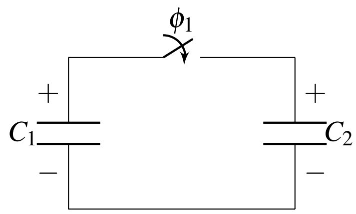
EECS 16A Fall 2020 Designing Information Devices and Systems I Discussion 9B

1. Capacitors and Charge Conservation

(a) Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and an open switch. Suppose that C_1 is initially charged to $+1\,\text{V}$ and that C_2 is charged to $+2\,\text{V}$. How much charge is on C_1 and C_2?



Answer:

$$q_1 = C_1 V_1 = 1\,\mu\text{C}$$
$$q_2 = 2\,\mu\text{C}$$

(b) Now the switch is closed (i.e. the capacitors are connected together.) What are the voltages across and the charges on C_1 and C_2?

Answer:

Charge is always conserved on a floating node.

Let $Q_{C_1,1}, Q_{C_2,1}$ be the charges on the capacitors after the switch is closed. There was $3\,\mu\text{C}$ of total charge on the top two plates of the capacitors initially, so we must have

$$Q_{C_1,1} + Q_{C_2,1} = 3\,\mu\text{C}$$

Further, the voltages on the capacitors must be the same, so:

$$\frac{Q_{C_1,1}}{C_1} = \frac{Q_{C_2,1}}{C_2}$$

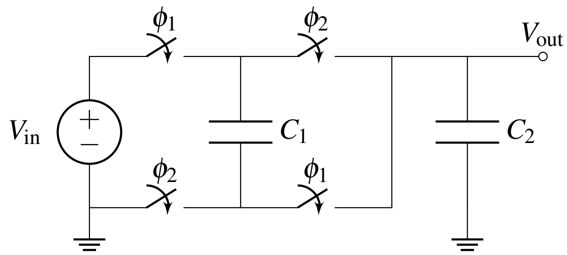
Solving this system gives:

$$Q_{C_1,1} = Q_{C_2,1} = 1.5\,\mu\text{C}$$

Comparing to the previous part, charge has moved from C_2 to C_1. This yields a voltage of $1.5\,\text{V}$.

2. Charge Sharing

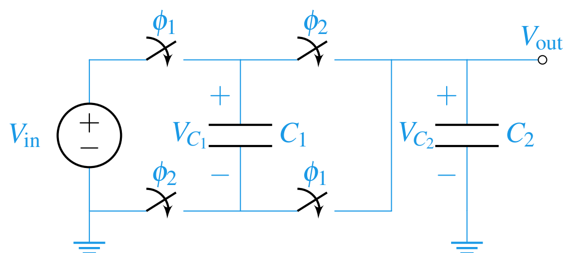
Consider the circuit shown below. In phase ϕ_1, the switches labeled ϕ_1 are on while the switches labeled ϕ_2 are off. In phase ϕ_2, the switches labeled ϕ_2 are on while the switches labeled ϕ_1 are off.



(a) Draw the polarity of the voltage (using + and − signs) across the two capacitors C_1 and C_2. (It doesn't matter which terminal you label + or −; just remember to keep these consistent through phase 1 and 2!)

Answer:

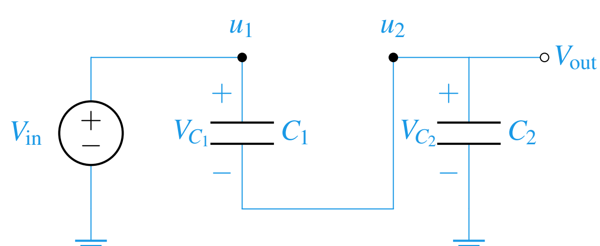
One way of marking the polarities is + on the top plate and − on the bottom plate of both C_1 and C_2. Let's call the voltage drop across C_1 V_{C_1} and across C_2 V_{C_2}.



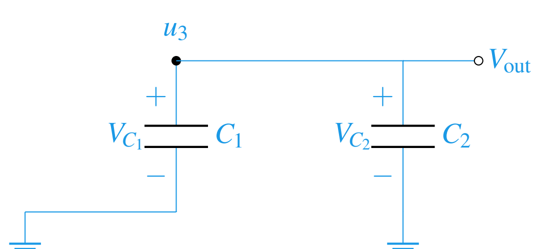
(b) Redraw the circuit in phase ϕ_1 and phase ϕ_2. Keep your polarity from part (a) in mind.

Answer:

Phase ϕ_1



Phase ϕ_2



(c) Find V_{out} in phase ϕ_2 as a function of V_{in}, C_1, and C_2.

Answer:

First, we must identify the floating node in phase ϕ_2. For this circuit, the floating node is u_3, as we can see that charge on the "+" plates of C_1 and C_2 cannot flow to ground.

Now that we know what plates are connected to our floating node, we must find the charge on those plates in phase ϕ_1. The two capacitors in series have a total capacitance of $C_{eq} = \frac{C_1 C_2}{C_1+C_2}$. We know that there is a voltage of V_{in} across this capacitor and thus $Q_{C_{eq}} = V_{\text{in}} \frac{C_1 C_2}{C_1+C_2}$ charge. Because they're in series, we know that the charge across the equivalent capacitance is the same as a charge across each individual capacitor. Since we are looking for the charge on the "+" terminals of those capacitors it will be:

$$\begin{aligned} Q_{u_3}^{\phi_1} &= Q_{C_1} + Q_{C_2} \\ &= 2Q_{C_{eq}} \\ &= 2V_{\text{in}} \frac{C_1 C_2}{C_1 + C_2} \end{aligned}$$

Similarly, we must find the charge on those plates in phase ϕ_2.

$$\begin{aligned} Q_{u_3}^{\phi_2} &= V_{C_1} C_1 + V_{C_2} C_2 \\ &= (u_3 - 0)C_1 + (u_3 - 0)C_2 \\ &= (V_{\text{out}} - 0)C_1 + (V_{\text{out}} - 0)C_2 \\ &= V_{\text{out}}(C_1 + C_2) \end{aligned}$$

Because of the conservation of charge, we can equate the total charge in phase ϕ_1 and phase ϕ_2.

$$\begin{aligned} Q_{u_3}^{\phi_1} &= Q_{u_3}^{\phi_2} \\ 2V_{\text{in}} \frac{C_1 C_2}{C_1 + C_2} &= V_{\text{out}}(C_1 + C_2) \\ V_{\text{out}} &= 2\frac{C_1 C_2}{(C_1 + C_2)^2} V_{\text{in}} \end{aligned}$$

(d) How will the charges be distributed in phase ϕ_2 if we assume $C_1 \gg C_2$?

Answer:

We know that the capacitors are in parallel in phase ϕ_2, so the voltage across both capacitors is the same. Considering that $Q = CV$, if $C_1 \gg C_2$, then $Q_1 \gg Q_2$.

