

# EECS 16A    Designing Information Devices and Systems I

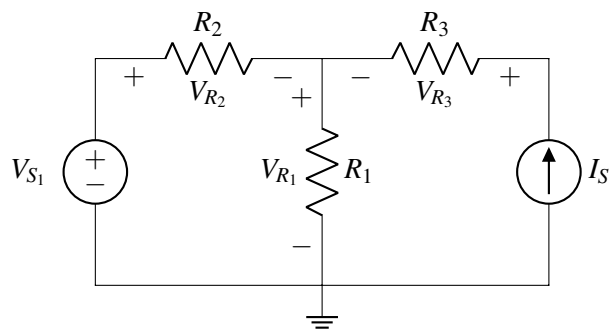
## Fall 2020    Discussion 8B

### 1. Superposition

For the following circuits:

- Use the superposition theorem to solve for the voltages across the resistors.
- For parts (b) and (c) only, find the power dissipated/generated by all components. Is power conserved?

(a)



**Answer:** Turning on only  $V_{S1}$ , we have the following voltages across the resistors:

$$V_{R1} = \frac{R_1}{R_1 + R_2} V_{S1}$$

$$V_{R2} = \frac{R_2}{R_1 + R_2} V_{S1}$$

$$V_{R3} = 0$$

Then turning only  $I_{S1}$ , we have the following voltages:

$$V_{R1} = \frac{R_1 R_2}{R_1 + R_2} I_{S1}$$

$$V_{R2} = -\frac{R_1 R_2}{R_1 + R_2} I_{S1}$$

$$V_{R3} = I_{S1} R_3$$

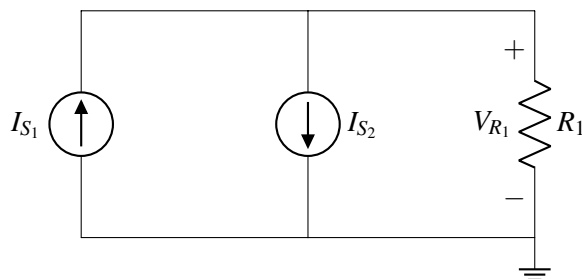
Using superposition we can sum up the contributions from both  $V_{S1}$  and  $I_{S1}$  to get:

$$V_{R1} = \frac{R_1}{R_1 + R_2} V_{S1} + \frac{R_1 R_2}{R_1 + R_2} I_{S1}$$

$$V_{R2} = V_{S1} - V_{R1} = \frac{R_2}{R_1 + R_2} V_{S1} - \frac{R_1 R_2}{R_1 + R_2} I_{S1}$$

$$V_{R3} = I_{S1} R_3$$

(b)

**Answer:**

- i. While we could apply the algorithm we have learned in class, let's see if there's a way to find the answer quicker than before. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know  $i_{R_1} = I_{S_1} - I_{S_2}$ . Applying Ohm's Law we find:

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

We could also solve this using superposition. Turning on  $I_{S_1}$  gives  $V_{R_1} = I_{S_1}R_1$ . Turning on  $I_{S_2}$  gives  $V_{R_1} = -I_{S_2}R_1$ . Finally, the total  $V_{R_1}$  is the sum of the individual  $V_{R_1}$ 's or

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = (I_{S_1} - I_{S_2})^2 R_1$$

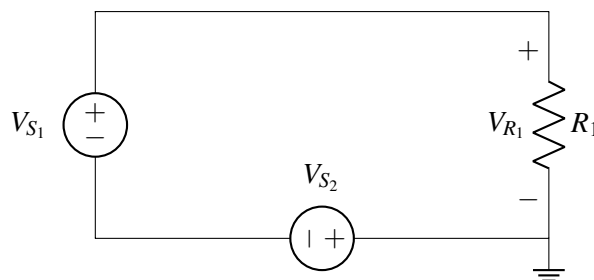
$$P_{I_{S_1}} = -I_{S_1} V_{R_1} = -(I_{S_1} - I_{S_2}) I_{S_1} R_1$$

$$P_{I_{S_2}} = I_{S_2} V_{R_1} = (I_{S_1} - I_{S_2}) I_{S_2} R_1$$

$$P_{R_1} + P_{I_{S_1}} + P_{I_{S_2}} = (I_{S_1} - I_{S_2})^2 R_1 - (I_{S_1} - I_{S_2}) I_{S_1} R_1 + (I_{S_1} - I_{S_2}) I_{S_2} R_1 = 0$$

Power is conserved.

(c)

**Answer:**

- i. Once again, we could apply the circuit analysis algorithm or find the answer directly. Notice the circuit only has one loop, so we can use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

We could also solve with superposition. Turning on  $V_{S_1}$  gives  $V_{R_1} = V_{S_1}$ . Turning on  $V_{S_2}$  gives  $V_{R_1} = -V_{S_2}$ . The overall voltage is then the sum.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = \frac{(V_{S_1} - V_{S_2})^2}{R_1}$$

$$P_{V_{S_1}} = -I_{R_1} V_{S_1} = -\frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1}$$

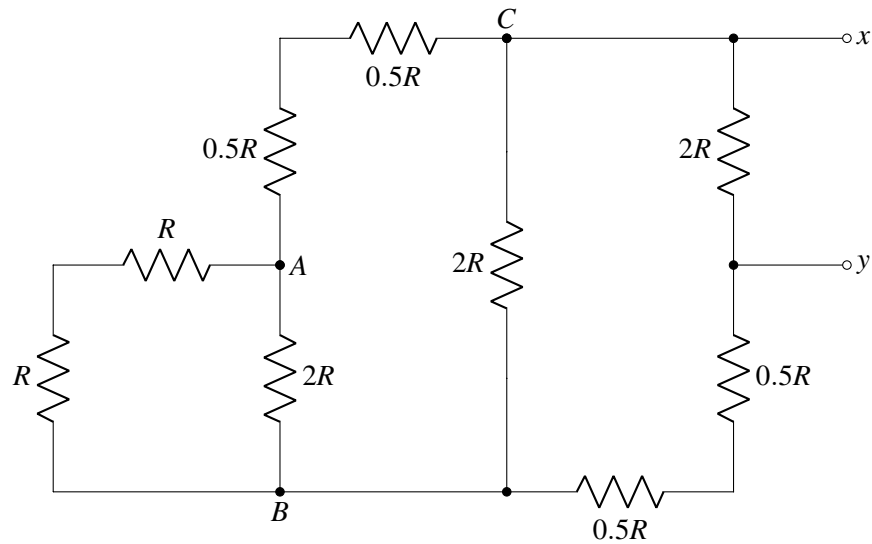
$$P_{V_{S_2}} = I_{R_1} V_{S_2} = \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{R_1} + P_{V_{S_1}} + P_{V_{S_2}} = \frac{(V_{S_1} - V_{S_2})^2}{R_1} - \frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1} + \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1} = 0$$

Power is conserved.

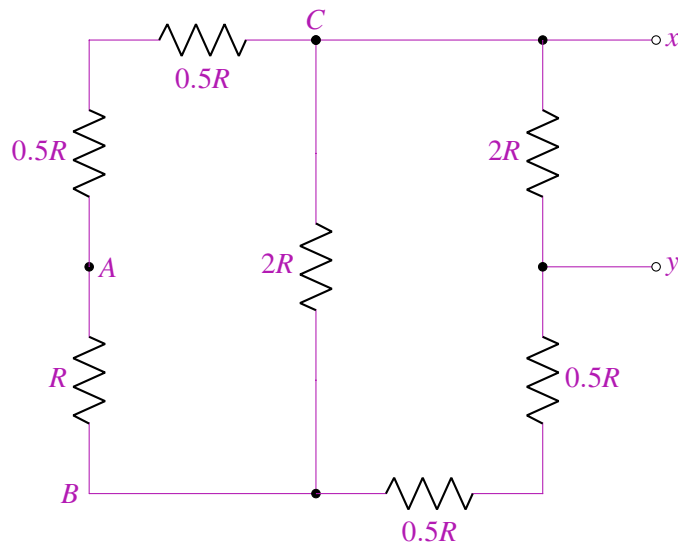
## 2. (Practice) Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals  $x$  and  $y$  using the resistor combination rules for series and parallel resistors.

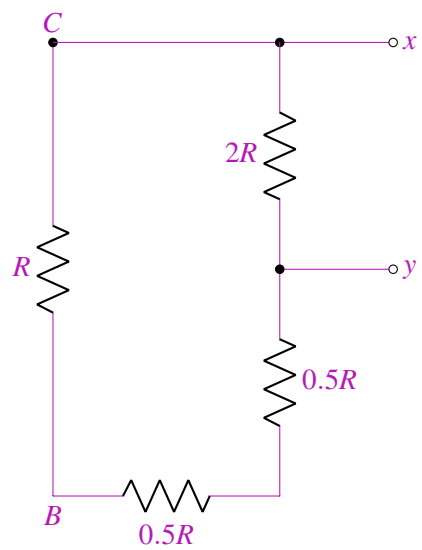
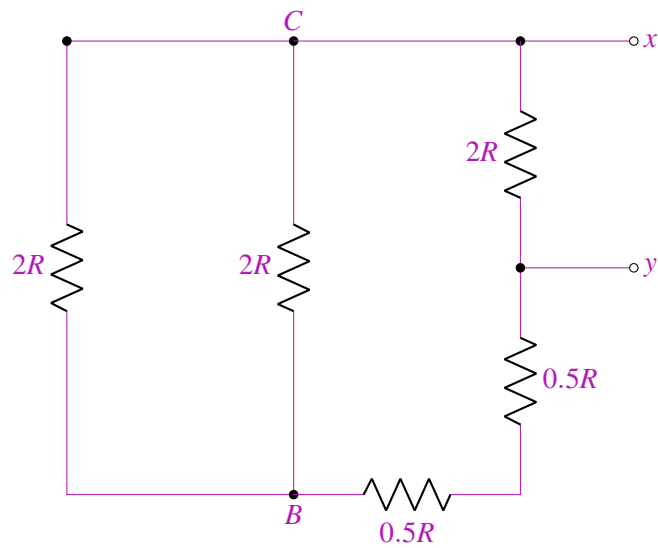


**Answer:**

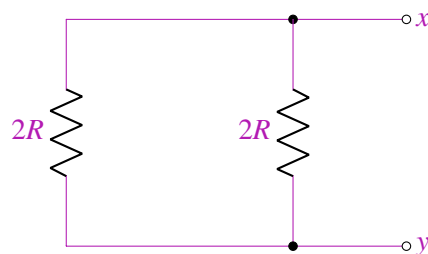
We find the equivalent resistance for the resistors from left to right. First we find  $R_{AB}$ .



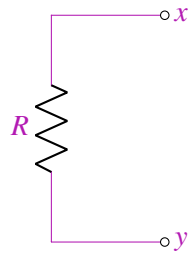
Then we can find  $R_{CB}$ .



If we move from node  $C$  counter-clockwise to node  $y$ , the resistance seen is  $R + 0.5R + 0.5R = 2R$ . Therefore we have,



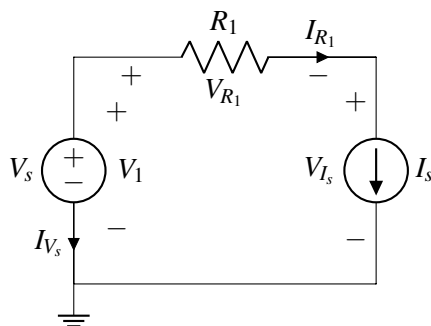
Now we can find  $R_{xy}$ .



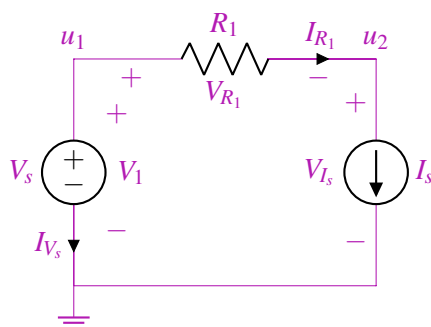
Therefore, the equivalent resistance is  $R_{xy} = R$ .

### 3. (Practice) Passive Sign Convention and Power v 2.0

Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let  $V_s = 5\text{ V}$ ,  $I_s = 0.5\text{ A}$  and  $R_1 = 5\ \Omega$ .



**Answer:** We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equations for nodes  $u_1, u_2$  in this circuit are:

$$I_{V_s} + I_{R_1} = 0$$

$$-I_{R_1} + I_s = 0$$

The Element equations for the voltage source and the resistor are:

$$u_1 - 0 = V_1 = V_s$$

$$V_{R_1} = u_1 - u_2 = I_{R_1} R_1$$

Finally, the KVL equation around the one loop formed by the circuit is:

$$V_1 - V_{R_1} - V_{I_s} = 0$$

Solving the above equations with  $V_s = 5\text{ V}$ ,  $I_s = 0.5\text{ A}$  and  $R_1 = 5\ \Omega$ :

$$I_{R_1} = 0.5\text{ A}$$

$$I_{V_s} = -0.5\text{ A}$$

$$V_{R_1} = 2.5\text{ V}$$

$$V_{I_s} = 2.5\text{ V}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = IV = I_{R_1}V_{R_1} = 0.5 \text{ A} \cdot 2.5 \text{ V} = 1.25 \text{ W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = IV = I_{V_s}V_1 = -0.5 \text{ A} \cdot 5 \text{ V} = -2.5 \text{ W}$$

Finally, we can solve for the power dissipated across the current source:

$$P_{I_s} = IV = I_sV_{I_s} = 0.5 \text{ A} \cdot 2.5 \text{ V} = 1.25 \text{ W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is actually *supplying* power to the circuit.

**Note:** In this case the current source is also dissipating power but it could be also supplying if the numbers were picked differently. For example, if  $I_s = 2\text{A}$  the same equations would give  $P_{R_1} = 20\text{W}$ ,  $P_{V_s} = -10\text{W}$ ,  $P_{I_s} = -10\text{W}$ . Also, numbers could have been selected such that the voltage source dissipated and the current source supplied power to the circuit.

This circuit can also be solved using NVA:

**Step 5:**

$$I_{R_1} = I_s \quad \text{KCL on } u_2$$

**Steps 6 & 7:**

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1}$$

**Steps 8 & 9:**

$$\frac{u_1 - u_2}{R_1} = I_s \Rightarrow u_2 = u_1 - I_s R_1 = V_s - I_s R_1$$

Since, we already know that  $u_1 = V_s$ , as it is set by the voltage source. Then we can write that:

$$V_{I_s} = u_2 = 2.5\text{V}, \quad V_{R_1} = u_1 - u_2 = 2.5\text{V}$$

$$I_{R_1} = I_s = 0.5\text{A}, \quad I_{V_s} = -I_{R_1} = -I_s = -0.5\text{A}$$