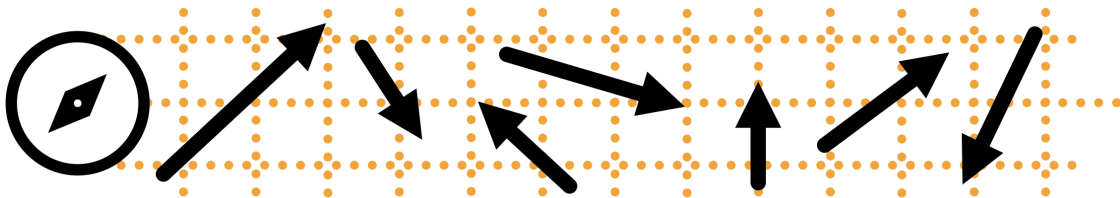


# EECS 16A    Designing Information Devices and Systems I

## Fall 2020    Discussion 1B

### 1. Vectors



A vector is an ordered list of numbers. For instance, a point on a plane  $(x,y)$  is a vector! We label vectors using an arrow overhead  $\vec{v}$ , and since vectors can live in ANY dimension of space we'll need to leave our notation general  $(x,y) \rightarrow \vec{v} = (v_1, v_2, \dots)$ . Below are few more examples (the left-most form is the general definition):

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \qquad \vec{a} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3 \qquad \vec{b} = \begin{bmatrix} 2.4 \\ 5.3 \end{bmatrix} \in \mathbb{R}^2$$

Just to unpack this a bit more,  $\vec{b} \in \mathbb{R}^3$  in english means "vector  $\vec{b}$  lives in 3-Dimensional space".

- The  $\in$  symbol literally means "in"
- The  $\mathbb{R}$  stands for "real numbers" (FUN FACT:  $\mathbb{Z}$  means "integers" like  $-2, 4, 0, \dots$ )
- The exponent  $\mathbb{R}^n \leftarrow$  indicates the dimension of space, or the amount of numbers in the vector.

One last thing: it is standard to write vectors in column-form, like seen with  $\vec{a}, \vec{b}, \vec{x}$  above. We call these *column vectors*, in contrast to horizontally written vectors which we call *row vectors*.

Okay, let's dig into a few examples:

(a) Which of the following vectors live in  $\mathbb{R}^2$  space?

$$i. \begin{bmatrix} 3 \\ 6 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \qquad iii. \begin{bmatrix} -4.76 \\ 1.32 \\ 0.01 \end{bmatrix} \qquad iv. \begin{bmatrix} -20 \\ 100 \end{bmatrix}$$

**Solution/Answer:**

i. Yes ii. No iii. No iv. Yes

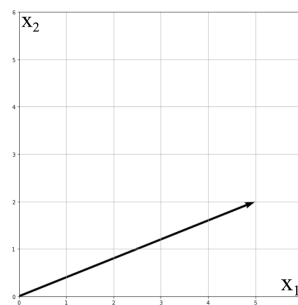
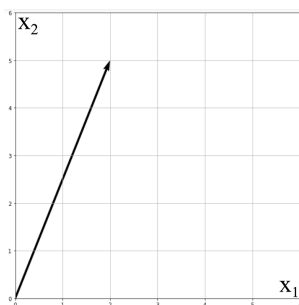
Remember  $\mathbb{R}^2$  means 2D space, which hosts vectors with 2 terms.

We count and see only i. and iv. have 2 terms.

(b) Graphically show the vectors (either in a sketch with axes, or a plot on a computer):

$$i. \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

**Solution/Answer:** Although these vectors look similar, remember that the ordering matters!

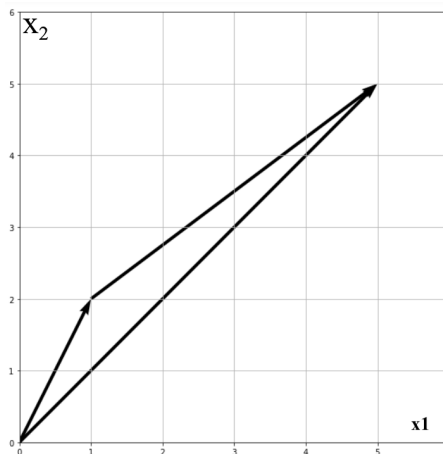


- (c) Compute the sum  $\vec{a} + \vec{b} = \vec{c}$  from the vectors below, and then graphically sketch or plot these vectors. (show them in a way that forms a triangle; also is there only one possible triangle?)

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

**Solution/Answer:**



Computation is done element-wise:

$$\vec{c} = \vec{a} + \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

- 2. Solving Systems of Equations** A system of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following systems of equations, state whether there is a unique solution, no solution, or an infinite number of solutions. If there are an infinite number of solutions give one possible solution.

- (a) Solve the following system. How many solutions does it have?

$$\begin{aligned} x + y &= 4 \\ x - y &= 2 \end{aligned}$$

**Solution/Answer:** Adding the two equations together, we get

$$2x = 4 + 2 = 6$$

This gives  $x = 3$ . Substituting back into the first equation, we get

$$3 + y = 4,$$

which gives us  $y = 1$ .

So there is a unique solution,  $x = 3$  and  $y = 1$ .

(b) Now write the system in augmented matrix form:

$$x + y = 4 \tag{1}$$

$$x - y = 2 \tag{2}$$

**Solution/Answer:**

Augmented matrix form is just a shorthand notation for representing the system. You can write:

$$\left[ \begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array} \right]$$

(c) Once in augmented matrix form we can use a systematic procedure called Gaussian Elimination to solve the system of equations. See what solution you get using Gaussian elimination.

**Solution/Answer:**

$$\left[ \begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array} \right]$$

We can subtract the first equation from the second equation to eliminate the  $x$  variable.

$$\left[ \begin{array}{cc|c} 1 & 1 & 4 \\ 1-1 & -1-1 & 2-4 \end{array} \right].$$

The first row here represents the equation (1).

$$x + y = 4.$$

The second row represents the equation (2) - (1):

$$(x - y) - (x + y) = 2 - 4.$$

The augmented matrix form represents this by grouping the  $x$  terms and the  $y$  terms together, i.e.

$$(x - x) - (y + y) = 2 - 4,$$

or

$$0 - 2y = -2.$$

The augmented matrix is written as:

$$\left[ \begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -2 & -2 \end{array} \right],$$

From here we read off that  $y = 1$ .

Now, we can back substitute into (1) to get that  $x = 3$ .

(d) Now consider the system

$$7x + y = 7 \quad (3)$$

$$42x + 6y = 42. \quad (4)$$

How many solutions does it have? Solve it first using any method, then write it as an augmented matrix and try to solve it.

**Solution/Answer:**

We can see that the first equation gives the second equation when multiplied by 6. But how would we do this systematically? We first write the matrix:

$$\left[ \begin{array}{cc|c} 7 & 1 & 7 \\ 42 & 6 & 42 \end{array} \right]$$

Then, taking six times the first row and subtracting it from the second gives:

$$\left[ \begin{array}{cc|c} 7 & 1 & 7 \\ 0 & 0 & 0 \end{array} \right].$$

So the second equation reduces to  $0 = 0$ . Therefore there are infinitely many solutions. How could we write out all the infinitely many solutions?

For every value of  $y$  we choose we can write a value of  $x$ .

So let  $y = a$ , where  $a$  is some arbitrary real number. We can use a shorthand notation to write this as  $a \in \mathbb{R}$ , where  $\mathbb{R}$  is the set of real numbers and  $\in$  means in.

Substituting this into the first equation we get:

$$7x + a = 7,$$

so

$$x = (7 - a)/7. \text{ This is sometimes called a parametric form of the solution with parameter } a.$$

If you wanted to write the set of solutions as a vector, you could write all vectors of the form

$$\begin{bmatrix} (7 - a)/7 \\ a \end{bmatrix}$$

are solutions.

(e) Now consider the system

$$7x + y = 7 \quad (5)$$

$$42x + 6y = 42 \quad (6)$$

$$7x + y = 6 \quad (7)$$

How many solutions does it have? Solve it first using any method, then write it as an augmented matrix and try to solve it.

**Solution/Answer:**

Here we see that (6) is a multiple of (5), however, (5) and (7) are inconsistent. So no solution exists.

But how would we do this systematically? We first write the matrix:

$$\left[ \begin{array}{cc|c} 7 & 1 & 7 \\ 42 & 6 & 42 \\ 7 & 1 & 6 \end{array} \right]$$

Then, taking six times the first row and subtracting it from the second gives:

$$\left[ \begin{array}{cc|c} 7 & 1 & 7 \\ 0 & 0 & 0 \\ 7 & 1 & 6 \end{array} \right].$$

The subtracting the first row from the second gives:

$$\left[ \begin{array}{cc|c} 7 & 1 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{array} \right].$$

Here the last row represents the equation  $0y = -1$ , which we know cannot have a solution. Hence, in this case the system has no solutions.