infty$ then $I_L = 0A$.

We know that $P_L = V_L I_L$. Therefore, $P_L = (100 \mu V) (0A) = 0W$.

(b) Find the value of $R_L$ that maximizes the current $I_L$ through resistor $R_L$. Calculate the values of $V_L$, $I_L$, and the power $P_L$ delivered to resistor $R_L$.

**Solution:**

We can again use the result $V_L = V_S \left( \frac{R_L}{R_S + R_L} \right)$ to calculate $I_L$ and find what $R_L$ should be to maximize $I_L$. We know that $I_L = \frac{V_L}{R_L} = \frac{V_S}{R_S + R_L}$. We can then see $R_L$ should be as small as possible (ideally $R_L = 0$) to maximize $I_L$. If $R_L = 0$ then it behaves like a wire, and there will be no voltage drop across it. Therefore, $I_L = \frac{V_S}{R_S} = \frac{100 \mu V}{50 \Omega} = 2 \mu A$. We know that $P_L = V_L I_L$. Therefore, $P_L = (0V)(2 \mu A) = 0W$.

(c) Find the value of $R_L$ that maximizes the power $P_L$ delivered to resistor $R_L$. Calculate the values of $V_L$, $I_L$, and the power $P_L$ delivered to resistor $R_L$. (**Hint:** The power optimization is best performed algebraically by setting the derivative of $P_L$ with respect to $R_L$ to zero. Alternatively you can do the optimization graphically. Plot $P_L$ versus $R_L$ and find the maximum.)
3. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don’t last a long time. For example, a Google Pixel phone, under typical usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality the voltage drops gradually, but let’s keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel’s battery has a battery capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage drops from 3.8V to zero.

(a) How long will a Pixel’s full battery last under typical usage conditions?

**Solution:**

300mW of power at 3.8V is about 79mA of current. Our 2770mAh battery can supply 79mA for
\[
\frac{2770 \text{mAh}}{79 \text{mA}} = 35 \text{h}, \text{ or about a day and a half.}
\]

(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has 1.602 × 10⁻¹⁹ C of charge.)

**Solution:**

One hour has 3600 seconds, so the battery’s capacity can be written as 2770mAh × 3600 s = 9.972 × 10⁶ mAs = 9972As = 9972C.

An electron has a charge of approximately 1.602 × 10⁻¹⁹ C, so 9972 C is \( \frac{9972 \text{C}}{1.602 \times 10^{-19} \text{C}} \approx 6.225 \times 10^{22} \) electrons. That’s a lot!

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a Ws.

**Solution:**

The battery capacity is 2770mAh at 3.8V, which means the battery has a total stored energy of
\[
2770 \text{mAh} \cdot 3.8 \text{V} = 10.5 \text{Wh} = 10.5 \text{Wh} \cdot 3600 \text{s} = 37.9 \text{kJ}.
\]
(d) Suppose PG&E charges $0.12 per kWh. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?

**Solution:**

2770mAh at 3.8V is 2770mAh · 3.8V = 10.5Wh, or 0.01kWh. At $0.12 per kWh, that is $0.12 · 0.01 per day, or $0.12 · 0.01 · 31 = $0.037, or about 4 cents a month. Compare that to your cell phone data bill! Whew!

(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor $R_{bat}$. We now wish to charge the battery by plugging into a wall plug. The wall plug can be modeled as a 5V voltage source and 200mΩ resistor, as pictured in Figure 2. What is the power dissipated across $R_{bat}$ for $R_{bat} = 1\, \text{mΩ}$, $1\, \Omega$, and $10\, \text{kΩ}$? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of $R_{bat}$?

![Figure 2: Model of wall plug, wire, and battery.](image)

**Solution:**

The energy stored in the battery is 2770mAh at 3.8V, which is 2.77Ah · 3.8V = 10.5Wh. We can find the time to charge by dividing this energy by power in W to get time in hours.

- **For $R_{bat} = 1\, \text{mΩ}$:**
  
  The total resistance seen by the battery is $1\, \text{mΩ} + 200\, \text{mΩ} = 201\, \text{mΩ}$ (because the wire and $R_{bat}$ are in series), so by Ohm’s law, the current is \( \frac{5V}{0.201\Omega} = 24.88\, \text{A} \). The voltage drop across $R_{bat}$ is (again by Ohm’s law) \( 24.88\, \text{A} · 0.001\, \Omega = 0.02488\, \text{V} \). Then power is \( 0.02488\, \text{V} · 24.88\, \text{A} = 0.619\, \text{W} \), and the total time to charge the battery is \( \frac{10.5\, \text{Wh}}{0.619\, \text{W}} = 17\, \text{h} \).

- **For $R_{bat} = 1\, \Omega$:**
  
  The total resistance seen by the battery is \( 1\, \Omega + 0.2\, \Omega = 1.2\, \Omega \), the current through the battery is \( \frac{5V}{1.2\, \Omega} = 4.167\, \text{A} \), and the voltage across the battery is by Ohm’s law \( 4.167\, \text{A} · 1\, \Omega = 4.167\, \text{V} \). Then the power is \( 4.167\, \text{A} · 4.167\, \text{V} = 17.36\, \text{W} \), and the total time to charge the battery is \( \frac{10.5\, \text{Wh}}{17.36\, \text{W}} = 0.6\, \text{h} \), about 36 min.

- **For $R_{bat} = 10\, \text{kΩ}$:**
  
  The total resistance seen by the battery is \( 10000\, \Omega + 0.2\, \Omega = 10000.2\, \Omega \), the current through the battery is \( \frac{5V}{10000.2\, \Omega} \approx 0.5\, \text{mA} \), and the voltage across the battery is by Ohm’s law \( 0.5\, \text{mA} · 10\, \text{kΩ} \approx 5\, \text{V} \) (up to 2 significant figures). Then the power is \( 5\, \text{V} · 0.5\, \text{mA} = 2.5\, \text{mW} \), and the total time to charge the battery is \( \frac{10.526\, \text{Wh}}{0.0025\, \text{W}} = 4210\, \text{h} \).

4. **Measuring Voltage and Current**

In order to measure quantities such as voltage and current, engineers use voltmeters and ammeters. A simple model of a voltmeter is a resistor with a very high resistance, $R_{VM}$. The **voltmeter measures the voltage**
across the resistance $R_{VM}$. The measured voltage is then relayed to a microprocessor (such as the MSP430s used in Lab).

This model of an voltmeter is shown in Figure 3. Let us explore what happens when we connect this voltmeter to various circuits to measure voltages.

Throughout this problem assume $R_{VM} = 1\text{M}\Omega$. Recall that the SI prefix $M$ or Mega is $10^6$.

![Figure 3: Our model of a voltmeter, $R_{VM} = 1\text{M}\Omega$](image)

(a) Suppose we wanted to measure the voltage across $R_2$ ($v_{out}$) produced by the voltage divider circuit shown in Figure 4 on the left. The circuit on the right in Figure 4 shows how we would connect the voltmeter across $R_2$. Assume $R_1 = 100\Omega$ and $R_2 = 200\Omega$.

First calculate the value of $v_{out}$. Then calculate the voltage the voltmeter would measure, i.e. $v_{meas}$.

![Figure 4: Left: Circuit without the voltmeter connected, Right: Voltmeter measuring voltage across $R_2$](image)

**Solution:** We start by finding $v_{out}$ in the circuit on the left. Recognizing that this circuit is a voltage divider, we can directly find the following:

$$v_{out} = \frac{R_2}{R_1 + R_2} \cdot 5V = \frac{200\Omega}{300\Omega} \cdot 5V = 3.3333V$$

Next we consider the circuit on the right. We start by combining the resistor $R_2$ and $R_{VM}$ since they are in parallel. Then we can apply the voltage divider formula to calculate the voltage across $R_{VM}$.

$$R_2 || R_{VM} = \frac{R_2 R_{VM}}{R_2 + R_{VM}} = \frac{1\text{M}\Omega \cdot 200\Omega}{1\text{M}\Omega + 200\Omega} = 199.96\Omega$$

$$v_{out} = \frac{R_2 || R_{VM}}{R_1 + R_2 || R_{VM}} \cdot 5V = \frac{199.96\Omega}{199.96\Omega + 100\Omega} \cdot 5V = 3.3331V$$
(b) Repeat part (a), but now \( R_1 = 10 \, M\Omega \) and \( R_2 = 10 \, M\Omega \). Is this voltmeter still a good tool to measure the output voltage?

**Solution:** We start by again finding \( v_{\text{out}} \) in the circuit on the left. Recognizing that this circuit is a voltage divider, we can directly find the following:

\[
v_{\text{out}} = \frac{R_1}{R_1 + R_2} \cdot 5 \text{V} = \frac{10 \, M\Omega}{20 \, M\Omega} \cdot 5 \text{V} = 2.5 \text{V}
\]

Next we consider the circuit on the right. We start by combining the resistor \( R_2 \) and \( R_{\text{VM}} \) since they are in parallel. Then we can apply the voltage divider formula to calculate the voltage across \( R_{\text{VM}} \).

\[
R_2 || R_{\text{VM}} = \frac{R_2 R_{\text{VM}}}{R_2 + R_{\text{VM}}} = \frac{10 \, M\Omega \cdot 1 \, M\Omega}{10 \, M\Omega + 1 \, M\Omega} = 0.909 \, M\Omega
\]

\[
v_{\text{out}} = \frac{R_2 || R_{\text{VM}}}{R_1 + R_2 || R_{\text{VM}}} \cdot 5 \text{V} = \frac{0.909 \, M\Omega}{0.909 \, M\Omega + 10 \, M\Omega} \cdot 5 \text{V} = 0.4166 \text{V}
\]

Since the resistors \( R_1 \) and \( R_2 \) are larger than \( R_{\text{VM}} \), using the voltmeter to measure element voltages significantly changes the value of \( V_{\text{out}} \). Thus our voltmeter is not a good tool to use to measure the voltage for this circuit.

(c) Now suppose we are working with the same circuit as in Part (a), but we know that \( R_2 = R_1 \). What is the maximum value of \( R_1 \) that ensures that the difference between voltage measurement of the voltmeter \( (v_{\text{meas}}) \) and the actual value \( (v_{\text{out}}) \) remains within \( \pm 10\% \) of \( v_{\text{out}} \)?

**Solution:** We will only have to consider the case where \( v_{\text{meas}} \) is less than \( v_{\text{out}} \), because the parallel combination of \( R_2 \) and \( R_{\text{VM}} \) can only make a resistor with total resistance smaller than \( R_2 \).

First, let’s symbolically represent what the outputs are in the two cases:

For the circuit without the voltmeter connected:

\[
V_{\text{out}} = \frac{R_2}{R_1 + R_2} \cdot V_s
\]

For the circuit with the voltmeter connected:

\[
R_{\text{VM}} || R_2 = \frac{R_{\text{VM}} R_2}{R_{\text{VM}} + R_2}
\]

\[
v_{\text{meas}} = \frac{R_{\text{VM}} R_2}{R_1 + \frac{R_{\text{VM}} R_2}{R_{\text{VM}} + R_2}} \cdot V_s = \frac{R_{\text{VM}} R_2}{R_1 (R_{\text{VM}} + R_2) + R_{\text{VM}} R_2} \cdot V_s
\]

Now we need:

\[
\frac{V_{\text{out}} - v_{\text{meas}}}{V_{\text{out}}} \leq \frac{1}{10}
\]

\[
\frac{R_2}{R_1 + R_2} \cdot V_s - \frac{R_{\text{VM}} R_2}{R_1 (R_{\text{VM}} + R_2) + R_{\text{VM}} R_2} \cdot V_s \leq \frac{1}{10}
\]

\[
\frac{R_2}{R_1 + R_2} - \frac{R_{\text{VM}} R_2}{R_1 (R_{\text{VM}} + R_2) + R_{\text{VM}} R_2} \leq \frac{1}{10}
\]

Since we know \( R_1 = R_2 \), we can simplify our final expression:
\[
\begin{align*}
\frac{1}{2} & - \frac{R_{VM}}{R_{VM} + R_2 + R_{VM}} \leq \frac{1}{10} \\
1 & - \frac{2R_{VM}}{2R_{VM} + R_2} \leq \frac{1}{10} \\
\frac{R_2}{2R_{VM} + R_2} & \leq \frac{1}{10} \\
\frac{9}{10} R_2 & \leq \frac{2}{10} R_{VM} \\
R_2 & \leq \frac{2}{9} R_{VM} \\
R_1 = R_2 & \leq 0.22 \text{ M}\Omega
\end{align*}
\]

(d) Using the combination of our voltmeter and an additional resistor \(R_x\), we can make an ammeter and measure the current through an element. Using the circuit shown in Figure 5 where \(R_x = 1 \Omega\), then the measured current through \(R_x\) is \(I_{\text{meas}} = \frac{V_{VM}}{R_x}\) where \(V_{VM}\) is the voltage across the voltmeter.

In Figure 6, the voltmeter is connected to measure the current through resistor \(R_1 = 1k\Omega\). For the circuit on the left, find the current through \(R_1\) without the voltmeter connected (i.e. \(I_1\)). Then, for the circuit on the right, find the current measured by the voltmeter when it is connected as an ammeter (i.e. \(I_{\text{meas}}\)).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{The voltmeter combined with resistor \(R_x\) to function as an ammeter (i.e. to measure current), \(R_{VM} = 1M\Omega\).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure6.png}
\caption{Circuits for Part (d) Left: Original circuit; Right: Circuit with the voltmeter connected as an ammeter.}
\end{figure}
Solution:

We start with the circuit on the left

\[ I_1 = \frac{5 \text{V}}{1\text{k}\Omega} = 5 \text{mA} \]

For the circuit on the right, we start by computing \( R_x || R_{VM} \).

\[ R_x || R_{VM} = \frac{R_x R_{VM}}{R_x + R_{VM}} = \frac{1 \Omega \cdot 1 \text{M} \Omega}{1 \Omega + 1 \text{M} \Omega} \approx 1 \Omega \]

Next, we compute the voltage across the \( R_x || R_{VM} \) combination. Notice this circuit is again a voltage divider.

\[ V_{R_{VM}} = \frac{R_x || R_{VM}}{R_1 + R_x || R_{VM}} \cdot 5 \text{V} = \frac{1 \Omega}{1 \text{k} \Omega + 1 \Omega} \cdot 5 \text{V} = 0.004995 \text{V} \]

The measured current is this voltage divided by the resistance \( R_x \).

\[ I_{\text{meas}} = \frac{V_{R_{VM}}}{R_x} = \frac{0.004995 \text{V}}{1 \Omega} \approx 5 \text{mA} \]

(e) What is the minimum value of \( R_1 \) that ensures the difference between current measurement \( (I_{\text{meas}}) \) and the actual value \( (I_1) \) stays within \( \pm 10\% \) of \( I_1 \)?

Solution:

Again, we will only consider the case where the measured current is smaller than the actual current, because the series combination of \( R_1 \) and \( R_x || R_{VM} \) can only create a resistor bigger than \( R_1 \). First let’s symbolically represent what the outputs are in the two cases:

For the circuit without the ammeter connected:

\[ I_1 = \frac{V_s}{R_1} \]

For the circuit with the ammeter connected:

\[ R_{VM} || R_x = \frac{R_{VM} R_x}{R_{VM} + R_x} \]

\[ V_{VM} = \frac{R_{VM} R_1}{R_1 + R_{VM} R_1} \cdot V_s \]

\[ I_{\text{meas}} = \frac{V_{VM}}{R_x} = \frac{R_{VM} R_1}{R_{VM} + R_1} \cdot \frac{V_s}{R_x} \]

Now we need:

\[ \frac{I_1 - I_{\text{meas}}}{I_1} \leq \frac{1}{10} \]

\[ \frac{V_s}{R_1} - \frac{R_{VM} R_1}{R_{VM} + R_1} \cdot \frac{V_s}{R_x} \leq \frac{1}{10} \]
We will approximate $\frac{R_{VM}R_x}{R_{VM}+R_x} = R_x = 1\, \Omega$.

$$1 - \frac{R_1}{1+R_1} = \frac{1}{1+R_1} \leq \frac{1}{10}$$

$9\, \Omega \leq R_1$

5. **Equivalent Resistance (PRACTICE)**

(a) Find the equivalent resistance looking in from points $a$ and $b$.

![Diagram of circuit with 5R and 3R resistors in parallel]

**Solution:**

$$R_{eq} = 5R \parallel 3R = \frac{5R \cdot 3R}{5R + 3R} = \frac{15}{8} R$$

(b) Find the equivalent resistance looking in from points $a$ and $b$.

![Diagram of circuit with 2R, R, and 3R resistors in series]

**Solution:** We find the equivalent resistance for the resistors from left to right.
(c) Find the equivalent resistance looking in from points $a$ and $b$.

**Solution:** Again, we find the equivalent resistance for the resistors from left to right.
6. Circuits Practice (PRACTICE)

(a) What does KVL tell you about $V_1$ and $V_2$ for any elements connected to the same pair of nodes?

Solution: Using KVL, we can write an equation. Going around the loop starting from the left node, we have:

$$V_1 - V_2 = 0$$

This says that $V_1 = V_2$ for any two elements connected to the same two nodes, provided they have the same voltage orientation.

(b) What does KCL tell you about $I_1$ and $I_2$ for any two elements connected to a node with nothing else connected to that node?

Solution: Using KCL, we can write an equation. Going around the loop starting from the left node, we have:

$$R_{eq} = 2R \parallel 4R = \frac{4}{3}R$$
**Solution:** Using KCL at the center node the relationship for the currents are:

\[ I_1 = I_2 \]

For any two elements that are connected to a node with nothing else connected to them, the current values will be identical, provided that the currents have the same direction.

(c) Find \( R_{ab} \), the equivalent resistance between terminals \( a \) and \( b \). Give your answer as a number, or an expression involving no more than one use of ||.

\[ a \quad 10\,\Omega \quad 10\,\Omega \quad b \]

\[ 60\,\Omega \quad 40\,\Omega \quad 10\,\Omega \quad 3\,\Omega \quad 7\,\Omega \]

**Solution:** The 3\( \Omega \) and 7\( \Omega \) resistors are in series. Combining them, we get 10\( \Omega \).

\[ a \quad 10\,\Omega \quad 10\,\Omega \quad b \]

\[ 60\,\Omega \quad 40\,\Omega \quad 10\,\Omega \quad 10\,\Omega \]

The two lower resistors with values of 10\( \Omega \) are in parallel. Combining them, we get 10\( || \)10\( \Omega \) = 5\( \Omega \).

\[ a \quad 10\,\Omega \quad 10\,\Omega \quad b \]

\[ 60\,\Omega \quad 40\,\Omega \quad 5\,\Omega \]

The upper two resistors with values of 10\( \Omega \) are also in parallel. We get another 5\( \Omega \) resistor.
Now, we can combine the two 5Ω resistors into one 10Ω resistor.

The equivalent of the 60Ω and 40Ω resistors in parallel are $R_{eq} = \frac{60Ω \cdot 40Ω}{60Ω + 40Ω} = 24Ω$.

The final equivalent resistance $R_{ab}$ can be written as $R_{ab} = 24Ω || 10Ω$, or as $R_{ab} = \frac{24 \cdot 10}{24 + 10}Ω = \frac{120}{17}Ω \approx 7.058Ω$.

(d) Find $I_x$. (Hint: Reference Discussion 7B)

**Solution:** By KCL at the right node of 10kΩ, the current that goes through 10kΩ is the same as the current that comes from the source. By KCL at the left node of 10kΩ, the current that goes through 10kΩ is the same as the sum of currents that go through 1kΩ and 3kΩ. We can consider the resistors 1kΩ and 3kΩ as a current divider, since the current that goes through 5kΩ is also $\frac{1}{2}A$. 

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The current $I_y$ will therefore be:

$$I_y = \frac{1 k\Omega}{1 k\Omega + 3 k\Omega} \cdot \frac{1 A}{2} = \frac{1}{4} \cdot \frac{1 A}{2} = \frac{1 A}{8}$$

To be consistent with the original current direction labeled, $I_x = -I_y = -\frac{1}{8} A$

7. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID’s. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.