

EECS 16A  
Fall 2019

Designing Information Devices and Systems I

Midterm 2 - VERSION B

Midterm 2 Solution - VERSION B

PRINT your student ID: \_\_\_\_\_

PRINT AND SIGN your name: \_\_\_\_\_, \_\_\_\_\_ \_\_\_\_\_  
(last name) (first name) (signature)

PRINT your discussion section and GSI(s) (the one you attend): \_\_\_\_\_

Name and SID of the person to your left: \_\_\_\_\_

Name and SID of the person to your right: \_\_\_\_\_

Name and SID of the person in front of you: \_\_\_\_\_

Name and SID of the person behind you: \_\_\_\_\_

**1. Tell us about something that makes you happy. (1 Point)**

**2. What courses are you thinking of taking next semester? (1 Point)**

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

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### 3. Power (15 points)

In the circuit shown in Figure 3.1 calculate the total power  $P_d$  **delivered** by source  $I_1$  to the rest of the circuit.

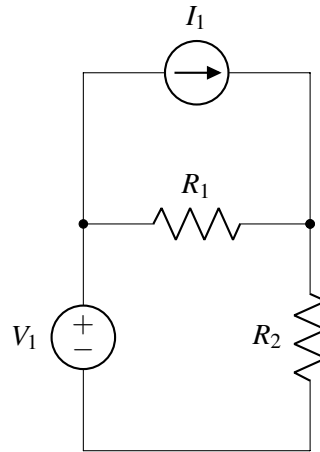
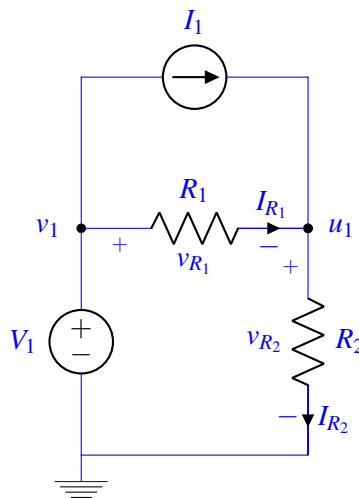


Figure 3.1: Circuit for power calculation

Use the following component values:  $R_1 = 1\ \Omega$ ,  $R_2 = 3\ \Omega$ ,  $V_1 = 5\text{V}$ ,  $I_1 = -1\text{A}$ .

**Solution:** The labeling steps (steps 1-4) of NVA analysis are shown in the circuit diagram below:



A ground node has been chosen, the node with known voltage is marked as  $v_1 = V_1$ , the node with unknown voltage is marked as  $u_1$ . Also, a direction has been picked for the currents flowing through resistors  $R_1$  and  $R_2$ , and the voltage across them is consequently labelled following passive sign convention. We will perform NVA to get the voltage at node  $u_1$ , and hence the power delivered to the circuit by the current source  $I_1$ . Applying KCL at node  $u_1$ :

$$I_1 + I_{R1} = I_{R2}$$

Ohm's law for resistors  $R_1$  and  $R_2$  gives:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{v_1 - u_1}{R_1} = \frac{V_1 - u_1}{R_1}$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{u_1}{R_2}$$

Plugging in the last two equations in the first and solving for  $u_1$  we get:

$$\begin{aligned} I_1 + \frac{V_1 - u_1}{R_1} &= \frac{u_1}{R_2} \\ \Rightarrow I_1 R_1 R_2 + V_1 R_2 - u_1 R_2 &= u_1 R_1 \\ \Rightarrow u_1 &= \frac{(I_1 R_1 + V_1) R_2}{R_1 + R_2} = 3\text{V} \end{aligned}$$

So finally the power **dissipated** by  $I_1$  is equal to:

$$P_{I_1} = I_1 V_{I_1} = I_1 (V_1 - u_1) = -1\text{A}(5\text{V} - 3\text{V}) = -2\text{W}$$

This means that the power delivered to the circuit by  $I_1$  is equal to:

$$P_{I_1, \text{del}} = 2\text{W}$$

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#### 4. Equivalent Circuit (15 points)

Your friend has characterized the circuit shown in Figure 4.1(a) in the lab by first connecting a voltmeter (represented in Figure 4.1(b) by the letter V in a circle) to terminals  $A - B$  to measure  $V_m$ , then disconnecting the voltmeter and connecting an am-meter (current meter, represented in Figure 4.1(b) by the letter A in a circle) to measure  $I_m$ .

Now they ask for your help designing an equivalent circuit model (looking from terminals  $A$  and  $B$ ) consisting only of a current source  $I_s$  and resistor  $R_s$ .

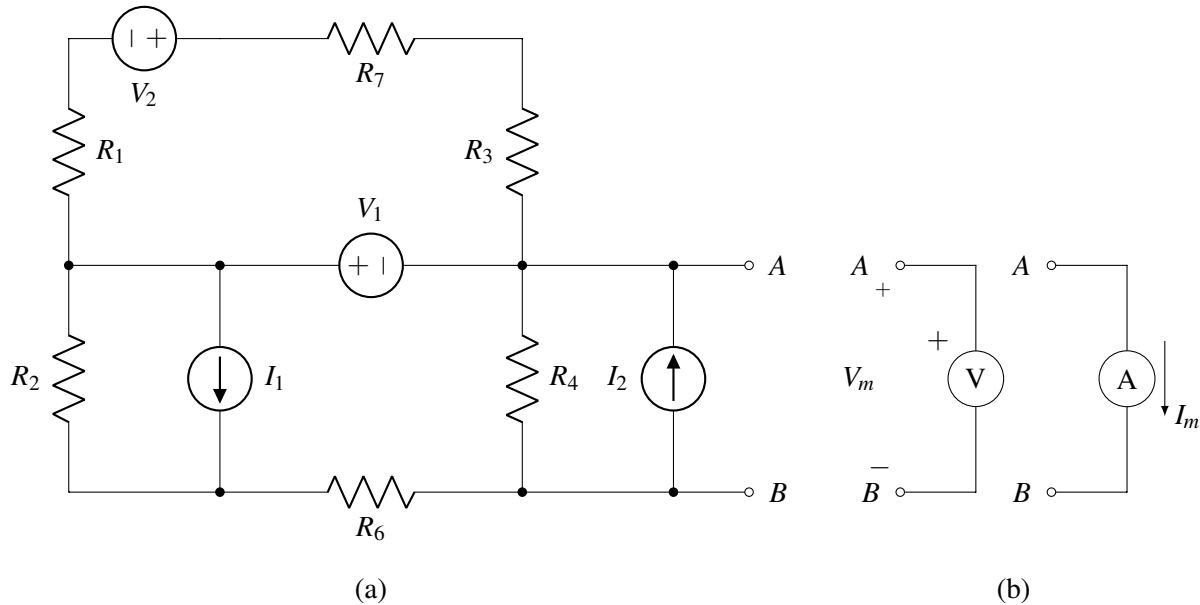


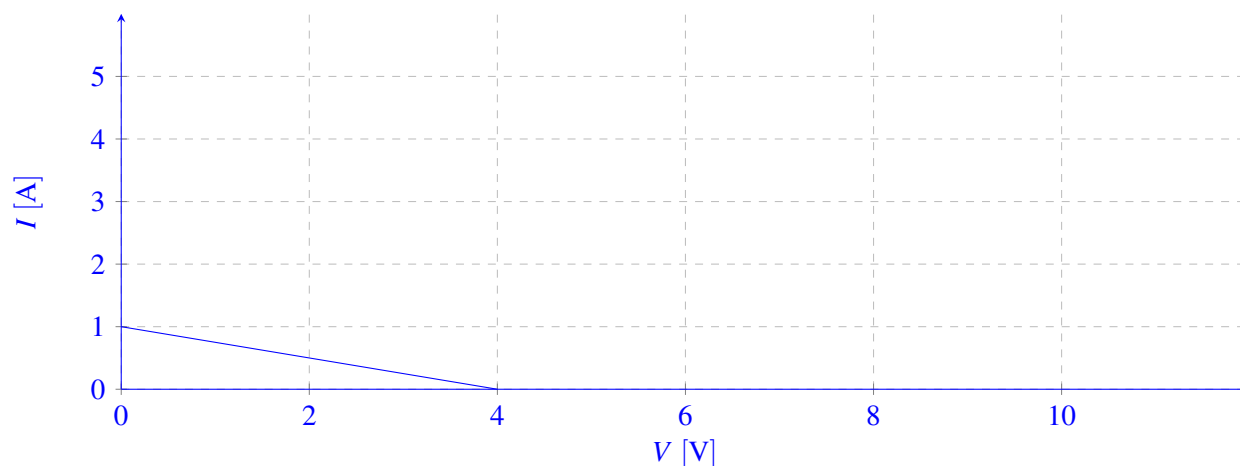
Figure 4.1: (a) Circuit tested in the lab, (b) Circuit used to measure  $V_m$  and  $I_m$

Use the following values:  $V_m = 4\text{V}$ ,  $I_m = 1\text{A}$ .

(a) Draw the  $I$ - $V$  characteristic between nodes  $A$ ,  $B$  of the circuit in Figure 4.1(a).

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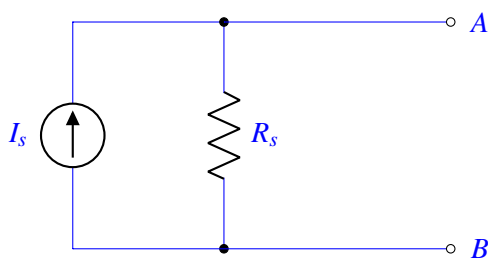
**Solution:**



- (b) Draw the circuit diagram of the equivalent model as seen from nodes  $A$ ,  $B$ . Clearly mark nodes  $A$  and  $B$ , current  $I_s$  (including direction), and resistor  $R_s$ .

**Solution:**

The current meter measures the short circuit current whereas the voltmeter measures the open circuit voltage across terminals  $A$  and  $B$ . Therefore, using the Norton equivalent model, we can model the circuit of Figure 4.1 (a) as a current source of value  $I_s = I_m$  in parallel with a Thevenin resistance  $R_s = R_{th} = \frac{V_m}{I_m}$ .



- (c) Calculate the numerical values of  $I_s$  and  $R_s$ . Mark your calculated component values in the circuit diagram drawn in part (b). Clearly mark the direction of the current.

**Solution:** We have  $I_s = I_m = 1A$  and  $R_s = R_{th} = 4\Omega$ .

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### 5. Next-Phone (15 points)

You have been hired by “Next-Phone”, a promising startup that has developed a 3D printer to produce individually customized smartphones.

Only one problem remains: designing accurate position sensing for the printhead. “No problem,” you tell your new boss, “I’ll take care of that!”

Figure 5.1 shows your design. The printhead is supported by two rollers that move the head in the  $x$  direction. Each roller runs on two conductive tracks with resistivities  $\rho_1$  and  $\rho_2$ , respectively, length  $L$ , and cross-sectional area  $A$ . The rollers are made of metal electrically connecting the strips. The printhead is an insulator (i.e. nonconductive material) so it can be modeled as an open-circuit. Roller 1 and Roller 2 are disconnected by the non-conductive Print Head. You connect two voltage sources of voltage  $V_S$  as shown in Figure 5.1. You then measure voltage  $V_{AB} = V_A - V_B$  to sense position  $x$ .

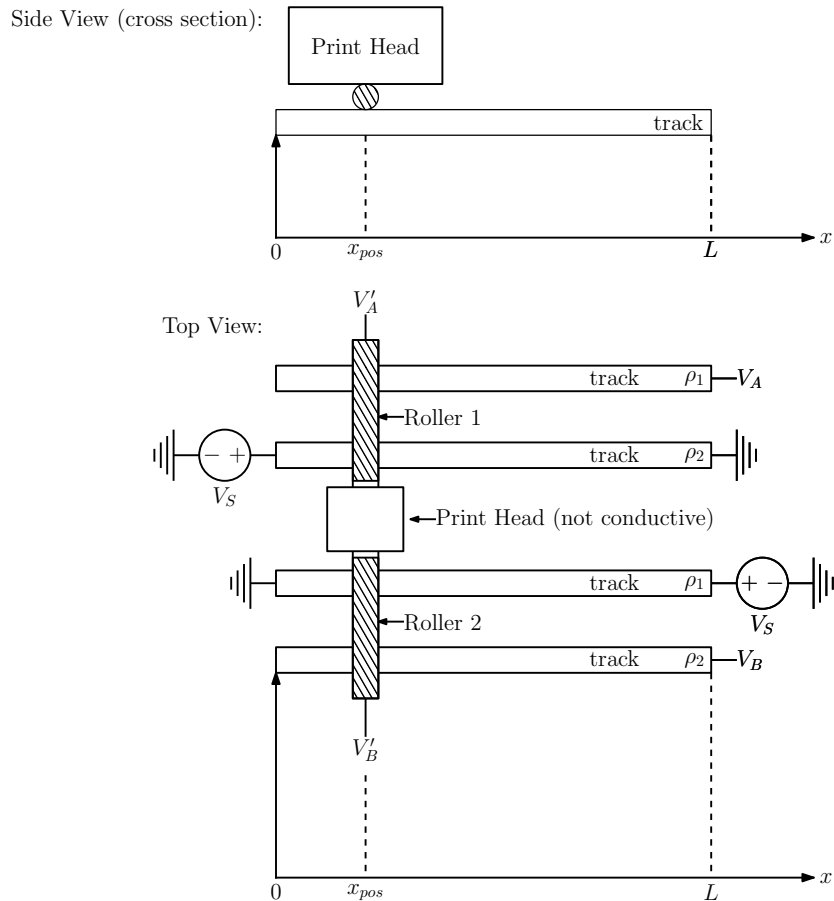
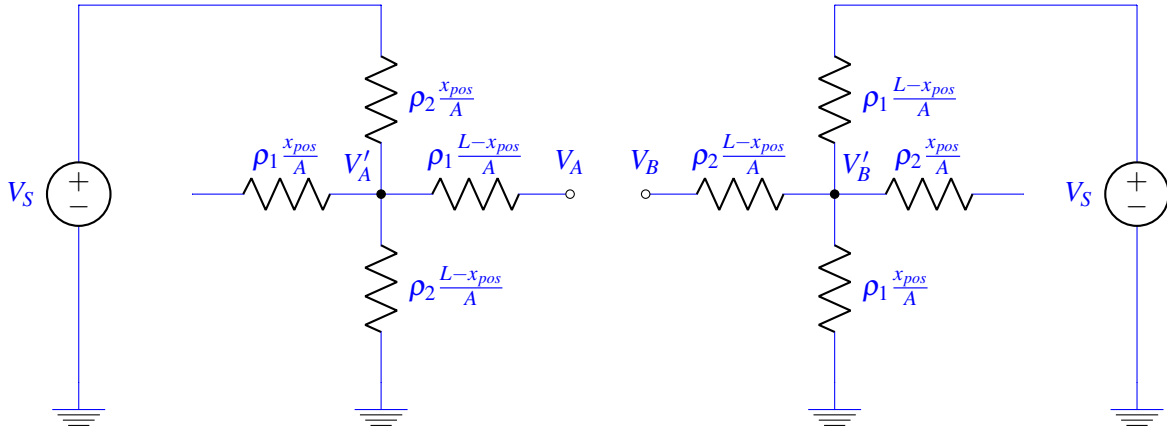


Figure 5.1: 3D printhead position sensor (top and side views)

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- (a) Draw an equivalent circuit diagram of Figure 5.1 consisting of sources, resistors, etc.

**Solution:** To draw the equivalent circuit diagram, we need to realize that the print head is essentially an open, and that the tracks can be modeled as resistors. So, the equivalent model is:



- (b) Derive algebraic expressions for  $V_{AA'} = V_A - V_{A'}$  and  $V_{BB'} = V_B - V_{B'}$  as a function of  $x_{pos}$ .

**Solution:** Since the resistors connecting nodes  $A$  and  $A'$  are essentially open we will have that  $V_{AA'} = 0$ . The same holds for nodes  $B$  and  $B'$ , so  $V_{BB'} = 0$ .

- (c) Find the value of voltage  $V_{AB}(x_{pos}) = V_A - V_B$  for  $V_S = 12\text{V}$ ,  $\rho_1 = 1\Omega\text{m}$ ,  $\rho_2 = 2\Omega\text{m}$ ,  $L = 400\text{mm}$ ,  $A = 1\text{cm}^2$ , and  $x_{pos} = 100\text{mm}$ .

**Solution:** Neglecting the open resistors in our model we only have two voltage dividers, so voltages  $V_A$ ,  $V_B$  are:

$$\begin{aligned} V_A &= \frac{L-x}{L} V_S \\ V_B &= \frac{x}{L} V_S \\ \Rightarrow V_A - V_B &= \frac{L-2x}{L} V_S \end{aligned}$$

Where we have used the fact that the resistivity and area parameters drop out from the divider equations. Substituting into the derived equation we get that:

$$V_A - V_B = \frac{400\text{mm} - 200\text{mm}}{400\text{mm}} 12\text{V} = 6\text{V}$$

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### 6. Current Sensor (19 points)

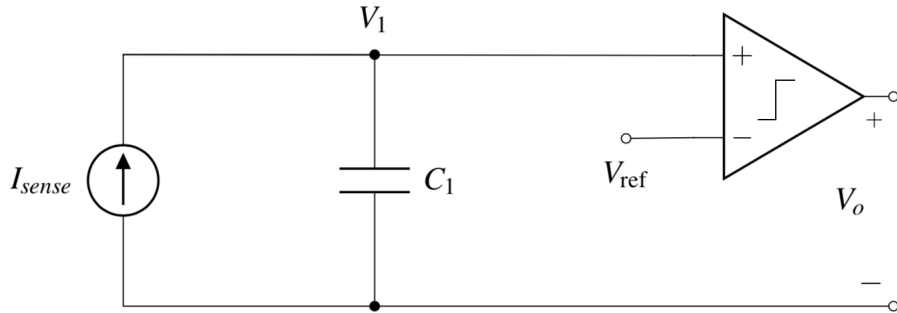


Figure 6.1: First version of the current sensor circuit

- (a) You've built a light sensor that outputs a current  $I_{sense}$  that is proportional to light intensity. Now you need a circuit that measures this current. Figure 6.1 shows the design of the first version of the current sensor circuit. At time  $t = 0$  capacitor  $C_1$  is discharged and a timer (stop watch) is started. When voltage  $V_1$  reaches  $V_{ref}$  the comparator "trips" and stops the timer at  $T_1$ . The output of the comparator is +1V if  $V_1 > V_{ref}$ , and 0V otherwise.

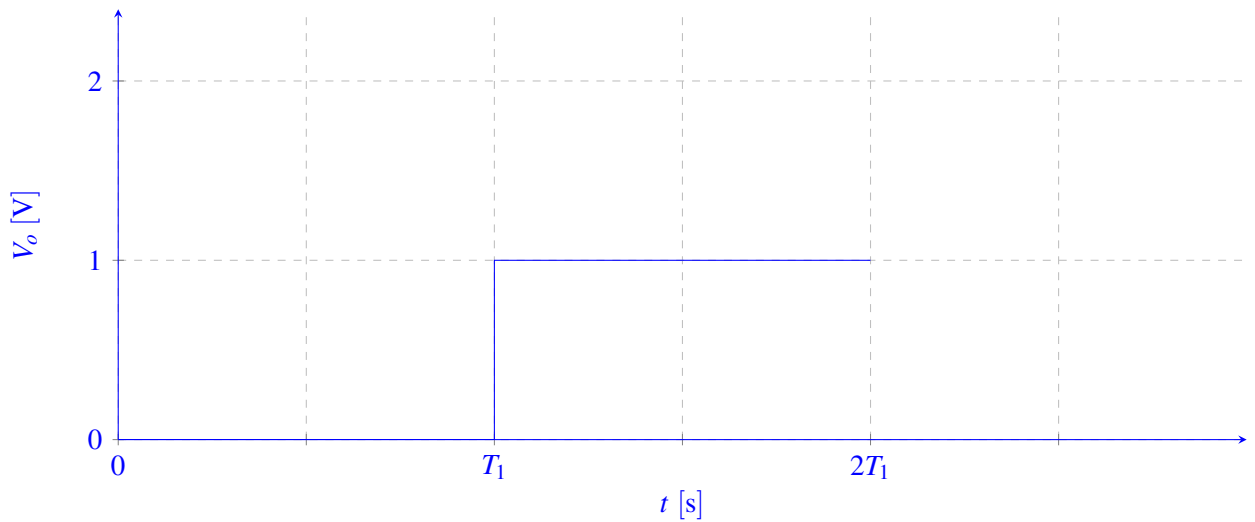
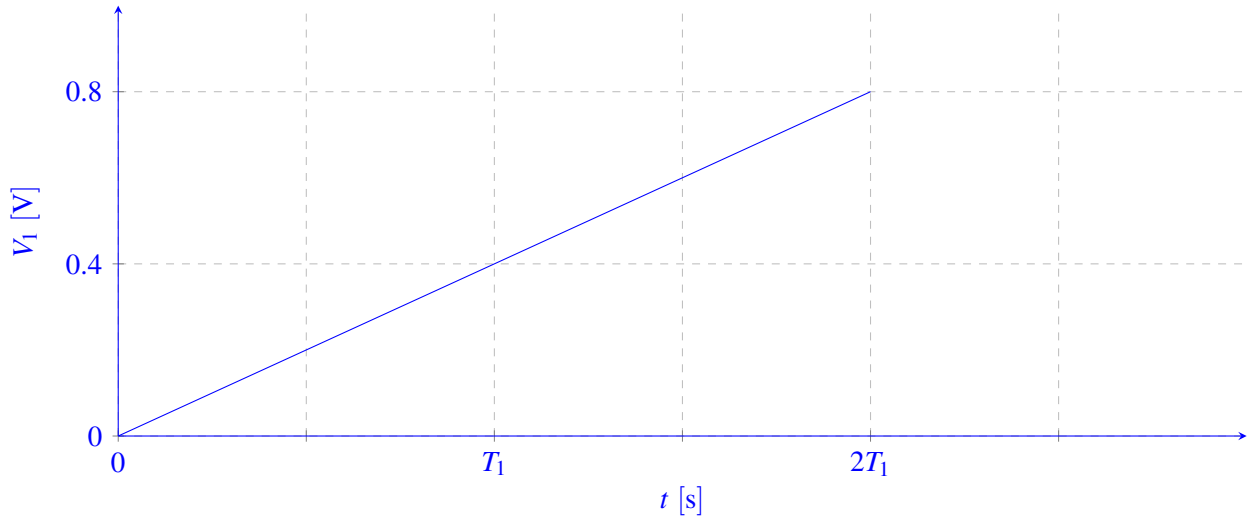
In the grids below, plot  $V_1$  and  $V_o$  as a function of time  $t$ , from  $t = 0$  to  $t = 2T_1$ .

Use  $I_{sense} = 0.2\text{A}$ ,  $C_1 = 1\text{F}$ , and  $V_{ref} = 0.4\text{V}$ .



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**Solution:**



- (b) Derive an expression for the time  $T_1$  measured by the timer as a function of  $I_{sense}$ ,  $C_1$ , and  $V_{ref}$ . Then, calculate its numerical value using the same component values as given in part (a).

**Solution:**

Current  $I_s$  flows through  $C_1$ .  $V_1(0) = 0$  because capacitor  $C_1$  is initially uncharged.

$$I_{sense} = C_1 \frac{dV_1(t)}{dt}$$

$$V_1(t) = \int \frac{I_{sense}}{C_1} dt = \frac{I_{sense}t}{C_1} + V_1(0) = \frac{I_{sense}t}{C_1}$$

Time  $T_1$  is the time needed for  $V_1$  to become equal to  $V_{ref}$  and the comparator to flip. Therefore,

$$T_1 = \frac{C_1 V_{ref}}{I_{sense}}$$

Plugging in numbers, we get:

$$T_1 = \frac{1F0.4V}{0.2A} = 2s$$

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- (c) Figure 6.2 shows an improved current sensor that does not depend on the value of capacitor  $C_1$ .

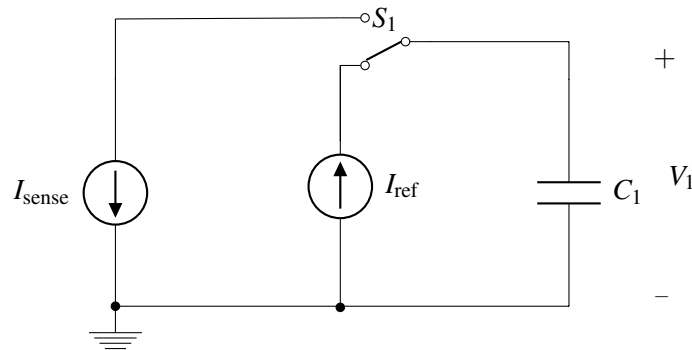


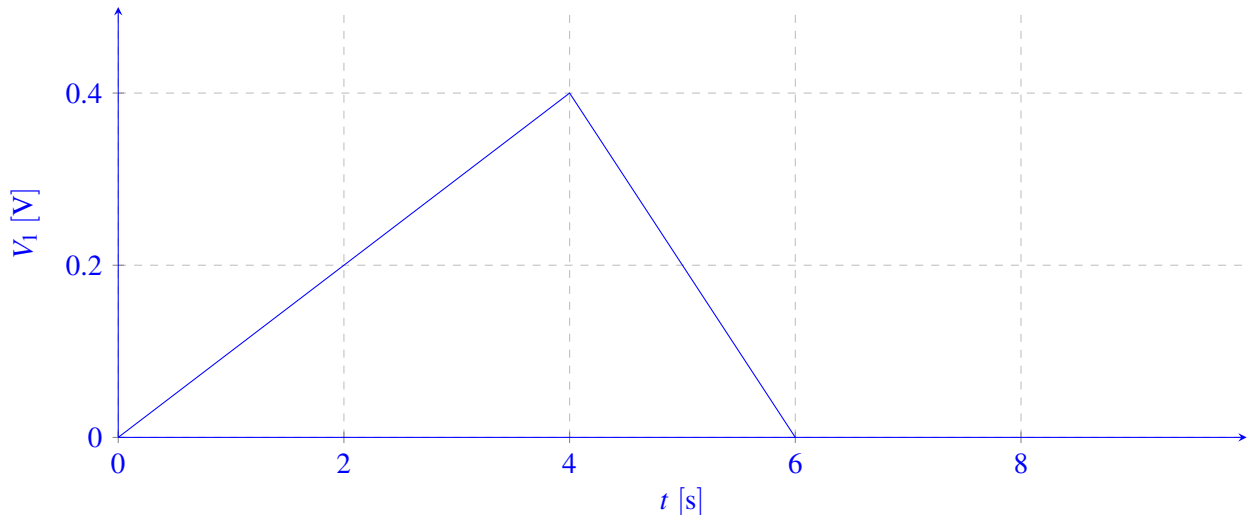
Figure 6.2: Improved Current Sensor

Capacitor  $C_1$  is discharged at time  $t = 0$  and switch  $S_1$  is connected to  $I_{ref}$ . When  $V_1$  reaches  $V_{ref}$  at time  $T_1$ , switch  $S_1$  is disconnected from  $I_{ref}$  and connected to  $I_{sense}$  instead.  $T_2$  is defined as the time from connecting  $S_1$  to  $I_{sense}$  to the moment  $V_1$  reaches 0V.

Plot  $V_1$  versus time from  $t = 0$  to  $t = T_1 + T_2$ .

Use  $I_{sense} = 0.2\text{A}$ ,  $C_1 = 1\text{F}$ ,  $V_{ref} = 0.4\text{V}$  and  $I_{ref} = 0.1\text{A}$ .

**Solution:**



- (d) Derive an expression for the ratio  $\frac{T_1}{T_2}$  (the times calculated in part (c)) as a function of  $I_{sense}$ ,  $I_{ref}$ ,  $C_1$ , and  $V_{ref}$ . Then, calculate its numerical value using the same component values as given in part (c).

**Solution:**

Initially, capacitance  $C_1$  is charged through the integration of  $I_{ref}$ .  $T_1$  is the time needed in order to  $V_1$  to become equal to  $V_{ref}$ . Therefore,

$$T_1 = \frac{C_1 V_{ref}}{I_{ref}}$$

$T_2$  is the time needed for  $V_1$  to get fully discharged or in other words, to go from  $V_{ref}$  to 0. Therefore,

$$T_2 = \frac{C_1 V_{ref}}{I_{sense}}$$

The ratio  $\frac{T_1}{T_2}$  is found to be:

$$\frac{T_1}{T_2} = \frac{I_{sense}}{I_{ref}}$$

Plugging in numbers, we get:

$$\frac{T_1}{T_2} = 2$$

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### 7. Capacitor Powered Quadcopter (15 points)

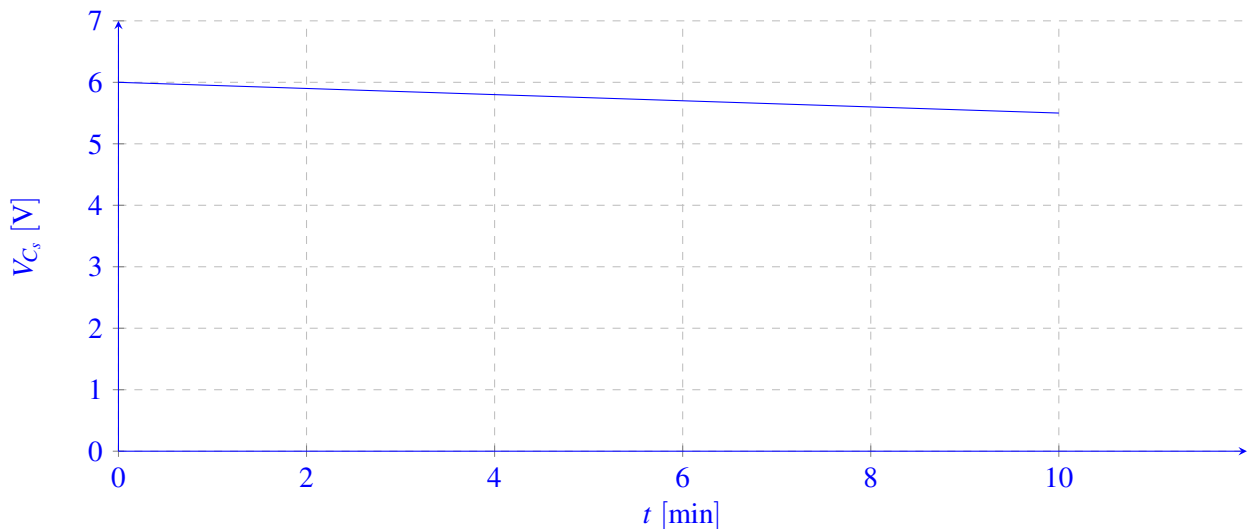
You've made a fun little quadcopter that juggles 10 colorful balls - all while flying in circles above the heads of its stunned audience that cannot get enough of the spectacle.

Unfortunately it takes quite a bit of time to recharge the battery after each demonstration. To shorten the time you decide to replace the battery with a capacitor  $C_s$  which can be charged virtually instantaneously.

The drone consumes a constant current  $I_d = 1\text{A}$ . The nominal supply voltage of the drone is  $V_{nom} = 6\text{V}$ , but it works with voltages as low as  $V_{min} = 5\text{V}$  (i.e. when the capacitor voltage drops below  $5\text{V}$  the drone crashes).  $V_{nom}$  is the initial voltage across the capacitor  $C_s$ .

- (a) Plot the voltage across the capacitor  $V_{C_s}$  as a function of time from 0 to 10 minutes in the graph space provided to you below. Use  $C_s = 1200\text{F}$ .

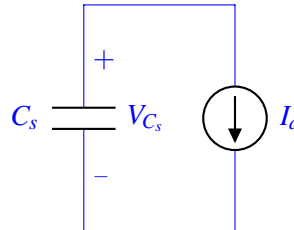
**Solution:**



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- (b) Calculate the minimum value of the capacitor  $C_s$  required to support 10 minutes of flying time.

**Solution:** Since the drone consumes a constant current  $I_d$ , our capacitor - drone system can be modeled as:



Hence, the capacitor will be discharged as follows:

$$I_d = C_s \frac{dV}{dt} = C_s \frac{\Delta V}{\Delta t}$$

$$\Delta t = C_s \frac{\Delta V}{I_d} = C_s \frac{V_{nom} - V_{min}}{I_d}$$

Since we want to support a minimum of 10 minute flight we need to have:

$$\Delta t \geq 10\text{min}$$

$$C_s \frac{V_{nom} - V_{min}}{I_d} \geq 10\text{min}$$

$$C_s \geq 10 \cdot 60 \cdot \frac{I_d}{V_{nom} - V_{min}} = 600\text{F}$$

- (c) Regardless of your answer in (b), assume  $C_s = 5\text{F}$ . Calculate the ratio of the energy  $E_2$  remaining in the capacitor at the end of the flight divided by the energy  $E_1$  initially stored in the capacitor  $C_s$  when it is fully charged.

**Solution:** The **ratio** of the energy  $E_2$  remaining in the capacitor at the end of the flight divided by the energy  $E_1$  initially stored in the capacitor  $C_s$  when it is fully charged does not depend on the actual value of the capacitor but only on the initial and final voltage since it is:

$$\frac{E_2}{E_1} = \frac{\frac{1}{2}C_s V_{min}^2}{\frac{1}{2}C_s V_{nom}^2}$$

$$\frac{E_2}{E_1} = \frac{V_{min}^2}{V_{nom}^2} = \frac{25}{36} \approx 0.69$$

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### 8. Fun with charge sharing (19 points)

- (a) In Figure 8.1, capacitors  $C_1$  and  $C_2$  are charged to  $V_1$  and  $V_2$  and switch  $S_1$  is open for time  $t < 0$ . At time  $t = 0$ , switch  $S_1$  is closed. Calculate  $V_1$  at time  $t > 0$ .

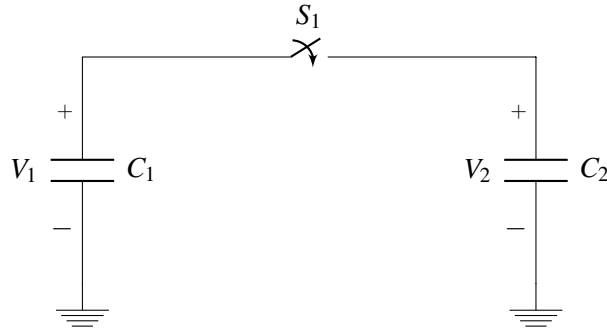


Figure 8.1: Capacitor Charge Sharing

Use the following values:  $C_1 = 1\text{F}$ ,  $C_2 = 2\text{F}$ ,  $V_1 = 3\text{V}$ ,  $V_2 = 3\text{V}$ .

#### Solution:

Let us define the initial charge on  $C_1$  as  $Q_{1i}$  and the initial charge on  $C_2$  as  $Q_{2i}$ . We know that  $Q_{1i} = C_1 V_{1i}$  and  $Q_{2i} = C_2 V_{2i}$ , where  $V_{1i}$  and  $V_{2i}$  are the initial voltages across  $C_1$  and  $C_2$ , respectively. (i.e. before switch  $S_1$  is closed). We know from conservation of charge that  $Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$ , where  $Q_{1f}$  and  $Q_{2f}$  are the final charge on  $C_1$  and  $C_2$ . (i.e. after switch  $S_1$  is closed). We can write this as:

$$(1) \quad C_1 V_{1i} + C_2 V_{2i} = Q_{1f} + Q_{2f}.$$

Additionally, we know that once switch  $S_1$  is closed, the voltage across  $C_1$  and  $C_2$  must be the same, because they are now in parallel with each other. Specifically,  $V_{1f} = V_{2f}$  where  $V_{1f}$  and  $V_{2f}$  are the final voltages across  $C_1$  and  $C_2$ , respectively. (i.e. after switch  $S_1$  is closed). Therefore,

$$C_1 V_{1i} + C_2 V_{2i} = (C_1 + C_2) V_{1f}$$

$$\text{At time } t > 0, V_{1f} = V_1 = \frac{C_1 V_{1i} + C_2 V_{2i}}{C_1 + C_2}$$

Plugging in numbers, we get:

$$V_{1f} = V_1 = 3\text{V}$$

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- (b) The circuit shown in Figure 8.2 operates in two phases. During phase 1, switches labeled  $S_1$  are closed and switches  $S_2$  are open. During phase 2, switches  $S_1$  are open and switches  $S_2$  are closed, as illustrated in the timing diagram shown in Figure 8.3.

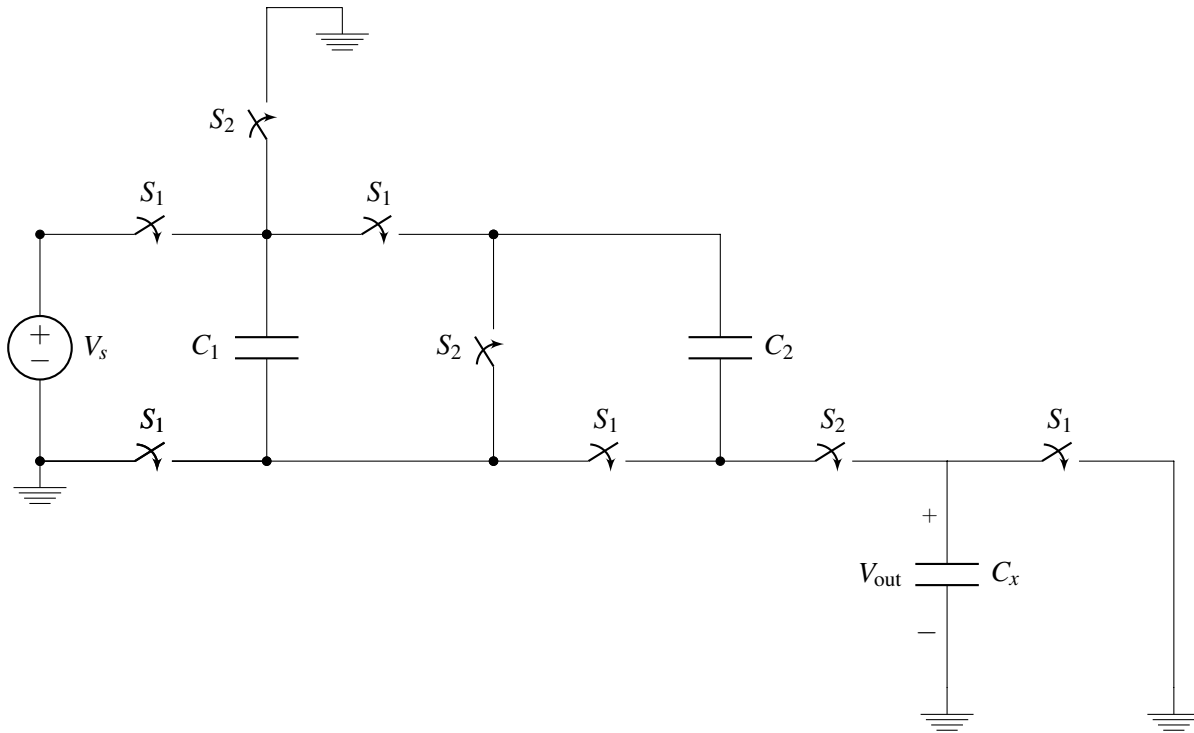


Figure 8.2: Capacitor Charge Sharing

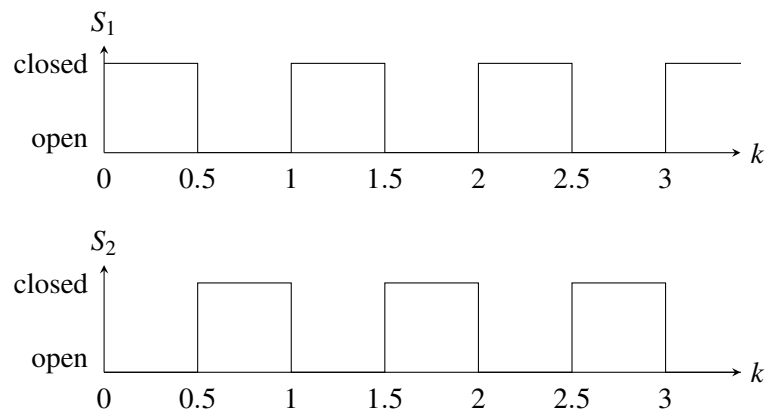


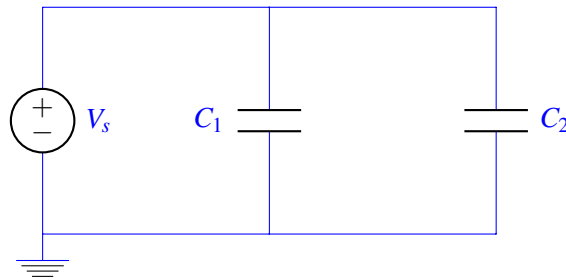
Figure 8.3: Timing diagram for switches



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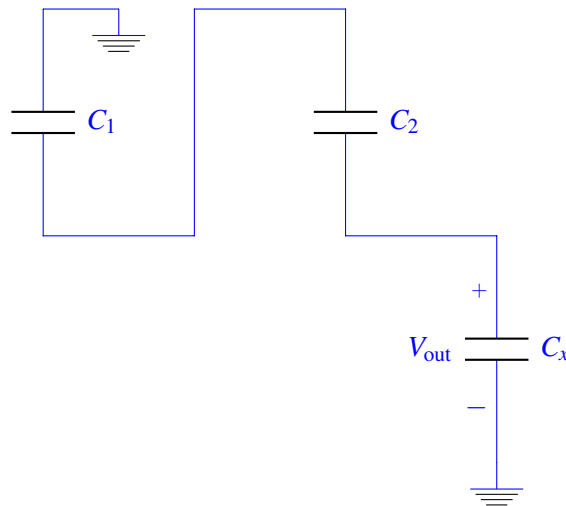
- i. Redraw the circuit during phase 1. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use  $C_1 = C_2 = C_0$ .

**Solution:**



- ii. Redraw the circuit during phase 2. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use  $C_1 = C_2 = C_0$ .

**Solution:**



- iii. Calculate the value of the voltage  $V_{out}$  during phase 2 as a function of  $C_0$ ,  $C_x$ , and  $V_s$ .

**Solution:**

In phase 1, the total charge is equal to  $Q_1 + Q_2 = -C_1V_s + C_2V_s$ .

In phase 2, from charge conservation, we get:

$$-C_1V_s + C_2V_s = C_1V_2 + C_2(V_2 - V_{out}) \quad (1)$$

Also, initially, the charge at the bottom plate of  $C_2$  was  $-C_2V_s$ . Therefore, from charge conservation, we get:

$$-C_2V_s = C_2(V_{out} - V_2) + C_xV_{out} \quad (2)$$

$$V_2 = \frac{(C_x + C_2)V_{out} + C_2V_s}{C_2} \quad (3)$$

Plugging (3) into (1),

$$V_{out} = \frac{-2C_1C_2V_s}{C_1C_x + C_1C_2 + C_2C_x} = \frac{-2V_s}{1 + \frac{2C_x}{C_0}}$$