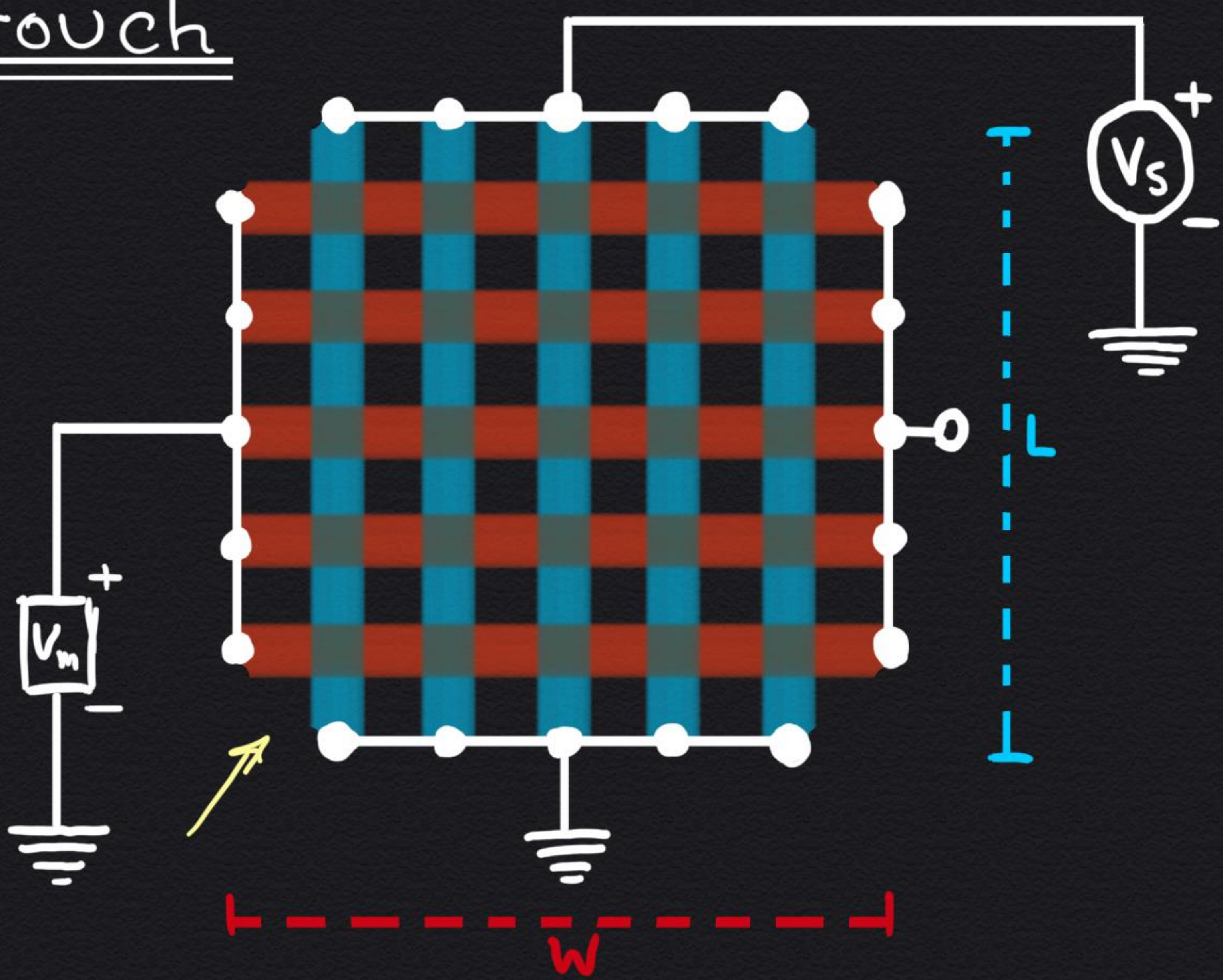
Discussion 8A notes

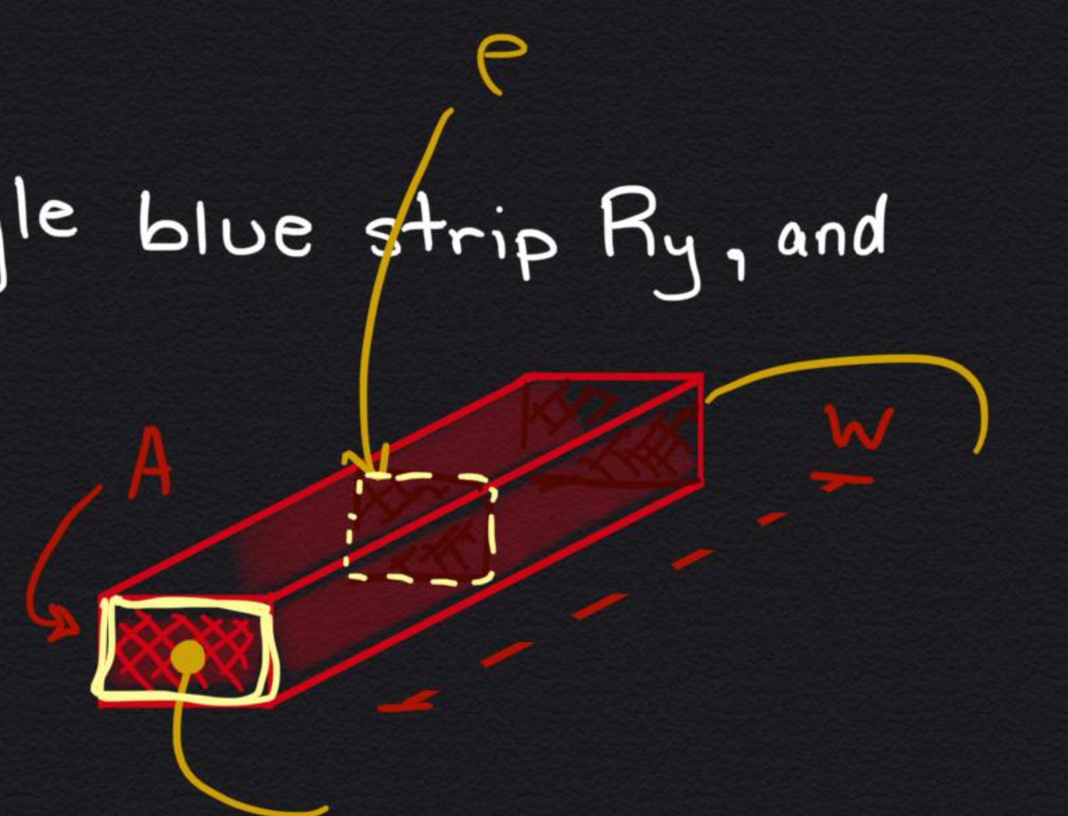
① Resist the touch

- Length L, width W.
- $N \times N$ stips (discretized)
- All strips have cross A, and resitivity ρ.
- Strips are split into $N+1$ segments.



(a) Find the resistance for a single blue strip R_y, and a red strip R_x.

$$R_x = \rho \frac{W}{A} \quad \Bigg\| \quad R_y = \rho \frac{L}{A}$$

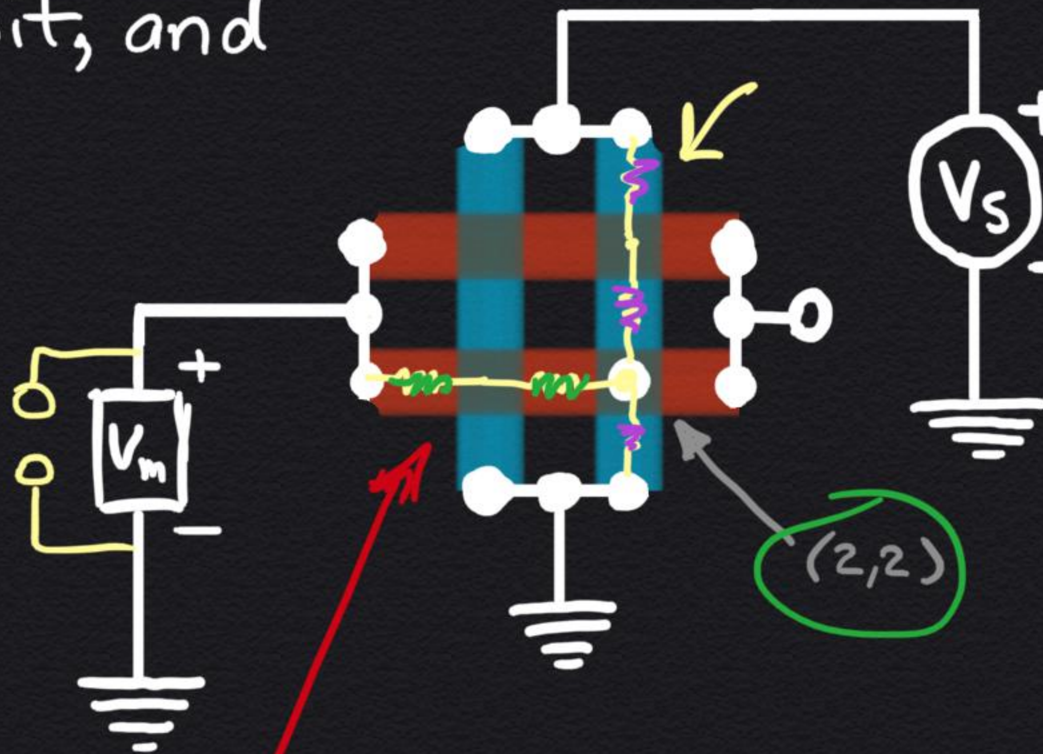


'A' ~ upping the cross-sectional area is like "opening more high-ways"

'L' ~ upping the length adds more resistor to go through.

(b) Now suppose our touchscreen has $N = 2$, and R_x, R_y are given so $V_s = 3V$, $R_x = 2\,k\Omega$, and $R_y = 2\,k\Omega$.

Draw an equivalent circuit, and find V_m:

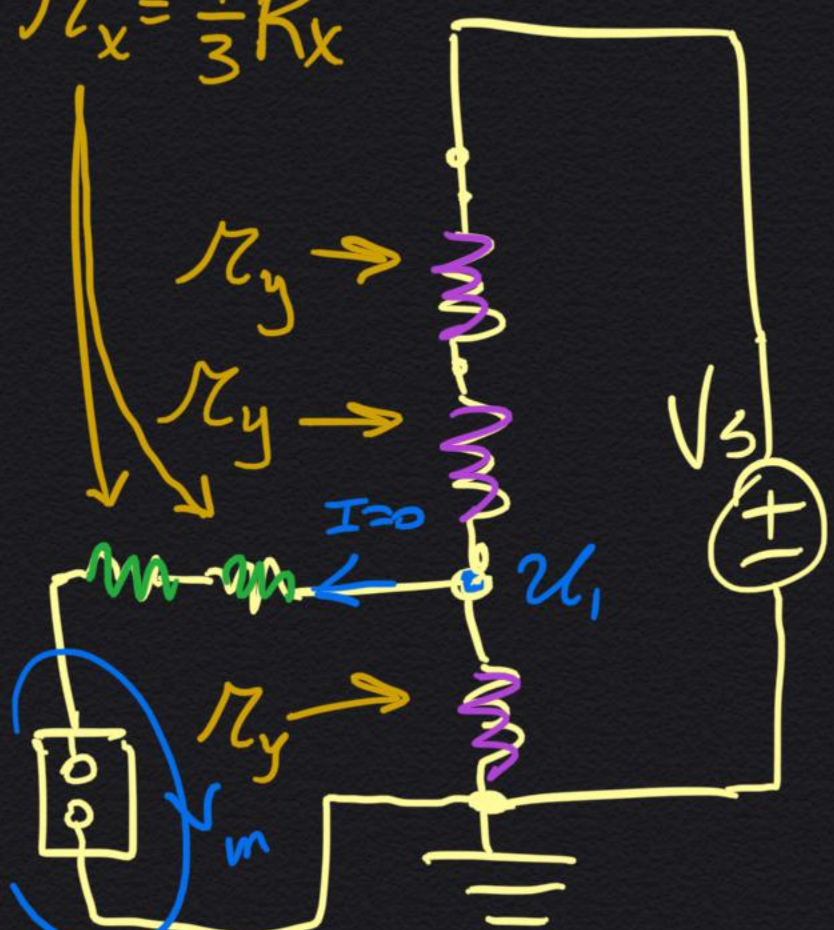


R_y $\qquad r_y = \frac{R_y}{2+1} = \frac{1}{3} R_y$

$r_x = \frac{1}{3} R_x$

R_x

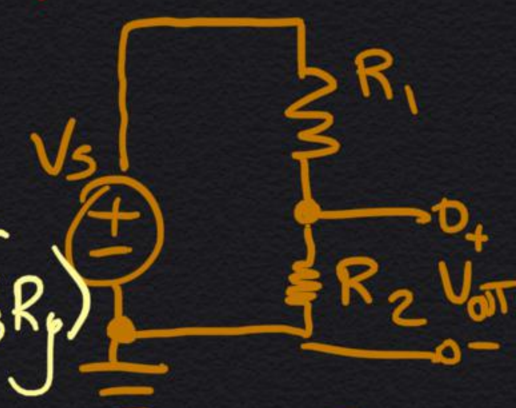
$r_x = \frac{R_x}{N+1} = \frac{1}{3} R_x$



$$u_1 = V_s \frac{r_y}{2r_y + r_y} = V_s \frac{\frac{1}{3} R_y}{(\frac{2}{3}R_y) + (\frac{1}{3}R_y)}$$

$$= \frac{1}{3} V_s$$

$$\boxed{V_m = u_1 = 1V}$$

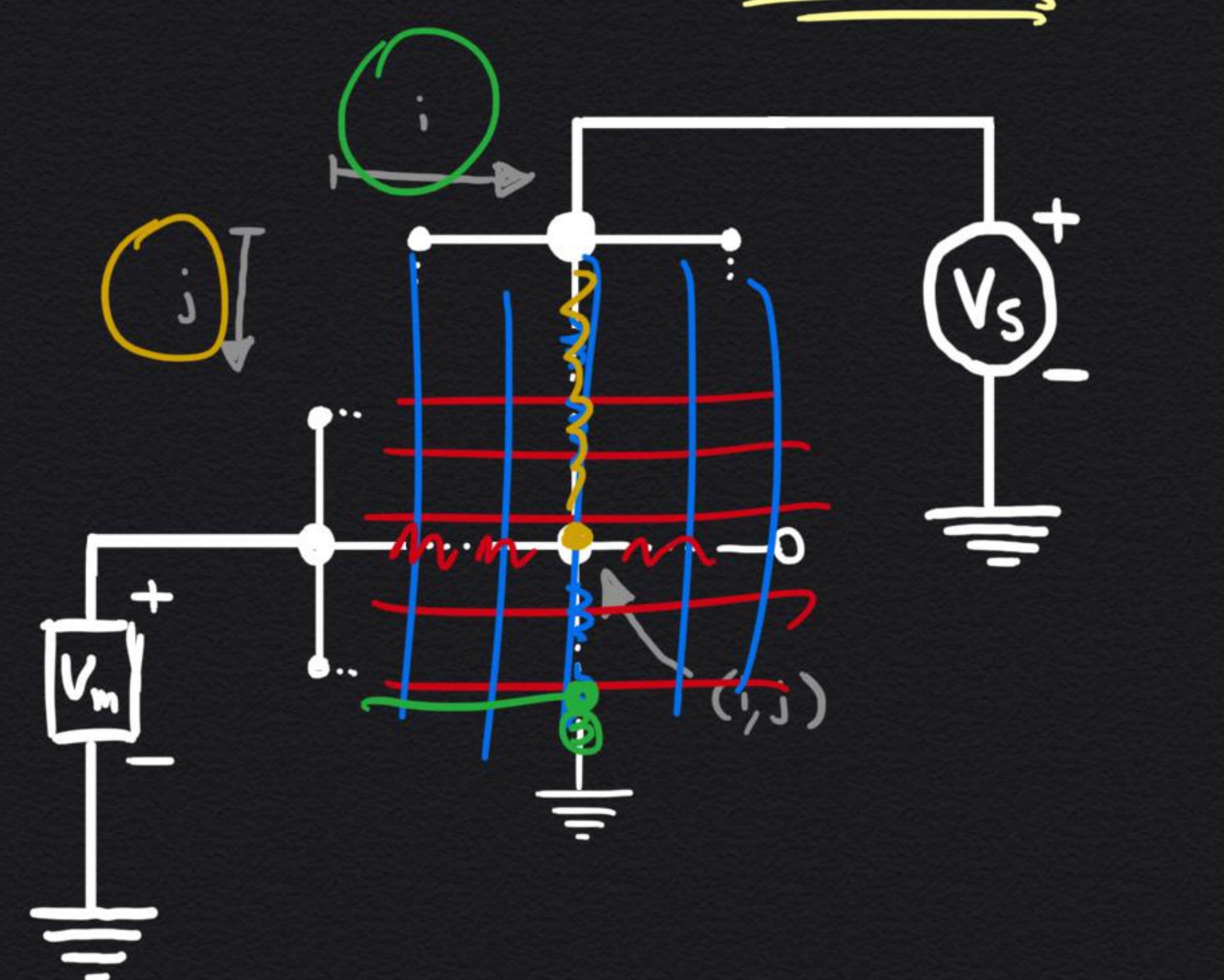


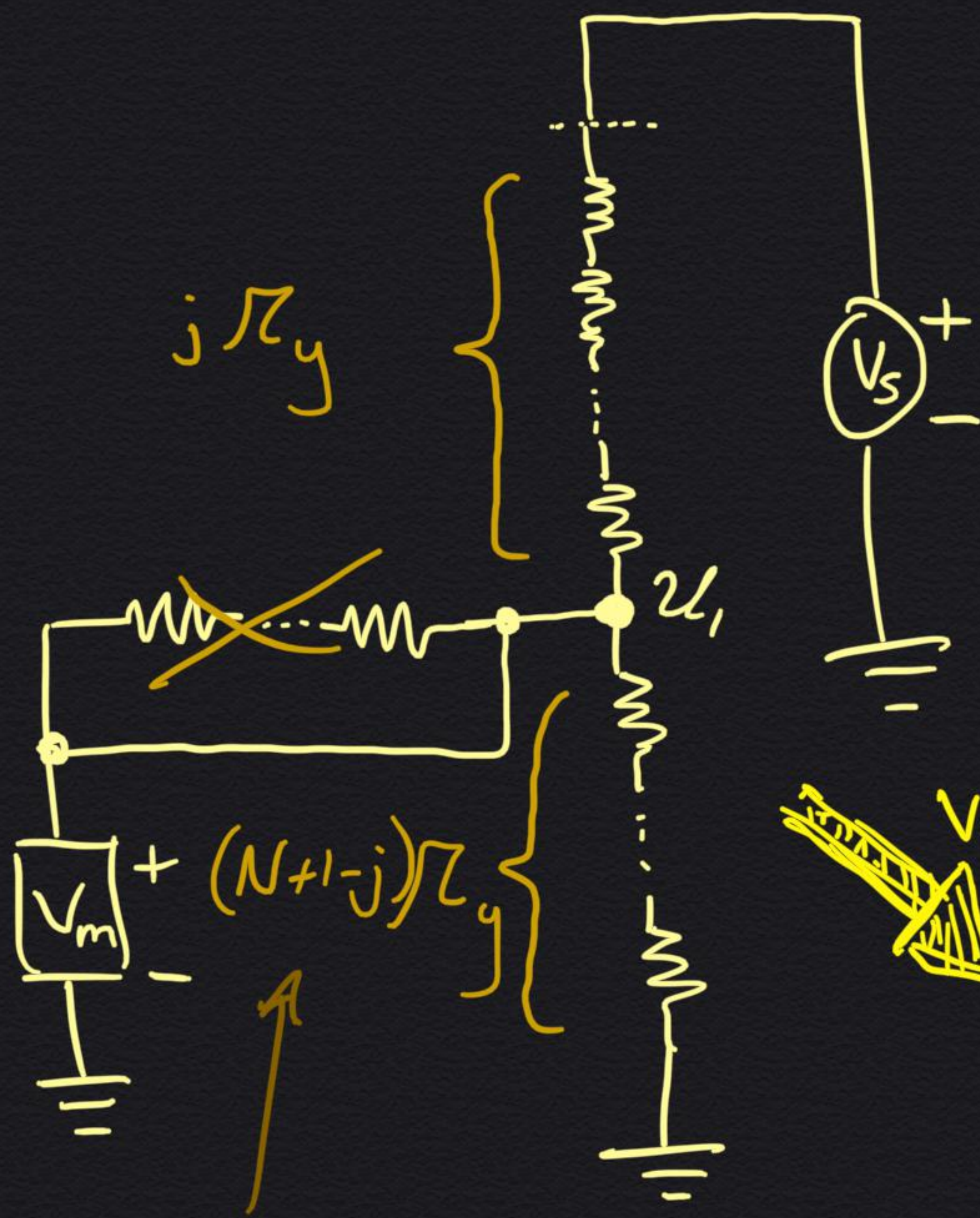
$V_{OUT} = V_s \left(\frac{R_2}{R_1 + R_2}\right)$

Voltmeter is like an open circuit, so no current and no voltage drop across green resistors.

(c) Let's generalize to the arbitrary $N \times N$ with node (i, j). Find V_m in terms of $V_s, N, i, j, \rho, L, W, A$.

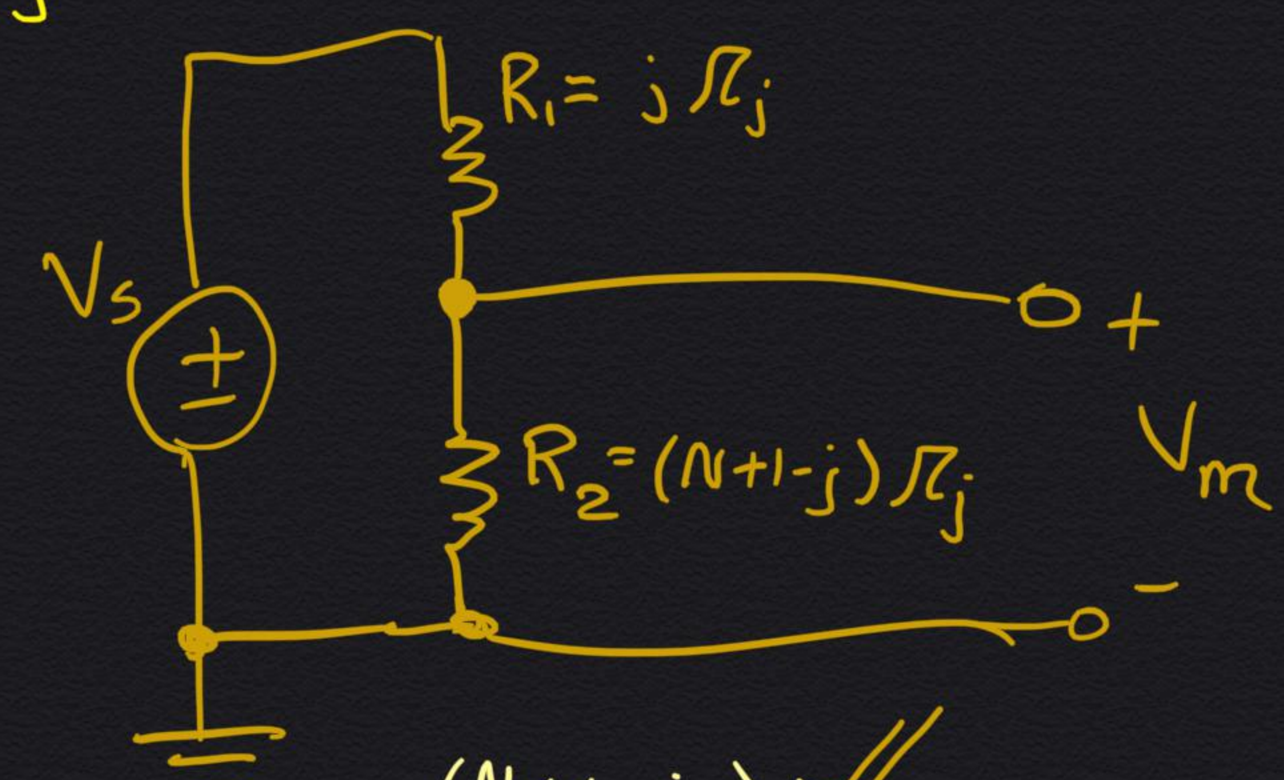
$$R_y = \rho \cdot \frac{L}{A} \quad \text{/\!/} \quad r_y = \frac{R_y}{N+1}$$





Since $j r_y + (N+1-j) r_y = (N+1) r_y$
$= R_y$ ✓

Voltage Divider!



$$V_m = V_s \cdot \left(\frac{R_2}{R_1 + R_2} \right) = V_s \cdot \frac{(N+1-j)\cancel{r_j}}{(N+1-\cancel{j}+\cancel{j})\cancel{r_j}}$$

$$= V_s \cdot \frac{N+1-j}{N+1} = V_s \left(\frac{N+1}{N+1} - \frac{j}{N+1} \right)$$

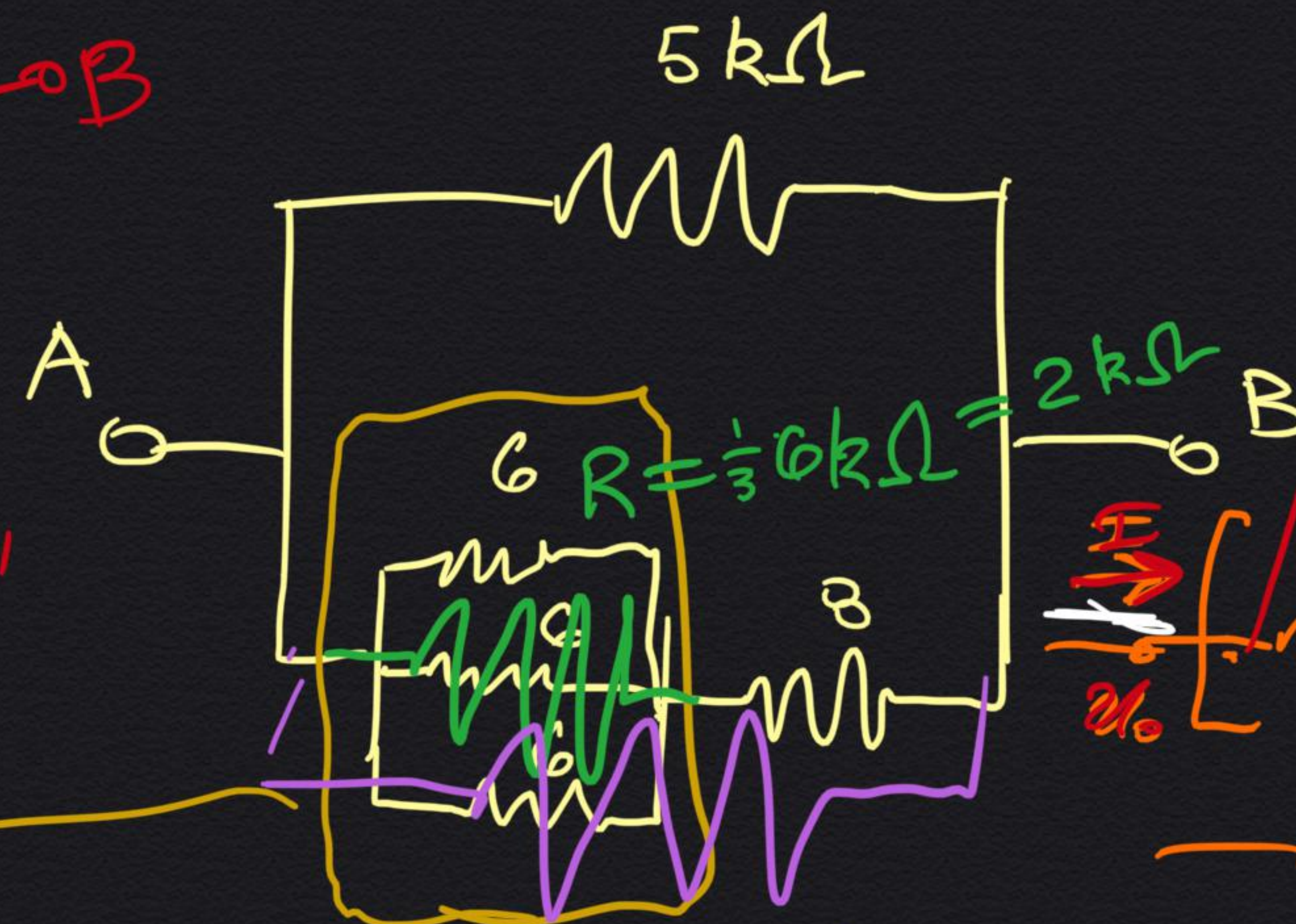
$$\boxed{V_m = V_s \left(1 - \frac{j}{N+1} \right)}$$

Yay! Check $j=1$ & $j=N$, should always be $0 < V_m < V_s$

$$R_{EQ} = \left(\frac{1}{5} + \frac{1}{10}\right)^{-1} \quad \left(\frac{2}{10}\right)$$

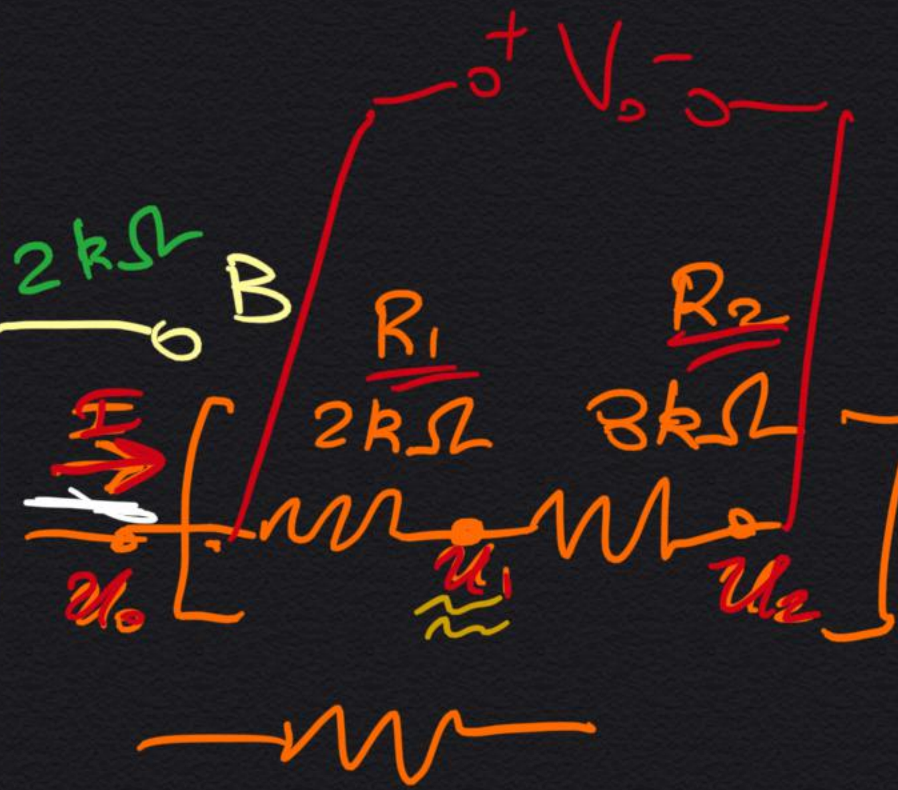
$$= \left(\frac{3}{10}\right)^{-1}$$

$$= \frac{10}{3}\ k\Omega$$



$R = \frac{1}{3} 6k\Omega = 2k\Omega$

$2 + 8 = 10k\Omega$

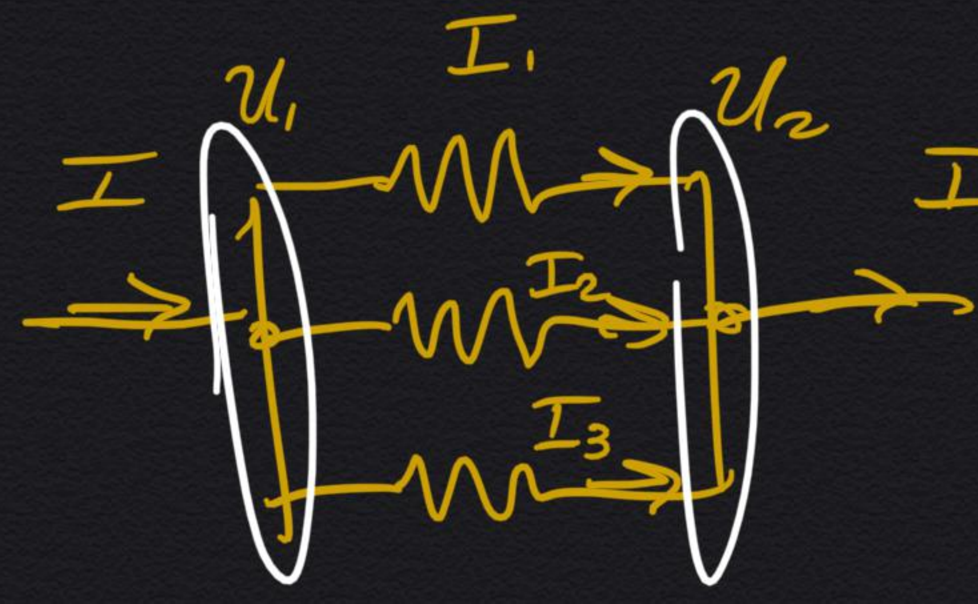


$$U_0 - U_1 = I R_1$$

$$U_1 - U_2 = I R_2$$

$$V_0 = U_0 - U_2$$

$$= I \underbrace{(R_1 + R_2)}_{R_{EQ}}$$



$$V_0 = u_1 - u_2 = I_1 R_1 = I_2 R_2 = I_3 R_3$$

$$I = I_1 + I_2 + I_3$$

$$= \left(\frac{V_0}{R_1} + \frac{V_0}{R_2} + \frac{V_0}{R_3} \right) = V_0 \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}_{1/R_{EQ}}$$

$$\frac{V_0}{R_{EQ}} = I = V_0 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$R_{EQ} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

when $R_1 = R_2 = R_3 \ldots$

$$= \left(\frac{3}{R_1} \right)^{-1} = \frac{R_1}{3} = \frac{1}{3} R_1$$