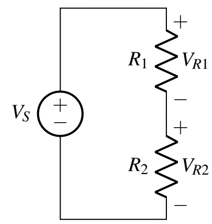
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 11A

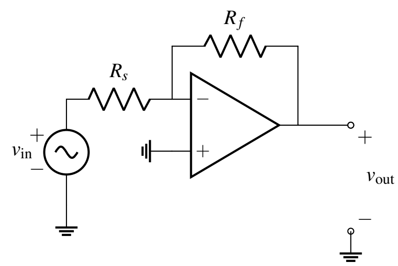
Reference: Op-Amp Example Circuits

Voltage Divider



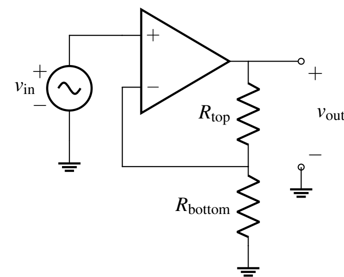
$$V_{R_2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amplifier



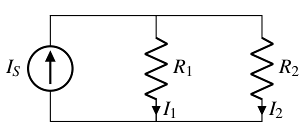
$$v_{\text{out}} = v_{\text{in}}\left(-\frac{R_f}{R_s}\right)$$

Noninverting Amplifier



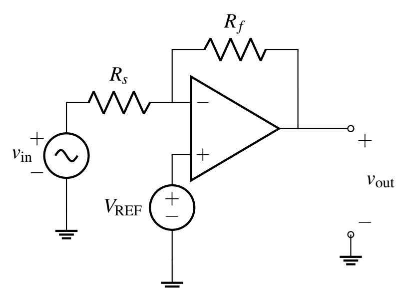
$$v_{\text{out}} = v_{\text{in}}\left(1+\frac{R_{\text{top}}}{R_{\text{bottom}}}\right)$$

Current Divider



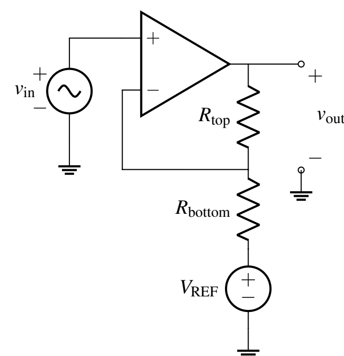
$$I_1 = I_S\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amplifier with Reference



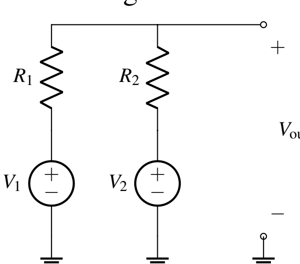
$$v_{\text{out}} = v_{\text{in}}\left(-\frac{R_f}{R_s}\right) + V_{\text{REF}}\left(\frac{R_f}{R_s}+1\right)$$

Noninverting Amplifier with Reference



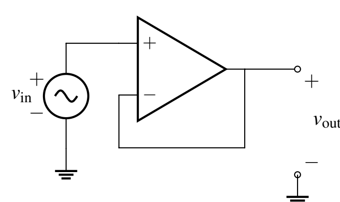
$$v_{\text{out}} = v_{\text{in}}\left(1+\frac{R_{\text{top}}}{R_{\text{bottom}}}\right) - V_{\text{REF}}\left(\frac{R_{\text{top}}}{R_{\text{bottom}}}\right)$$

Voltage Summer



$$V_{\text{out}} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

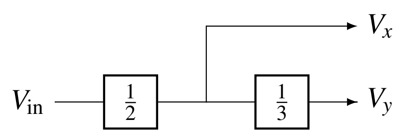
Unity Gain Buffer



$$v_{\text{out}} = v_{\text{in}}$$

1. Modular Circuit Buffer

Let's try designing circuits that perform a set of mathematical operations using op-amps. While voltage dividers on their own cannot be combined without altering their behavior, op-amps can preserve their behavior when combined and thus are a perfect tool for modular circuit design. We would like to implement the block diagram shown below:



In other words, create a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{\text{in}}$ and $V_y = \frac{1}{3}V_x = \frac{1}{6}V_{\text{in}}$.

(a) Draw two voltage dividers, one for each operation (the $1/2$ and $1/3$ scalings). What relationships hold for the resistor values for the $1/2$ divider, and for the resistor values for the $1/3$ divider?

Answer: Recall our voltage divider consists of V_{in} connected to two resistors (R_1, R_2) in series with the output voltage between ground and the central node. This yields the formula

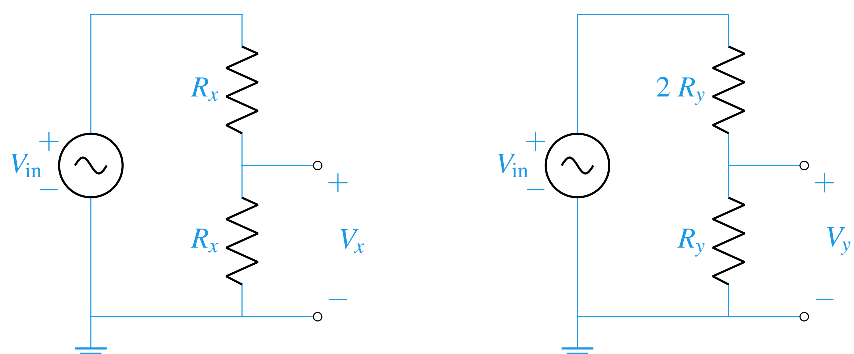
$$V_{\text{out}} = \left(\frac{R_2}{R_1 + R_2}\right) V_{\text{in}}.$$

For the $1/2$ operation (V_x output) we recognize

$$\frac{1}{2} = \left(\frac{R_2}{R_1 + R_2}\right) \quad \longrightarrow \quad R_1 + R_2 = 2R_2 \quad \longrightarrow \quad R_1 = R_2 \equiv R_x.$$

For the $1/3$ operation (V_y output) we recognize

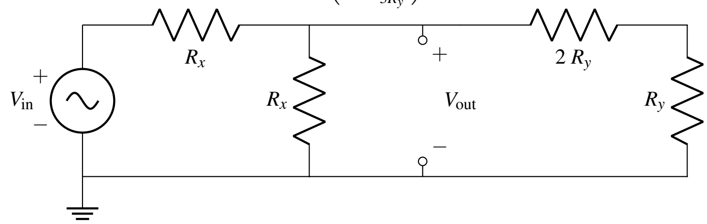
$$\frac{1}{3} = \left(\frac{R_2}{R_1 + R_2}\right) \quad \longrightarrow \quad R_1 + R_2 = 3R_2 \quad \longrightarrow \quad \frac{R_1}{2} = R_2 \equiv R_y.$$



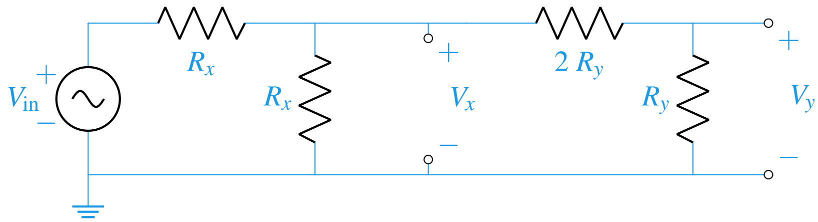
(b) If you combine the voltage dividers, made in part (a), as shown by the block diagram (output of the $1/2$ voltage divider becomes the source for the $1/3$ voltage divider circuit), do they behave as we hope (meaning $6V_{\text{in}} = 3V_x = V_y$)?

HINT: The following circuit and formula may be handy:

$$V_{out} = \left(\frac{1}{2+\frac{R_x}{3R_y}}\right)V_{in}$$



Answer: Combining the voltage divider circuits yield



To quickly access this combined system, we may identify V_x as the result of a new equivalent voltage divider (recognizing the R_y resistors in series and that series is in parallel with R_x). The load resistor becomes $R_{eq} = \frac{3R_xR_y}{R_x+3R_y}$. This yields

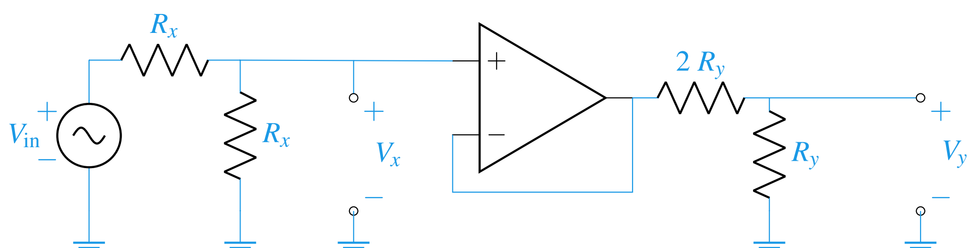
$$V_x = \left(\frac{R_{eq}}{R_x+R_{eq}}\right)V_{in} = \left(\frac{1}{2+\frac{R_x}{3R_y}}\right)V_{in} \qquad V_y = \frac{1}{3}V_x = \left(\frac{1}{6+\frac{R_x}{R_y}}\right)V_{in}$$

From this stage it is evident that combining our dividers changes their behavior (although they preserve behavior in the limit $R_y >> R_x$).
The new values for V_x, V_y are dependent on values from both dividers, which means they can't be treated independently! □.

(c) Perhaps we could use an op-amp (in negative-feedback) to achieve our desired behavior.
Modify the implementation you tried in part (b) using a negative feedback op-amp in order to achieve the desired V_x, V_y relations $V_x = (1/2)V_{in}$ and $V_y = (1/3)V_x = (1/6)V_{in}$.

HINT: Place the op-amp in between the dividers such that the V_x node is an input into the op-amp, while the source of the 2nd divider is the output of the op-amp!

Answer: Use the op-amp as a voltage buffer.
This means we short the op-amp's negative input to its output, since the positive input must now match its output (by the golden rules).



Since no current flow into the positive op-amp input, we've successfully isolated the dividers so they can be used in a modular fashion! □

NOTE: The V_x, V_y outputs from this configuration would change with the addition of a load on either terminal. As a follow-up, think about ways to make each output agnostic to the loads attached!

2. Modular Op-Amp Circuits

Let's expand our toolbox of op-amp circuits that perform mathematical operations by designing blocks that implement the following operations

(a) Scale the input voltage so that: $V_{\text{out}} = +5\ V_{\text{in}}$

(b) Scale and invert the input voltage so that: $V_{\text{out}} = -2\ V_{\text{in}}$

(c) Sum two input voltages together so that: $V_{\text{out}} = V_{\text{in}_1} + V_{\text{in}_2}$

Use the reference above for help!
Would connecting any of these blocks together modify their intended functionality?

Answer:

(a) Since our scaling of $+5$ is positive, we employ a non-inverting amplifier without a V_{REF} supply:

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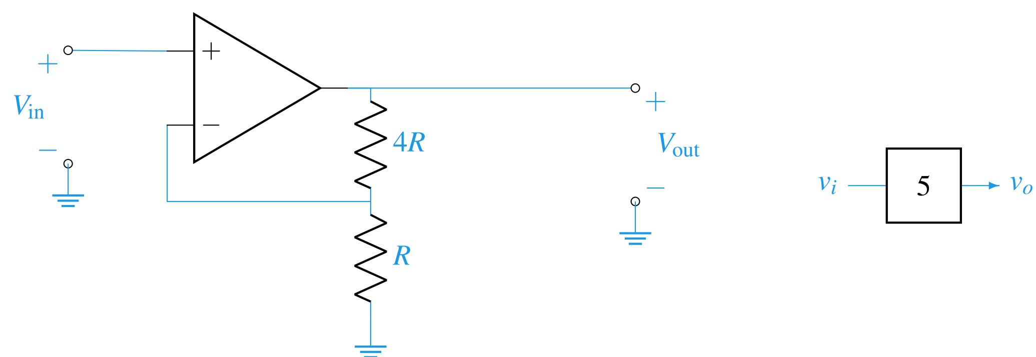
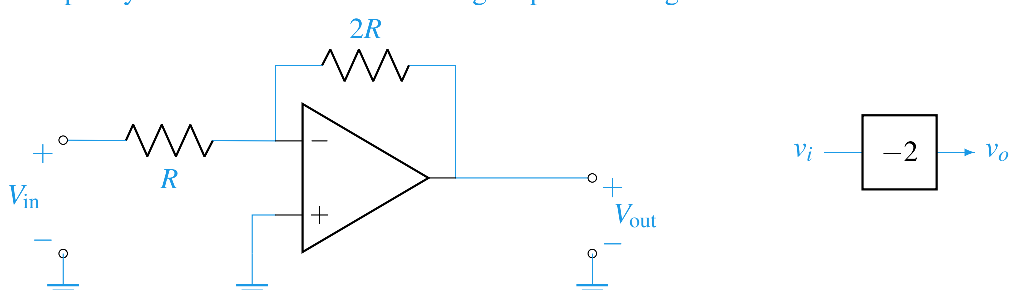
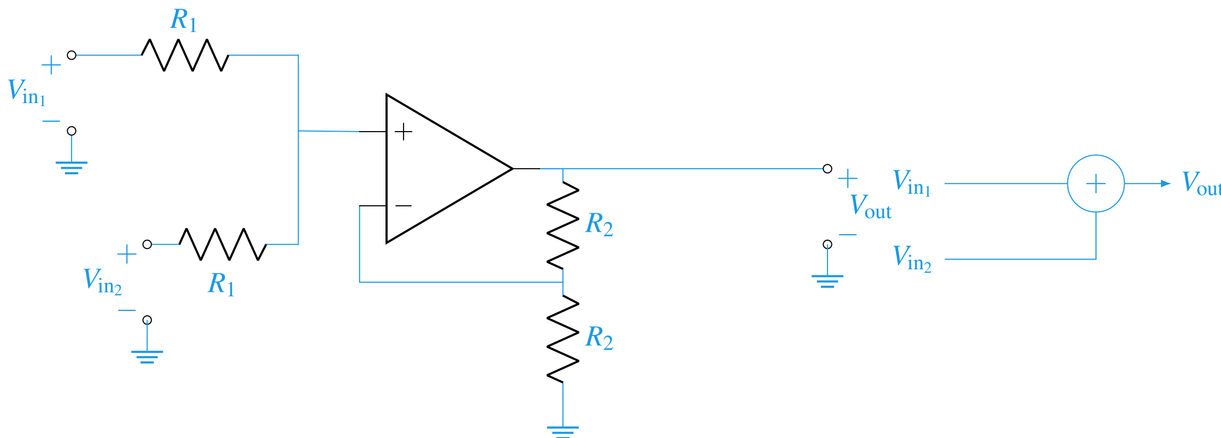


(b) To scale the input by -2 we must use the inverting amplifier configuration:



(c) This one is tricky!
The voltage summing circuit (in the reference) can get us close, but the resulting voltage still must be scaled by $+2$ using a non-inverting amplifier. Further, we need an op-amp in the circuit anyway to serve as a buffer! The best mindset for this problem is to think:
can we use a voltage divider involving the two input voltages as an input for a non-inverting amplifier?

Conceptually we can imagine a $1/2$ voltage divider where the usual ground node is replaced by a source, say V_{in_2}. Then our divider equation becomes $V_{\text{out}} = \frac{1}{2}(V_{\text{in}_1} + V_{\text{in}_2})$. Next we can feed that input into a non-inverting amplifier circuit:



We check if our circuit modules can be combined together by examining how the circuit output voltages change with an attached load resistor. Since every circuit has at least 1 fixed input (a node that is isolated from V_{out} entirely), and the golden rules ($u_+ = u_-$) apply for negative-feedback circuits, the circuits' functionalities are unimpeded by the load.