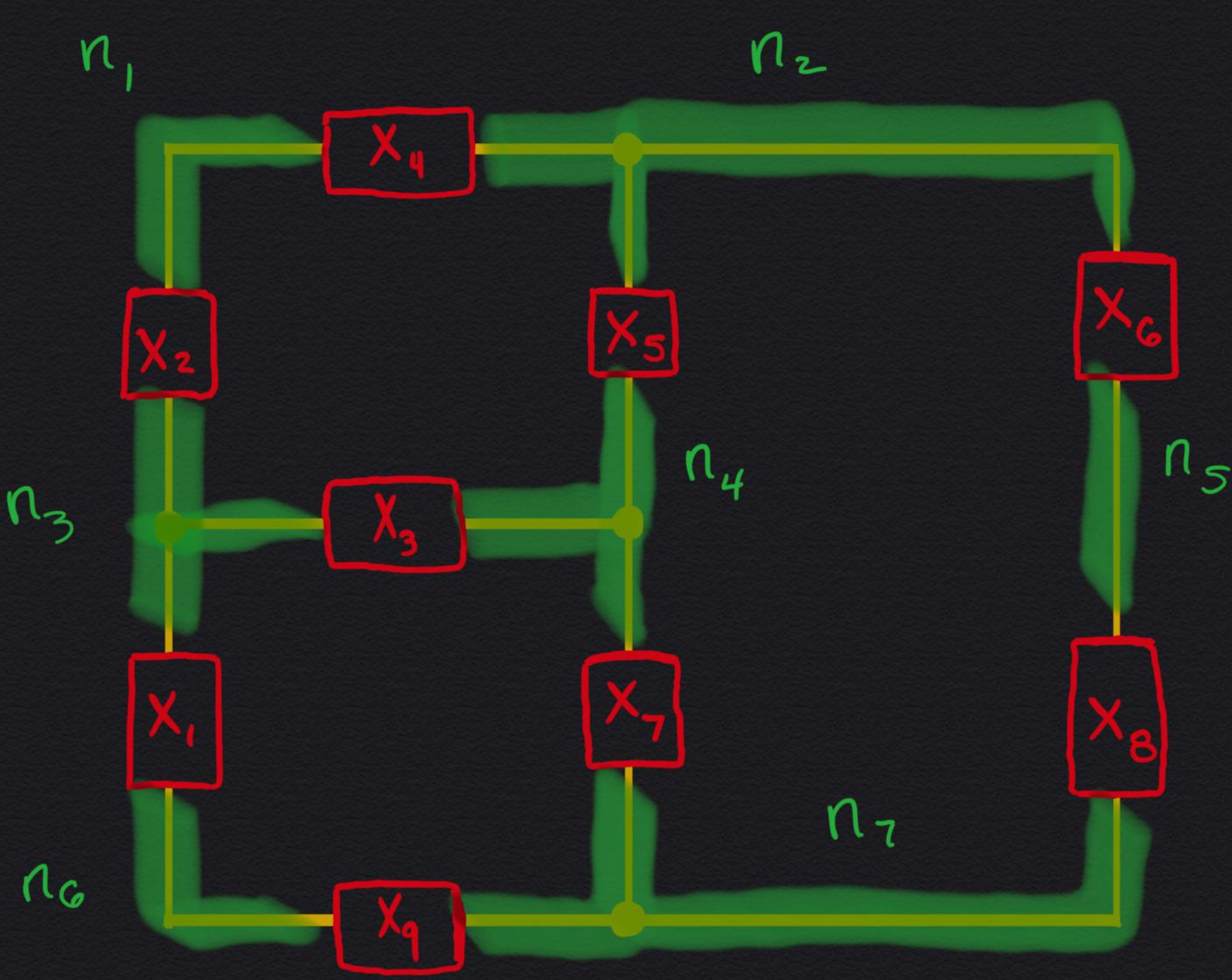
(1) In the circuit below, label & count all nodes and branches:

↳ branches: Nontrivial Circuit Elements

↳ nodes: Regions of "equi-potential"

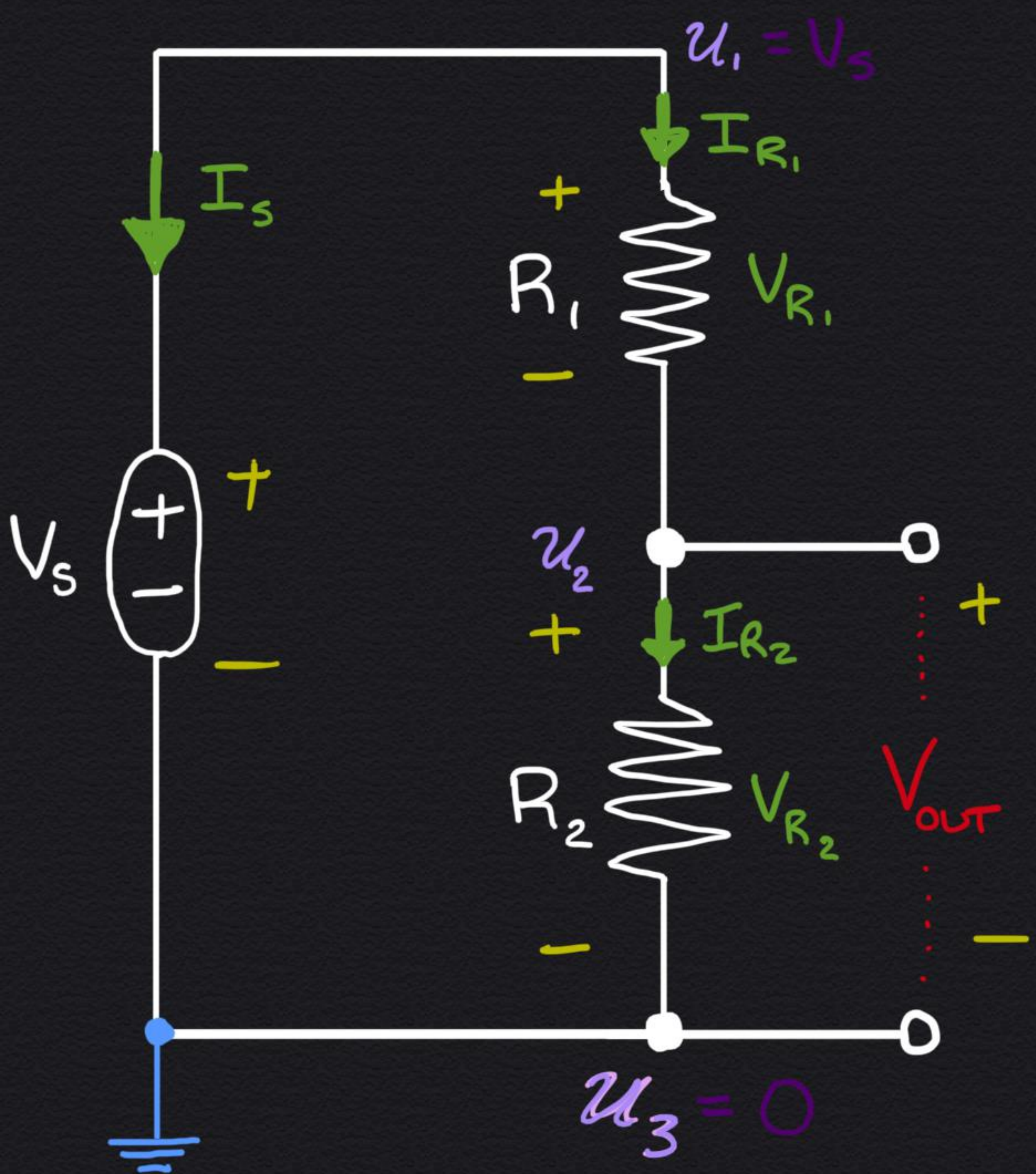
(Basically all connected wiring which is all at the same voltage.)



There are 7 nodes.

There are 9 branches.

② Voltage Divider! (Re-run)



Step 0: Knowns (except V_{OUT})

Step 1: Set ground

Step 2/3: Label node voltages

Step 4: Label element voltages and currents, including passive sign convention.

Note: Only the things in red and white on the circuit are known/mandatory/unambigous. The rest is the engineer's choice, but make sure YOUR CHOICES ARE CONSISTENT!

5] Write the KCL equations:

$$-I_{R_1} - I_S = 0 \rightarrow I_{R_1} = -I_S$$

$$+I_{R_1} - I_{R_2} = 0 \rightarrow I_{R_1} = I_{R_2}$$

Sum of the currents in/out of each junction, + sign for 'in', and − sign for out.

KCL says this sum must be zero at all junctions

↑ Note this another independent convention choice, since multiplying both sides of each equation would change nothing!!

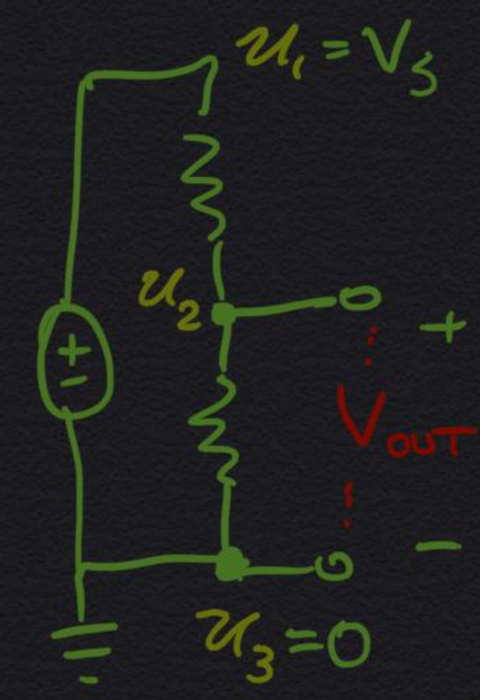
6] Write currents in terms of element voltages & characteristics, using Ohm's law $V = IR$.

$$I_{R_1} \cdot R_1 = V_{R_1}$$

$$I_{R_2} \cdot R_2 = V_{R_2}$$

7] Substitute in for known voltages

$$\underset{V_S}{u_1},\ \underset{V_{OUT}}{u_2},\ \underset{0}{u_3}$$



$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1} = \frac{V_S - V_{OUT}}{R_1}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_2 - u_3}{R_2} = \frac{V_{OUT}}{R_2}$$

$$V_{OUT} = u_2 - u_3, \quad u_3 = 0$$

(Everything here in this color of green is technically step 9.)

8] Plug into KCL equations

$$I_{R_1} = \frac{u_1 - u_2}{R_1} = -I_S$$

$$I_{R_1} = I_{R_2}$$

$$\frac{u_1 - u_2}{R_1} = \frac{u_2 - u_3}{R_2}$$

9] Solve for our node voltages u_j, $j=1,2,3$ from known circuit variables:

$u_1 - u_2$ → $V_S - V_{OUT}$; $u_2 - u_3$ → $V_{OUT} - 0$

$$\frac{V_S - V_{OUT}}{R_1} = \frac{V_{OUT} - 0}{R_2}$$

$$\frac{V_S}{R_1} = V_{OUT}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V_S = V_{OUT}\left(1 + \frac{R_1}{R_2}\right)$$

multiply by $1 = \frac{R_2}{R_2}$

$$V_{OUT} = V_S\left(\frac{1}{1 + R_1/R_2} \cdot \frac{R_2}{R_2}\right)$$

$$\boxed{V_{OUT} = V_S\left(\frac{R_2}{R_1 + R_2}\right)}$$

Notes:

1. If $R_2 = 0$, then $V_{OUT} = 0$
2. If $R_1 = 0$, then $V_{OUT} = V_S$
3. If $R_1 = R_2 = 0$, then you've blown up the battery! Lulz, actully most batteries have a tiny resistance inside, so you'd probably notice the battery get really hot, and die.
   - Some fancy power supplies will be designed to have a device inside that sets a limit to the current through it.
4. If $R_1 < 0$ or $R_2 < 0$, then you've broken the laws of physics...