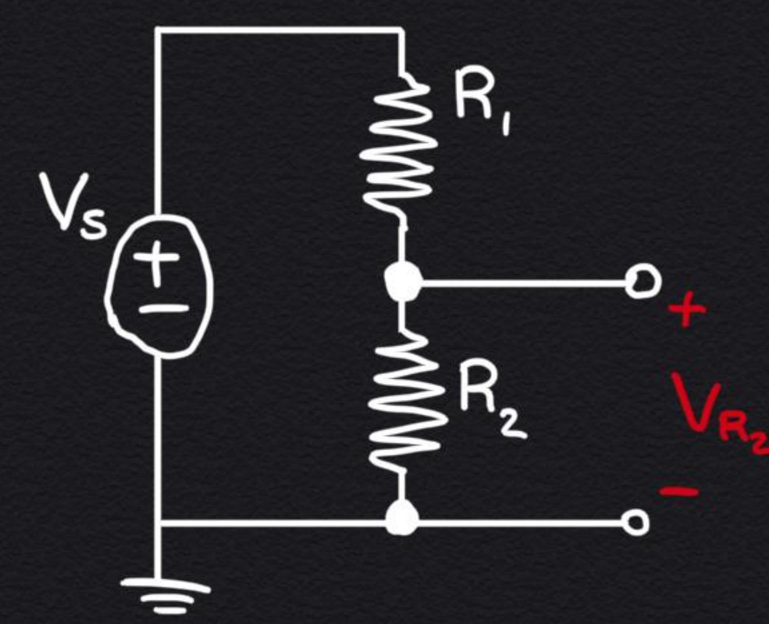
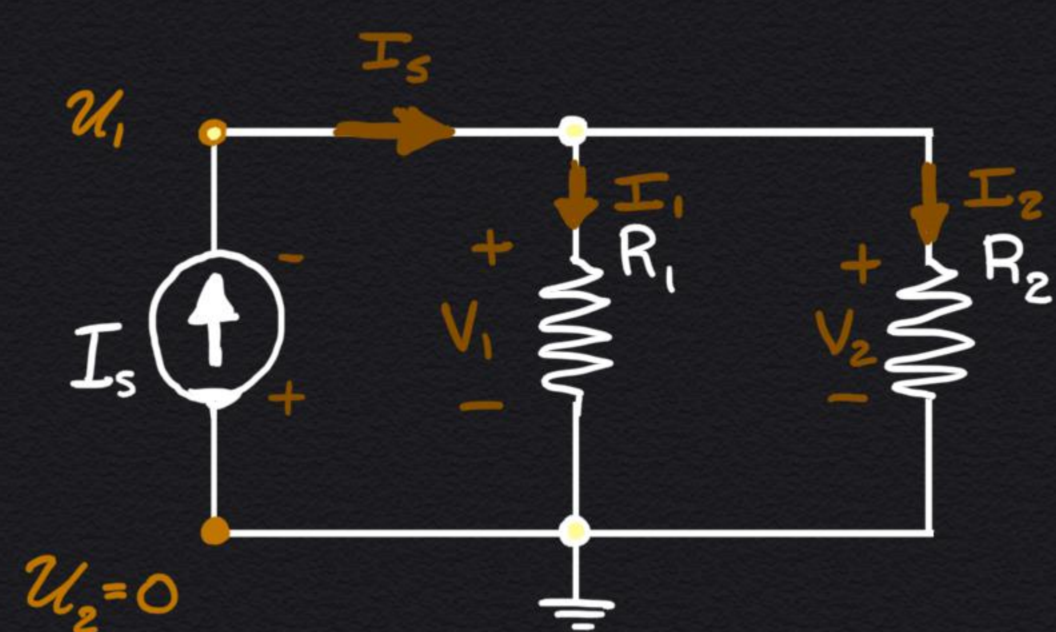
1 Current Divider

Previously we played with the voltage divider (shown right) and found the relationship $V_{R_2} = \left(\frac{R_2}{R_1+R_2}\right) V_s$.



Now we are going to make a similar derivation for a current divider. For the diagram below, find the current I_2 through R_2 as a function of I_s:



Step 1: Set ground ✓

Step 2/3: Label nodes

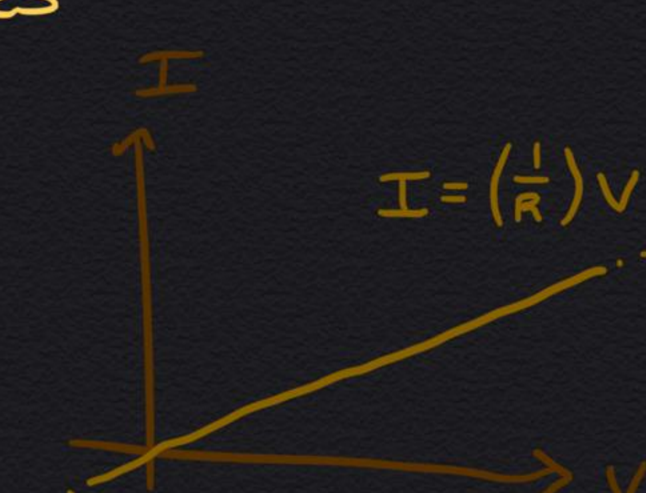
Step 4: Label currents and element voltages

Step 5: Using KCL

$$+I_s - I_1 - I_2 = 0$$

Step 6: Substitute in element voltages (using Ohm's law)

$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_2}{R_2}$$



Step 7: Plug in node voltages (might use KVL):

$u_2 = 0$

$V_1 = u_1 - u_2$

$V_2 = u_1 - u_2$

$$I_1 = \frac{u_1 - u_2}{R_1} = \frac{u_1}{R_1}$$

KVL: $+V_1 - V_2 = 0$ ✓

$$I_2 = \frac{u_1 - u_2}{R_2} = \frac{u_1}{R_2}$$

Step 8/9: Plug step 7 into KCL.
Solve for any unknown nodes, and currents.

$$I_s - \frac{u_1}{R_1} - \frac{u_1}{R_2} = 0 \longrightarrow I_s = u_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= u_1\left(\frac{R_2 + R_1}{R_1 R_2}\right)$$

$$\boxed{u_1 = I_s\left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

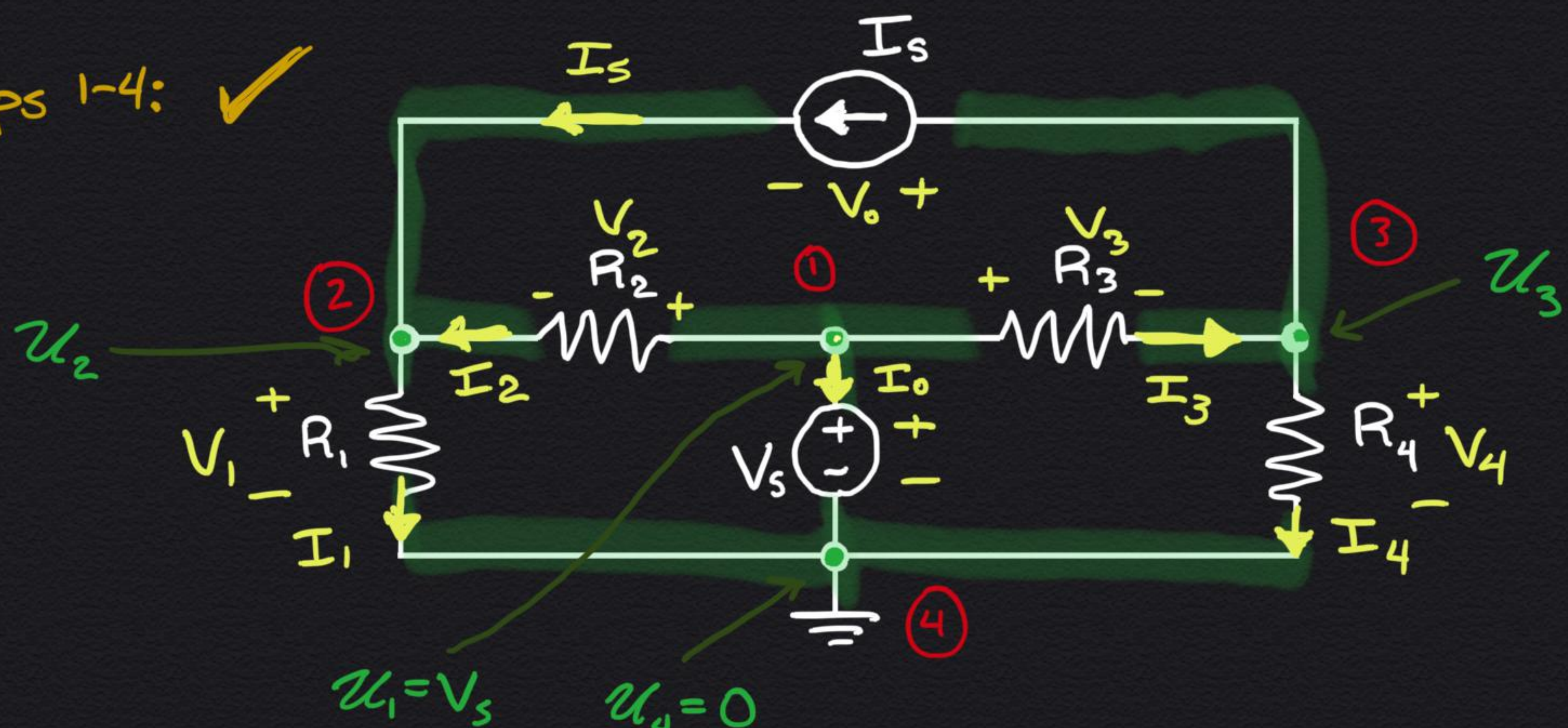
$$I_2 = \frac{u_1}{R_2} \longrightarrow \boxed{I_2 = I_s\left(\frac{R_1}{R_1 + R_2}\right)}$$

$$I_1 = \frac{u_1}{R_1} \longrightarrow I_1 = I_s\left(\frac{R_2}{R_1 + R_2}\right)$$

② Circuit Practice

Given the circuit below, solve for (a) all node voltages and (b) find I_3 through R_2 (from V_s):

Steps 1-4: ✓



Step 5: KCL

① $-I_0 - I_2 - I_3 = 0$

② $I_2 + I_s - I_1 = 0$

③ $I_3 - I_s - I_4 = 0$

④ $I_1 + I_4 + I_0 = 0$

Step 6/7: Voltage Plug-in

$$I_1 = \frac{V_1}{R_1} = \frac{\mathcal{U}_2 - \mathcal{U}_4}{R_1} = \frac{\mathcal{U}_2}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{\mathcal{U}_1 - \mathcal{U}_2}{R_2} = \frac{V_s - \mathcal{U}_2}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{\mathcal{U}_1 - \mathcal{U}_3}{R_3} = \frac{V_s - \mathcal{U}_3}{R_3}$$

$$I_4 = \frac{V_4}{R_4} = \frac{\mathcal{U}_3 - \mathcal{U}_4}{R_4} = \frac{\mathcal{U}_3}{R_4}$$

Step 8/9: Solve by plugging into KCL

$$-I_0 - I_2 - I_3 = 0$$
$$I_2 + I_s - I_1 = 0$$
$$I_3 - I_s - I_4 = 0$$
$$I_1 + I_4 + I_0 = 0$$

$$\frac{\mathcal{U}_2}{R_1} + \frac{\mathcal{U}_3}{R_4} + I_0 = 0$$

$$\Downarrow$$

$$\frac{\mathcal{U}_2}{R_1} + \frac{\mathcal{U}_3}{R_4} - I_2 - I_3 = 0$$

$$\Downarrow$$

$$\frac{\mathcal{U}_2}{R_1} + \frac{\mathcal{U}_3}{R_4} - \frac{V_s - \mathcal{U}_2}{R_2} - \frac{V_s - \mathcal{U}_3}{R_3} = 0$$

$$\Downarrow$$

$$\left[\mathcal{U}_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + \mathcal{U}_3\left(\frac{1}{R_4}+\frac{1}{R_3}\right) - V_s\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = 0\right]$$

This was all good work, but it will be easier instead to use the other KCL expressions.

$$\frac{V_s - \mathcal{U}_2}{R_2} + I_s - \frac{\mathcal{U}_2}{R_1} = 0$$

$$\left(\frac{V_s}{R_2} + I_s\right) - \mathcal{U}_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = 0 \Longrightarrow \boxed{\mathcal{U}_2 = \left(\frac{V_s}{R_2} + I_s\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

$$\frac{V_s - U_3}{R_3} - I_s - \frac{U_3}{R_4} = 0$$

$$\left(\frac{V_s}{R_3} - I_s\right) - U_3\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = 0 \Rightarrow \boxed{U_3 = \left(\frac{V_s}{R_3} - I_s\right)\left(\frac{R_3 R_4}{R_3 + R_4}\right)}$$

- Plug in U_3 to get I_3, this can be tough!

$$I_3 = \frac{V_s - U_3}{R_3} = \frac{1}{R_3}\left[V_s - \left(\frac{V_s}{R_3} - I_s\right)\left(\frac{R_3 R_4}{R_3 + R_4}\right)\right]$$

$$= \frac{V_s}{R_3} - \frac{V_s}{R_3}\left(\frac{R_4}{R_3 + R_4}\right) + I_s\left(\frac{R_4}{R_3 + R_4}\right)$$

$$= \frac{V_s R_3 + \cancel{V_s R_4} - \cancel{V_s R_4} + I_s R_3 R_4}{R_3 (R_3 + R_4)}$$

$$\boxed{I_3 = \frac{V_s + I_s R_4}{R_3 + R_4}}$$