

PRINT your name and student ID: _____

Extra page for scratchwork.

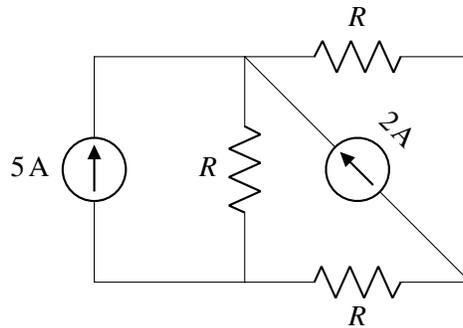
If you want any work on this page to be graded, please refer to this page on the problem's main page.

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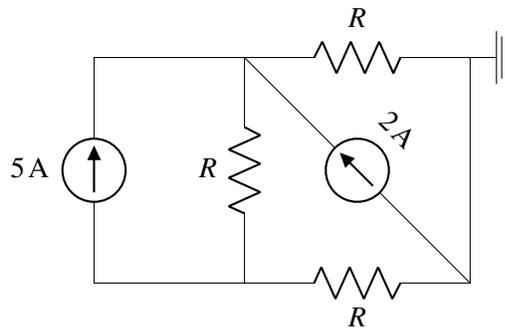
3. Nodal Analysis (6 Points)

Your friends, Anant and Elad, are attempting to solve the circuit below using the nodal analysis technique you learned in lecture. However, they got stuck on some steps and need your help!

In the following parts, they want to know whether their work is correct or not. For each part, *circle the correct answer and include a brief justification (fewer than 20 words) explaining your choice.*



- (a) (2 Points) Elad first grounds the circuit, such that it looks like the one below. Anant, who is used to circuit diagrams with the ground at the bottom of the circuit, wonders if we can put the ground off to the side.



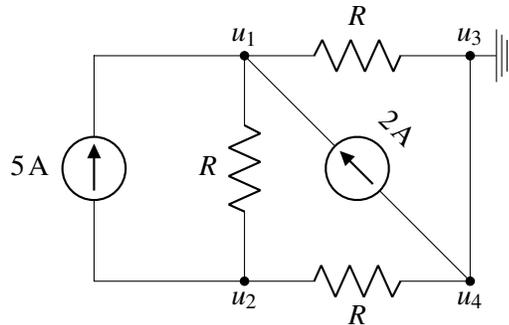
Did Elad choose to label the ground node at a valid location?

YES

NO

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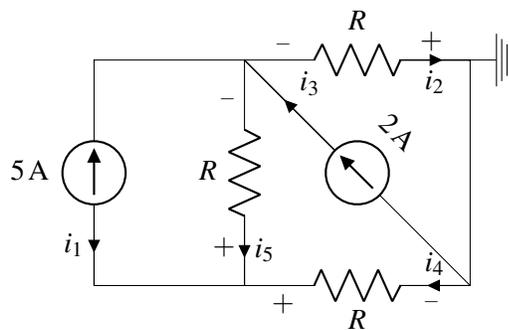
- (b) (2 Points) Anant then adds four labels u_1 through u_4 . Are any of these labels redundant (i.e., are any of the nodes in the circuit labeled more than once)?



YES

NO

- (c) (2 Points) Elad then labels the currents, i_1 through i_5 , and adds the $+/-$ signs for the resistors while attempting to obey passive sign convention. Did he follow passive sign convention correctly for all of the resistors?



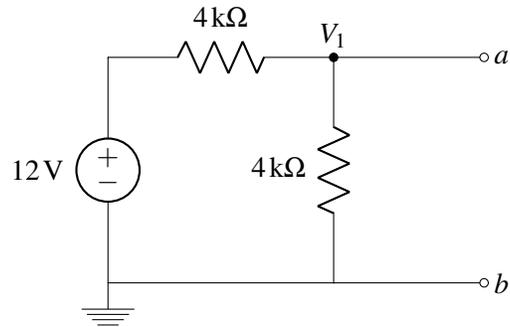
YES

NO

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4. Thévenin and Norton Circuits (13 Points)

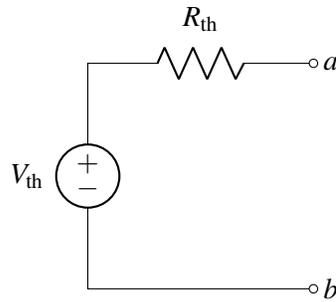
Consider the following circuit:



(a) (3 Points) *Find the voltage V_1 (relative to ground).*

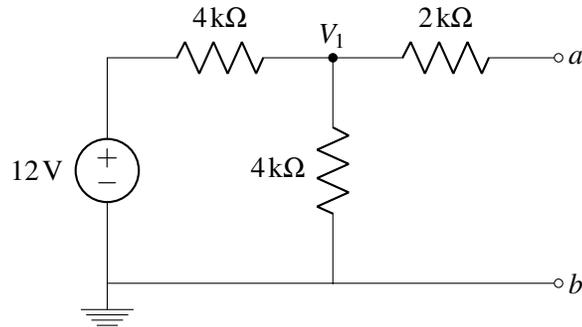
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- (b) (4 Points) Calculate R_{th} and V_{th} such that the Thévenin equivalent circuit shown below matches the I - V characteristics of the original circuit between the a and b terminals.

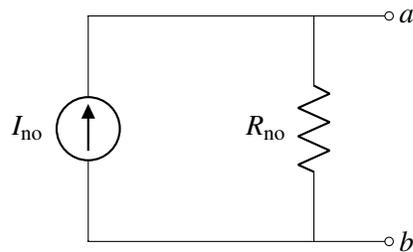


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- (c) (6 Points) As shown below, we will now consider what happens when we add another resistor to the original circuit.



Find the values of R_{no} and I_{no} such that the Norton equivalent circuit shown below matches the I - V characteristics of this new circuit between the a and b terminals.

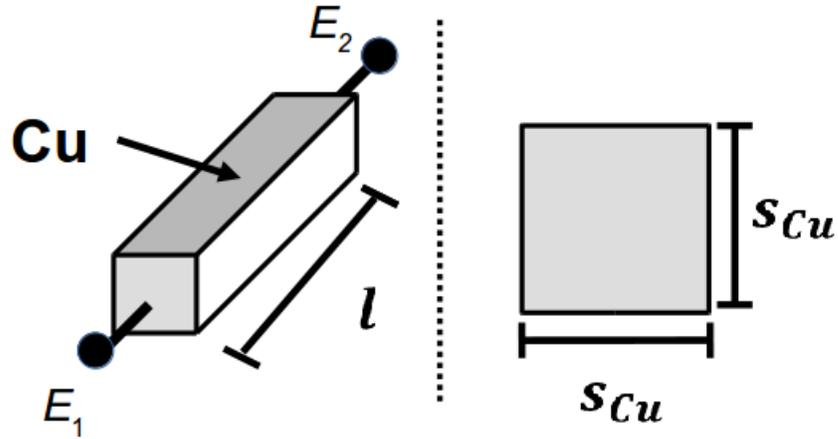


Hint: Your result from part (b) might be useful.

5. Wire we doing this... (13 Points)

A common structure used in the field of nanotechnology research is something called a core-shell nanowire. This consists of a physical structure that has a core made of one material and a shell made of another, where current flows through both parts. Note that the following figures are not drawn to scale.

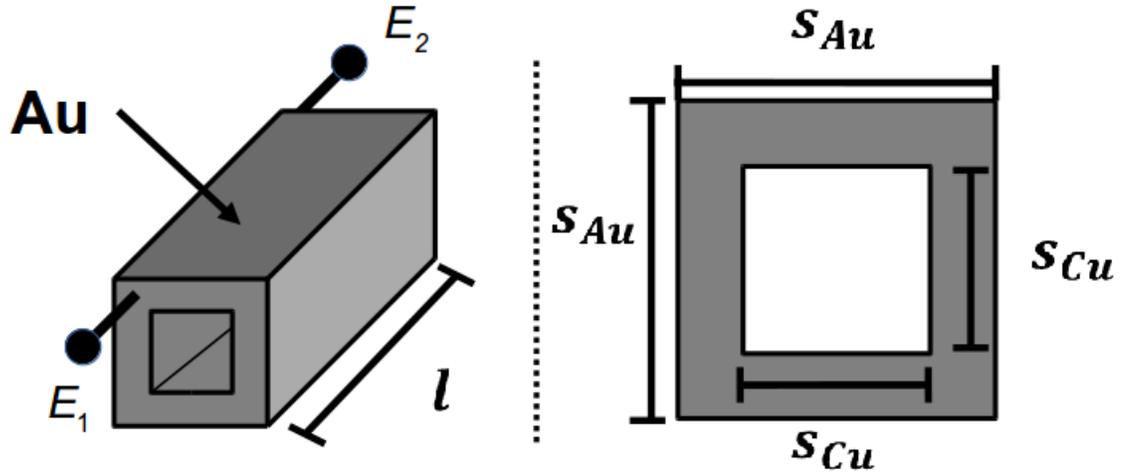
- (a) (3 Points) A copper (Cu) structure with a square cross-section is shown below. Given the material parameters, *calculate the resistance* R_{Cu} of the structure between E_1 and E_2 .



ρ_{Cu}	$1 \times 10^{-8} \Omega m$
s_{Cu}	5 nm
l	75 nm

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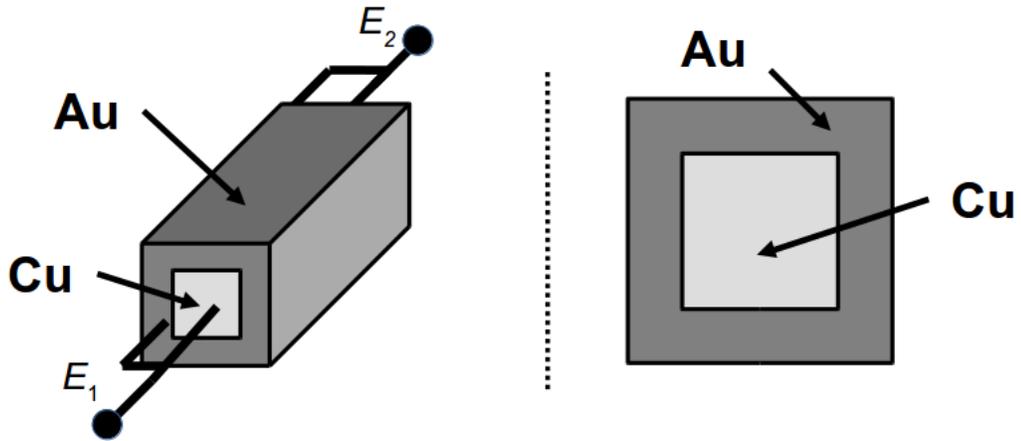
- (b) (4 Points) A gold (Au) structure in the shape of a shell is shown below. Given the material parameters, calculate the resistance R_{Au} of the Au structure between E_1 and E_2 .



ρ_{Au}	$2 \times 10^{-8} \Omega m$
S_{Au}	10 nm
l	75 nm

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- (c) (3 Points) Now the two structures are combined together, such that they make one structure, with the outside shell made of Au and the inside made of Cu. This is called a core-shell nanowire. Assuming that you are contacting the full ends of the nanowire (i.e., E_1 and E_2 are both connected with ideal wires to the faces of the Cu and Au structure), *model the nanowire as a set of resistors*, using R_{Au} for the resistance of the Au layer and R_{Cu} for the resistance of the Cu layer.

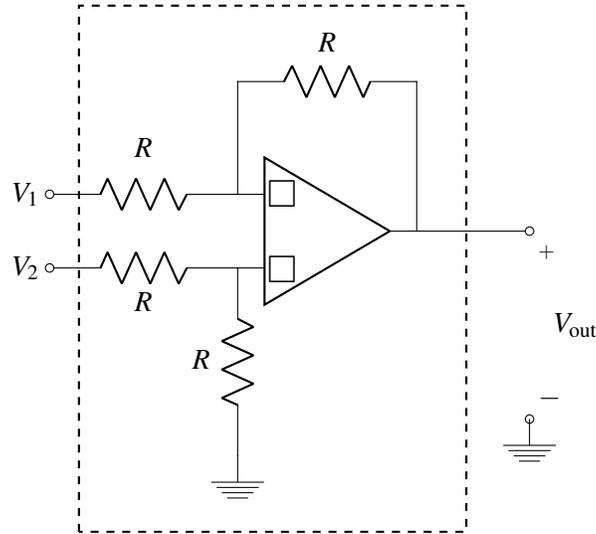


- (d) (3 Points) Based on your model from part (c), find the equivalent resistance R_{wire} between E_1 and E_2 .

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6. Do You See The Difference? (11 Points)

Consider the following circuit:



- (a) (4 Points) *Label the '+' and '-' terminals of the op-amp above so that it is in negative feedback.*
- (b) (7 Points) *Assuming that the op-amp is in negative feedback, use the Golden Rules (combined with any other analysis technique) to find V_{out} in terms of R , V_1 , and V_2 .*

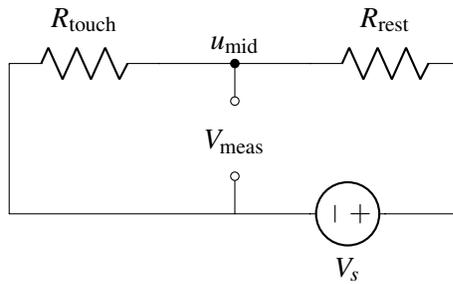
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7. A New Feature You Didn't Even Know You Wanted! (14 Points)

An up-and-coming computer company, Orange Inc., is trying to design a touchscreen bar to incorporate into their new laptop, right above the keyboard. Let's help them analyze their existing design to see where their design has gone wrong!

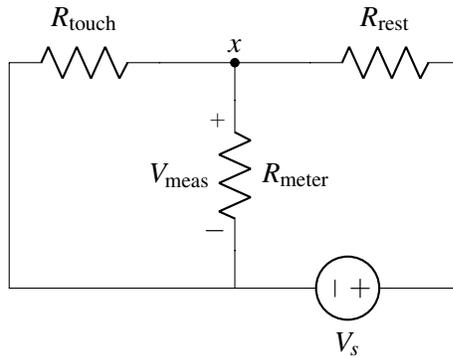
- (a) (7 Points) Orange Inc.'s touchscreen is small enough that we are only interested in the horizontal position of the touch and hence can use the 1D touchscreen circuit model shown below, where the u_{mid} node is labeled at the point the touch occurs. The touchscreen bar has a total length of 10cm, but due to some disputes with their supplier, Orange Inc. has not been able to find out what the resistivity of the touchscreen material is. Despite this, your colleague claims that they can still predict the relationship between V_{meas} and the position where a customer touched the bar. Is your colleague correct? *Circle your answer.*

If you answered that your colleague is correct, *provide an expression for V_{meas} as a function of V_s and the position of the touch x* (measured in cm relative to the left side of the circuit). If you answered that your colleague is incorrect, *provide an expression for V_{meas} as a function of V_s , R_{touch} , and R_{rest} .*



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- (b) (7 Points) It turns out that Orange Inc's problems aren't limited to their touchscreen materials – the device they use to measure the voltage V_{meas} has a finite but known resistance R_{meter} associated with it. Connecting the measurement device to the touchscreen results in the circuit model shown below. Without knowing the value of the resistivity of the material (which, as a reminder, would affect the values of R_{touch} and R_{rest}), can you compute the value of V_{meas} ? *Justify your answer by providing an expression for V_{meas} as a function of R_{touch} , R_{rest} , R_{meter} , and V_s .*



8. Force Touch (22 Points)

So far, our capacitive touchscreens have been able to measure the presence or absence of a touch, but with some modifications, we can actually measure how hard the finger is pressing (i.e., force) as well. Figure 8.1 shows this type of touch screen without any touch and with the finger pressing on it; the more force the finger applies to the screen, the more the distance between the two metal plates decreases.

Assume that the insulator in between the plates has some permittivity ϵ_1 and that the top metal plate has an area A . With no force applied on the screen, the top and bottom plates are a distance d apart. When a force is applied, the distance becomes d' ($< d$). Suppose when a finger is touching the screen, it creates a capacitance $C_{F,E_{top}}$ between itself and the top plate and a capacitance $C_{F,E_{bottom}}$ between itself and the lower plate.

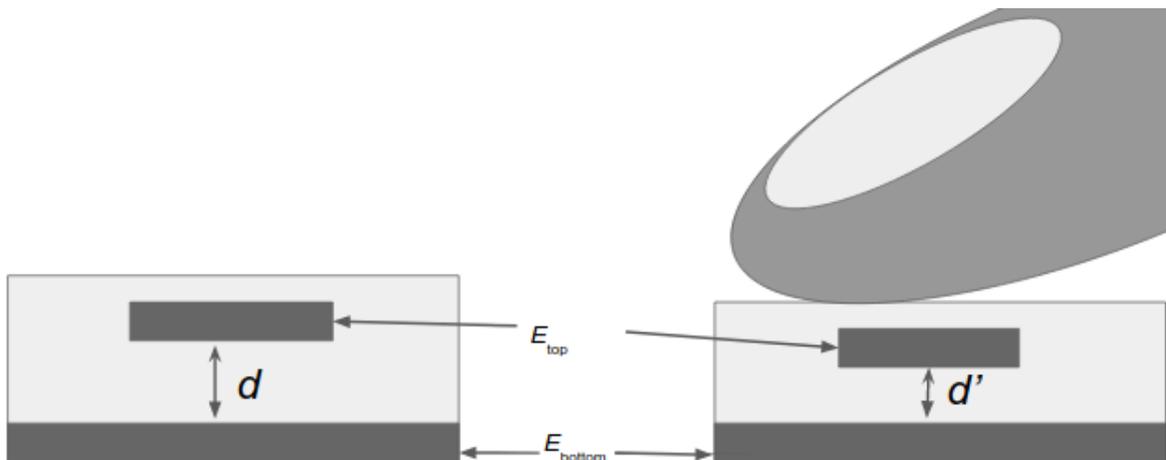


Figure 8.1: Sensor configurations.

- (a) (3 Points) With no finger touching or applying any force, *find the capacitance $C_{no\ touch}$ between the top metal plate and the bottom metal plate. Express your answer in terms of ϵ_1 , d , and A .*

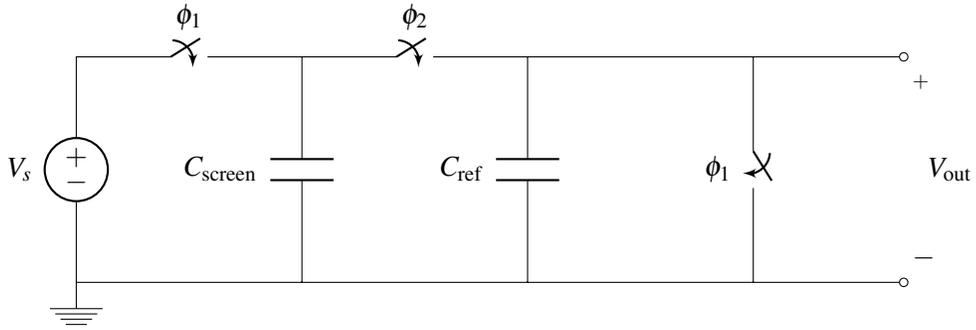
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(b) (4 Points) Now suppose that a finger that is touching the screen applies some force on our screen. *Draw a circuit model* including all of the capacitors connected to either E_{top} or E_{bottom} . *Label all elements* in your model.

(c) (4 Points) Assuming that $C_{F,E_{\text{top}}} = C_{F,E_{\text{bottom}}} = 0\text{F}$, *find the equivalent capacitance, C_{force} , between E_{top} and E_{bottom}* . Express your answer in terms of ϵ_1 , d' , and A .

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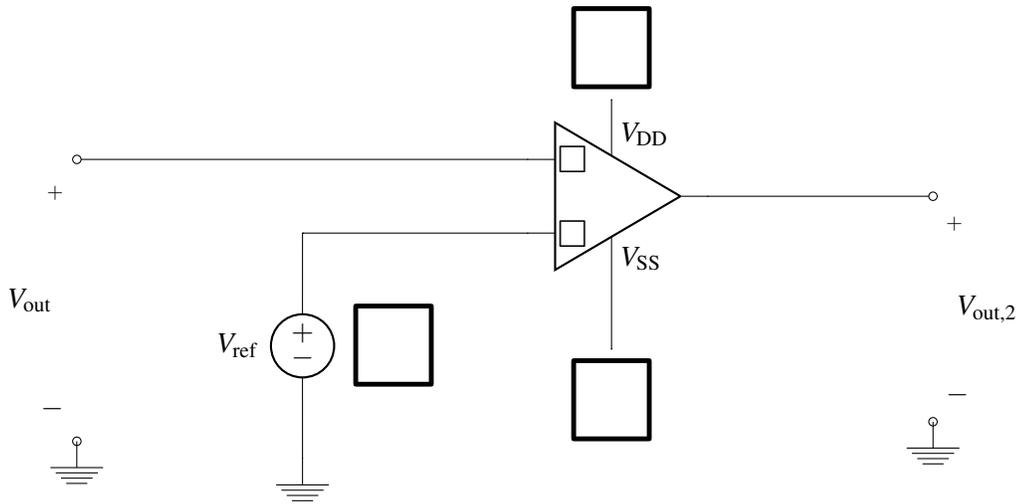
- (d) (4 Points) We connect our structure to the circuit shown below, where your answer to part (c) is now some C_{screen} (which represents the equivalent capacitance between E_{top} and E_{bottom}). The circuit cycles through two phases. In phase 1, switches labeled ϕ_1 are **on**, and in phase 2, switches labeled ϕ_2 are **on**. Derive the value of V_{out} during phase 2 in terms of C_{screen} , C_{ref} and V_s .



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- (e) (7 Points) In the previous circuit, if the finger is pressing with a certain force F' and $V_s = 5\text{ V}$, assume that $V_{\text{out}} = 2.5\text{ V}$ during phase 2. We want to design a circuit that outputs 0 V when we apply more force than F' and 3.3 V when we apply less force than F' .

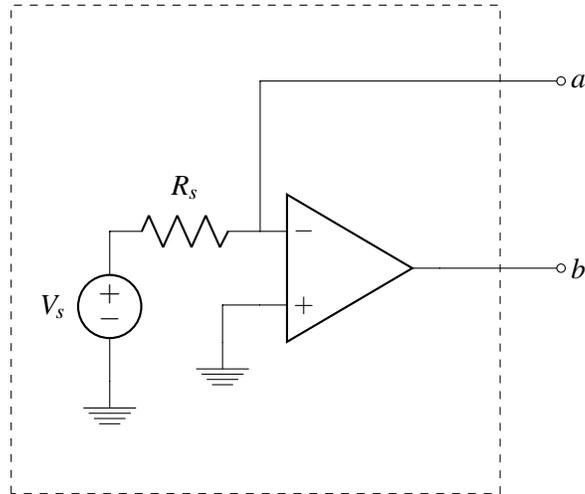
In the circuit below, label the terminals of the op-amp, indicate what you will connect its supplies to, and pick a value for V_{ref} such that $V_{\text{out},2} = 0\text{ V}$ when more force than F' is applied and $V_{\text{out},2} = 3.3\text{ V}$ when less force than F' is applied.



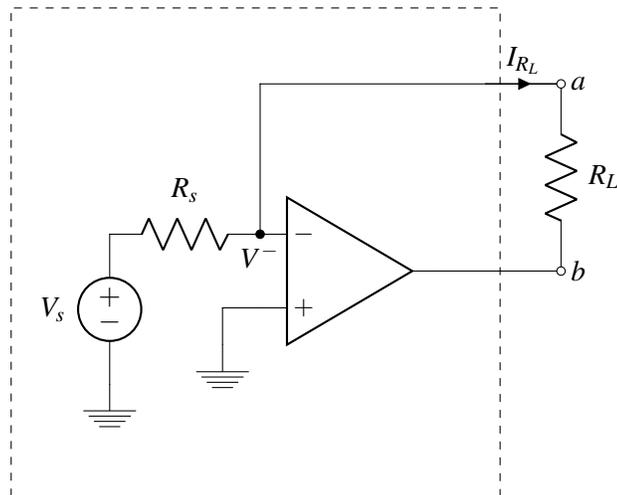
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9. Op-Amp Fun (11 Points + 5 Points)

Consider the following circuit:



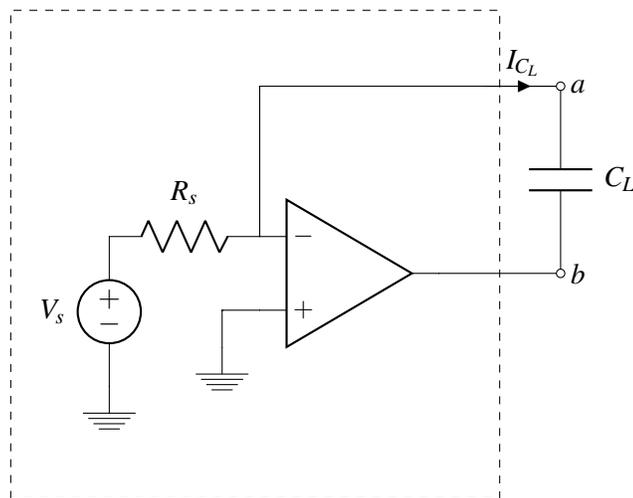
- (a) (3 Points) Suppose that we connect a resistor across the terminals a and b as shown in the circuit below. Find the voltage V^- at the inverting input terminal of the op-amp relative to ground.



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(b) (3 Points) Find the current I_{R_L} through the resistor R_L as a function of V_s , R_s , and R_L .

(c) (5 Points) Suppose that you replace R_L with a capacitor C_L as shown below. Find the current I_{C_L} flowing through the capacitor C_L as a function of V_s , R_s , and C_L .



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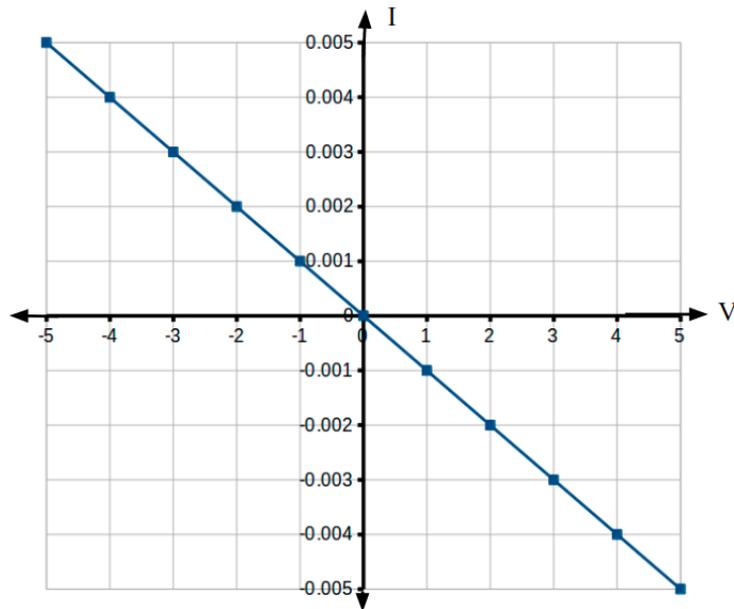
- (d) (Bonus: 5 Points) Assume that at time $t = 0$, the voltage across C_L is equal to 0 V. *Derive an expression for the voltage V_b at node b (relative to ground) as a function of V_s , R_s , C_L , and t .*

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10. Are You Resistive? (19 Points + 5 Points)

Bob is a quality control engineer, and his job is to document and analyze the test results of resistors made by his company.

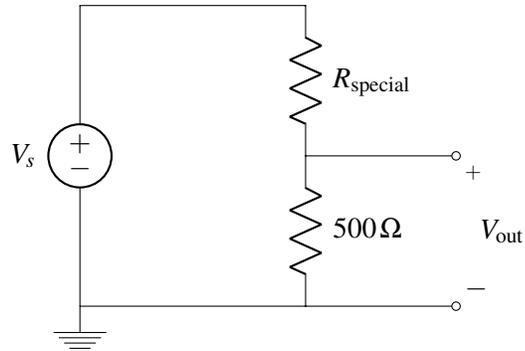
- (a) (8 Points) One day, Bob was testing the I - V curves of the resistors, and he saw something surprising for one particular resistor R_{special} . Based on this I - V characteristic, *find the value of R_{special}* .



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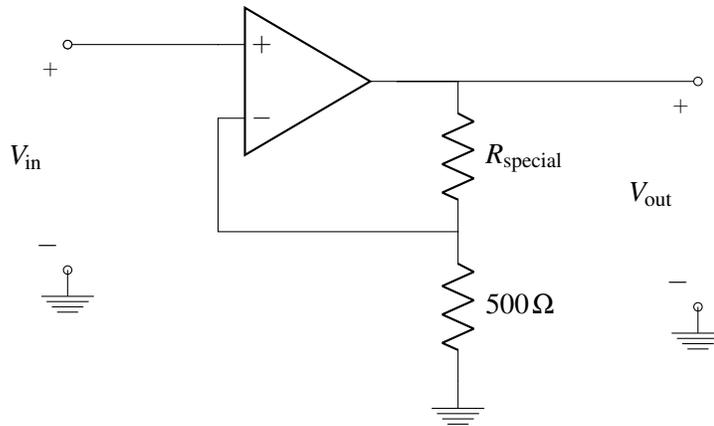
- (b) (5 Points) As shown below, Bob draws a voltage divider circuit using R_{special} , a 500Ω resistor, and a constant voltage source V_s on a sheet of paper.

Find V_{out} in terms of V_s using your value of R_{special} from part (a).



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- (c) (6 Points) Bob now uses the divider with an op-amp in a non-inverting amplifier configuration as shown below. *Is the op-amp below in positive or negative feedback? Make sure to justify your answer.*



- (d) (Bonus: 5 Points) Bob actually builds the circuit from part (c), and he finds that $V_{out} = 3V_{in}$. Based on this result, *did Bob measure $R_{special}$ correctly? Briefly justify your answer.*

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Extra page for scratchwork.

If you want any work on this page to be graded, please refer to this page on the problem's main page.

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Doodle page!

Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

Read the following instructions before the exam.

You have 120 minutes for the exam. There are 10 problems of varying numbers of points. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can.

There are 111 points possible (with an additional 10 possible bonus points) on this exam. Partial credit will be given for substantial progress on each problem.

Distribution of the points:

Problem	1	2	3	4	5	6	7	8	9	10
Points	1	1	6	13	13	11	14	22	11	19

The exam is printed double-sided. Do not forget the problems on the back sides of the pages!

There are 26 pages on the exam, so there should be 13 sheets of paper in the exam. Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

Write your name and your student ID on each page before time is called. If a page is found without a name and a student ID, we are not responsible for identifying the student who wrote that page.

You may consult TWO handwritten 8.5" × 11" note sheets (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed. **No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.**

Please write your answers legibly in the spaces provided on the exam; we will not grade outside a problem's designated space unless you specifically tell us where to find your work. In general, show all of your work in order to receive full credit.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

Our advice to you: if you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

Good luck!