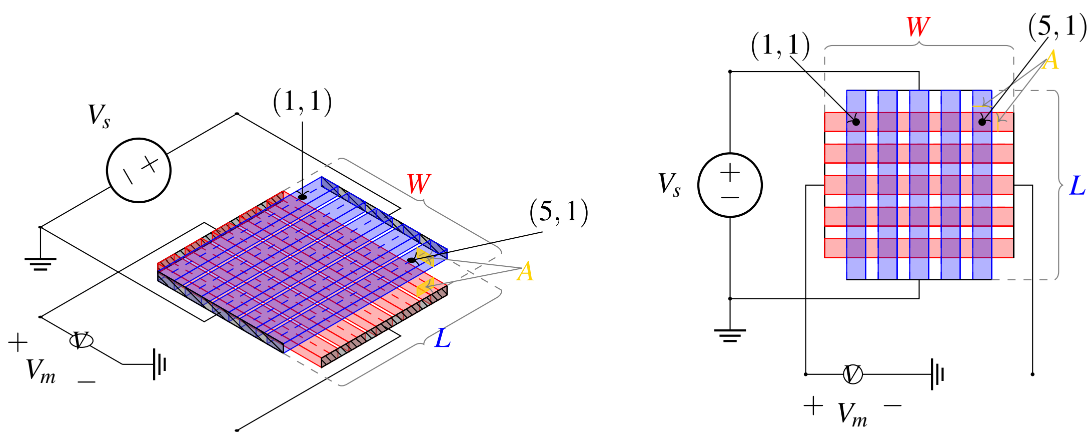
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 8A

1. Resist the Touch



(a) 5×5 2-D Resistive Touch Screen Oblique View (b) 5×5 2-D Resistive Touch Screen Top View

Figure 1: $N \times N$ Resistive Touch Screen, $N = 5$

In this question we will be re-examining the 2-dimensional resistive touchscreen. This touchscreen, is slightly different to the one shown in lecture and more like the one we will be examining in lab.

The touchscreen has length L and width W and is composed of a rigid bottom-layer and a flexible top-layer. Instead of a having a two continuous resistive sheets on the top and bottom layers, this is a simpler implementation with N vertical strips of conductive material in the top layer and N horizontal strips of conductive material in the bottom layer. The strips of a single layer are all connected by an ideal conducting plate on each side. All strips have resistivity, ρ, and cross-sectional area, A.

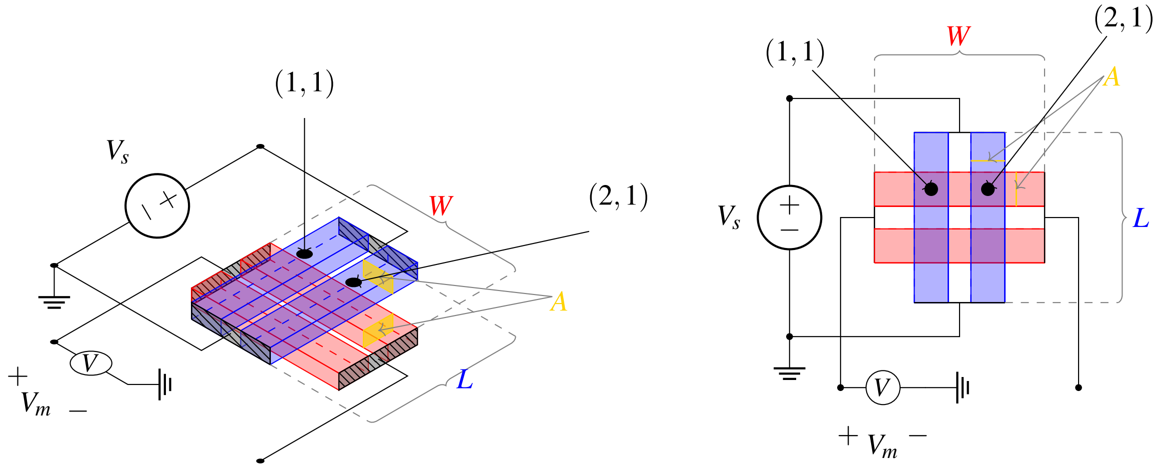
Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally, and that the upper left touch point in Figure 1(b) is position $(1,1)$, and the upper right touch point is $(N,1)$. The spacing between the strips in the top layer is $\frac{W}{N+1}$, and the spacing between the strips in the bottom layer is $\frac{L}{N+1}$.

(a) Find the resistance R_y for a single vertical blue strip and R_x for a single horizontal red strip, as a function of the screen dimensions W and L, the strip resistivity ρ, and the cross-sectional area A.

Answer: The equation for resistance is $R = \frac{\rho l}{A}$

Therefore for the bottom red horizontal resistive strips we have, $R_x = \frac{\rho W}{A}$.

For the top blue vertical resistive strips, $R_y = \frac{\rho L}{A}$.



(a) 2×2 2-D Resistive Touch Screen Oblique View (b) 2×2 2-D Resistive Touch Screen Top View

Figure 2: 2 × 2 Resistive Touch Screen

(b) Consider a 2×2 example for the touchscreen circuit, shown in Figure 2.

Assume that we connect a voltage source V_s, between the top and bottom terminals of the blue strips, and a voltmeter V_m to one of the left or right terminals as depicted in the diagram.

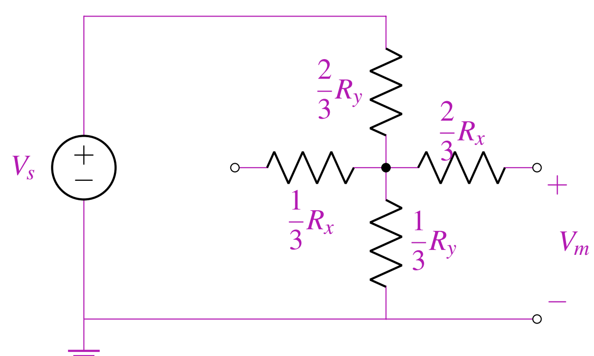
If $V_s = 3\,\text{V}$, $R_x = 2000\,\Omega$, and $R_y = 2000\,\Omega$, draw the equivalent circuit for when the point $(2,2)$ is pressed and solve for the measured voltage, V_m, with respect to ground.

Reminder: all top layer resistive strips and bottom layer resistive strips are spaced apart equally, and that the upper left touch point is position $(1,1)$. The spacing between the strips in the top layer is $\frac{W}{N+1}$, and the spacing between the strips in the bottom layer is $\frac{L}{N+1}$.

Answer:

Since all of the resistive strips are equally spaced, the resistor above point $(2,2)$ on the vertical blue strip becomes $\rho\frac{\frac{2L}{3}}{A} = \frac{2}{3}\rho\frac{L}{A} = \frac{2}{3}R_y$ and the resistor below point $(2,2)$ on the vertical strip becomes $\rho\frac{\frac{L}{3}}{A} = \frac{1}{3}\rho\frac{L}{A} = \frac{1}{3}R_y$.

The bottom layer red resistors, although they must be drawn in the equivalent circuit, do not affect the measured voltage, V_m, as there are open circuits, leading to no currents through those resistors, and therefore no voltage drops over either of them.



Observing that the rightmost top layer blue resistive strip forms a voltage divider, and remembering that there is no voltage drop across the dangling R_x resistors, we can determine V_m using the voltage divider equation.

Therefore, $V_m = V_{(2,2)} = V_s \frac{\frac{1}{3}R_y}{\frac{1}{3}R_y + \frac{2}{3}R_y} = \frac{1}{3}V_s = 1\text{V}.$

(c) Suppose a touch occurs at coordinates (i, j) for an arbitrary $N \times N$ touchscreen, and the voltage source and meter are connected as in the figures. A 5×5 example is shown in Figure 1(b). Find an expression for V_m as a function of V_s, N, i, and j. Again, the upper left corner is the coordinate $(1, 1)$ and the upper right coordinate is $(N, 1)$

Answer: The voltage does not depend on the x coordinate, as the meter is connected to the red dangling resistors along the horizontal. We will only be able to detect changes in the y coordinate. If the touch point occurs at (i, j), the i-th blue vertical bar from the left will be split into lengths of $L_{top} = \frac{j}{N+1}L$ and $L_{bottom} = \frac{N+1-j}{N+1}L$ at the j-th touch point from the top. The voltage divider takes the voltage over the bottom resistor, so we will see $V_m = \frac{L_{bottom}}{L}V_s = \frac{\frac{N+1-j}{N+1}L}{L}V_s = \frac{N+1-j}{N+1}V_s$

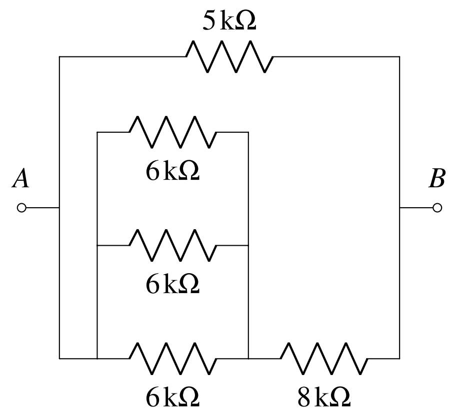
(d) Optional / Fun: Experiment with the TinkerCad models below to validate the theoretical results you just derived.

TinkerCad model of 2×2 equivalent circuit: https://www.tinkercad.com/things/0wIXz3MkD7B

TinkerCad model of 3×2 equivalent circuit: https://www.tinkercad.com/things/k5oolj2tUEN

2. Practice: Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals A and B using the resistor combination rules for series and parallel resistors.



Answer:

$$5\,\mathrm{k\Omega} \parallel ((6\,\mathrm{k\Omega} \parallel 6\,\mathrm{k\Omega} \parallel 6\,\mathrm{k\Omega}) + 8\,\mathrm{k\Omega}) = 5\,\mathrm{k\Omega} \parallel (2\,\mathrm{k\Omega} + 8\,\mathrm{k\Omega}) = 5\,\mathrm{k\Omega} \parallel 10\,\mathrm{k\Omega} = 3.33\,\mathrm{k\Omega}$$