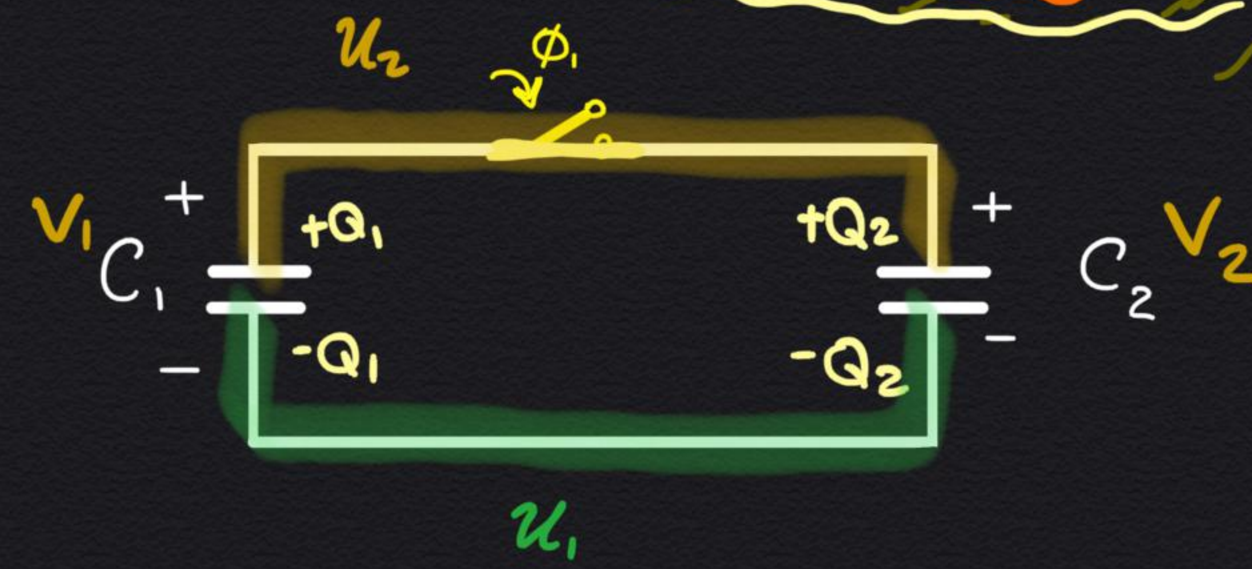
Discussion 9B Notes

① Charge conservation

Consider this circuit with $C_1 = C_2 = 1\,\mu F$.

Before ϕ_1 is closed, C_1 is charged with $+1V$ and C_2 with $+2V$.



a) What are the initial charges on C_1 & C_2?

$$\left[C = \frac{Q}{V} \right]$$

$$Q_1 = C_1 V_1 = (1\,\mu F)(1\,V) = 1\,\mu C \equiv 1\times10^{-6}\,C$$
$$Q_2 = C_2 V_2 = (1\,\mu F)(2\,V) = 2\,\mu C$$

✓

b) After closing ϕ_1, what are the charges and voltages across C_1 & C_2?

$$Q_{Tot} = Q_1^{(\phi 1)} + Q_2^{(\phi 1)} = \underline{3\,\mu C = Q_1 + Q_2}$$

$$V = \frac{Q}{C} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\left[\begin{array}{l} Q_1 = 3\,\mu C - Q_2 \\ Q_1 = \cancelto{1}{\left(\frac{C_1}{C_2}\right)} Q_2 \end{array} \right]$$

$$2Q_1 = 3\,\mu C$$

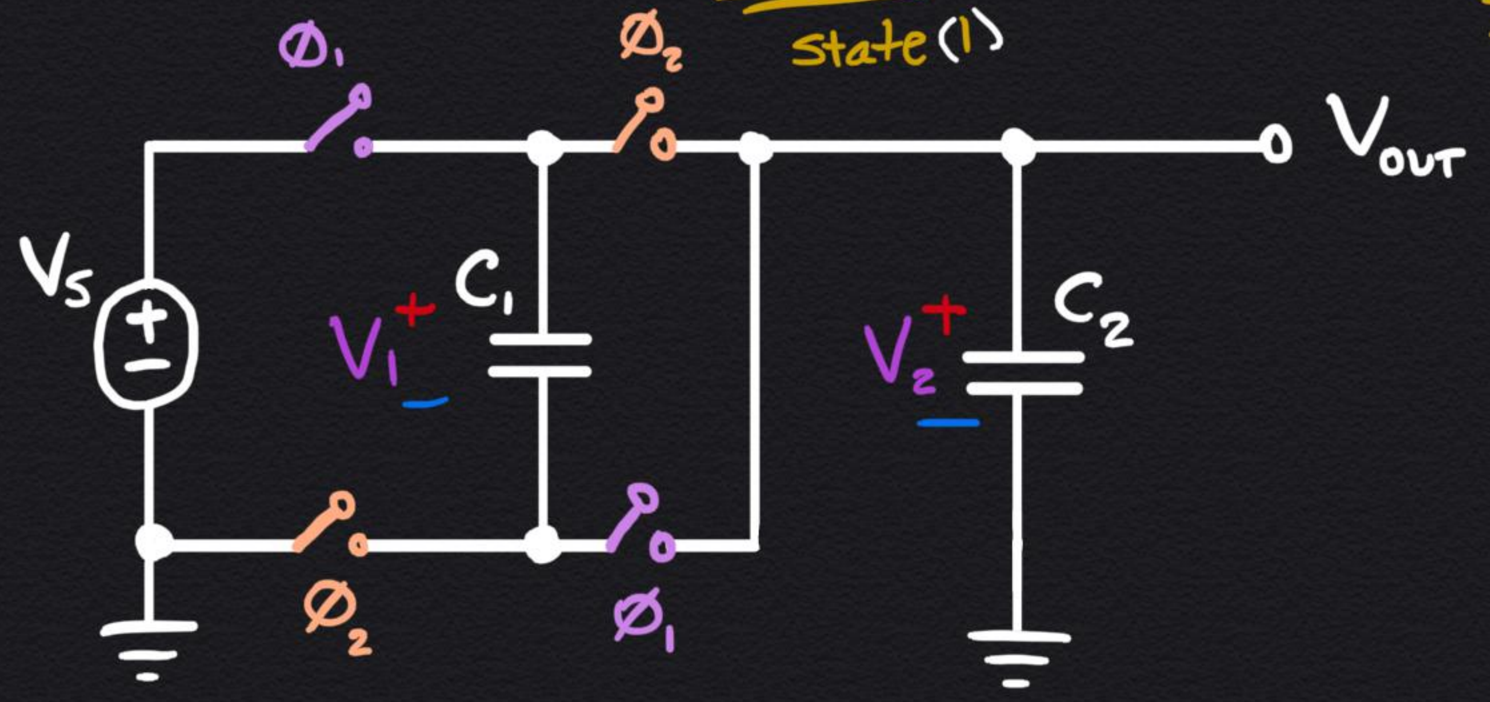
✓

$$Q_1 = Q_2 = 1.5\,\mu C$$

② Charge sharing

Consider the following circuit.

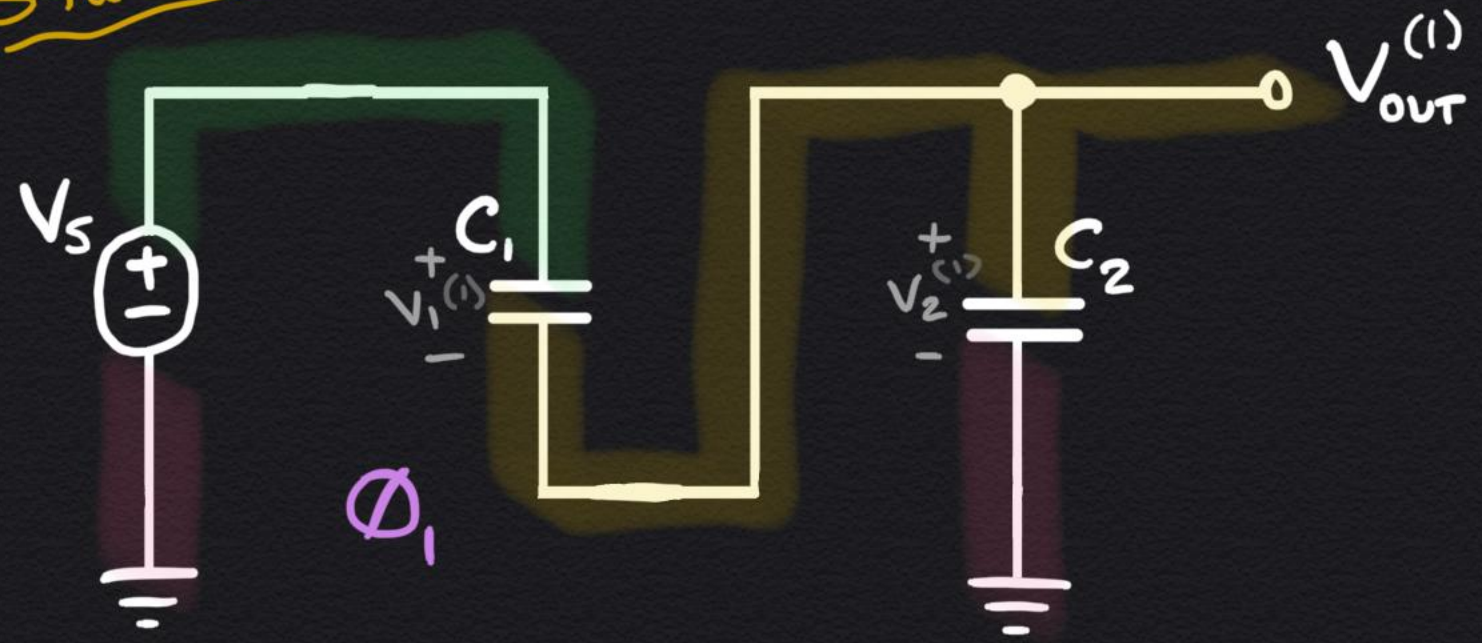
(We will go: Ø₁ open, Ø₂ open → Ø₁ closed, Ø₂ open (state (1)) → Ø₁ open, Ø₂ open → Ø₁ open, Ø₂ closed (state (2)))

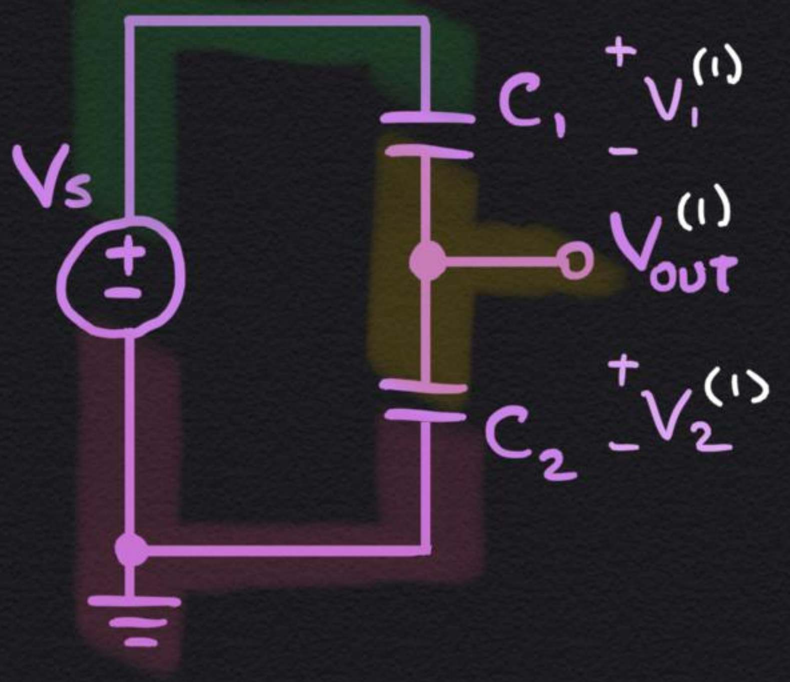


a) Draw voltage polarities on each capacitor:
(Make sure to stay consistent btwn phases!)

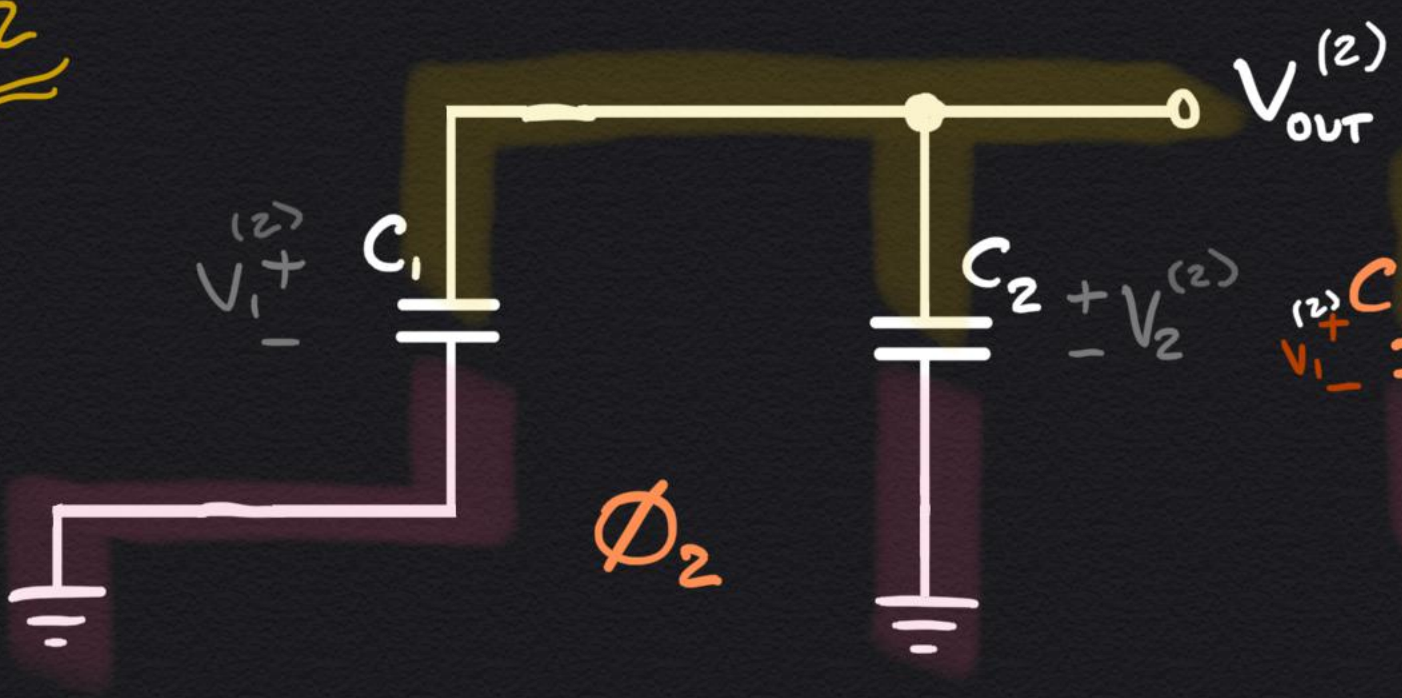
b) Draw the equivalent circuits for Ø₁ & Ø₂:

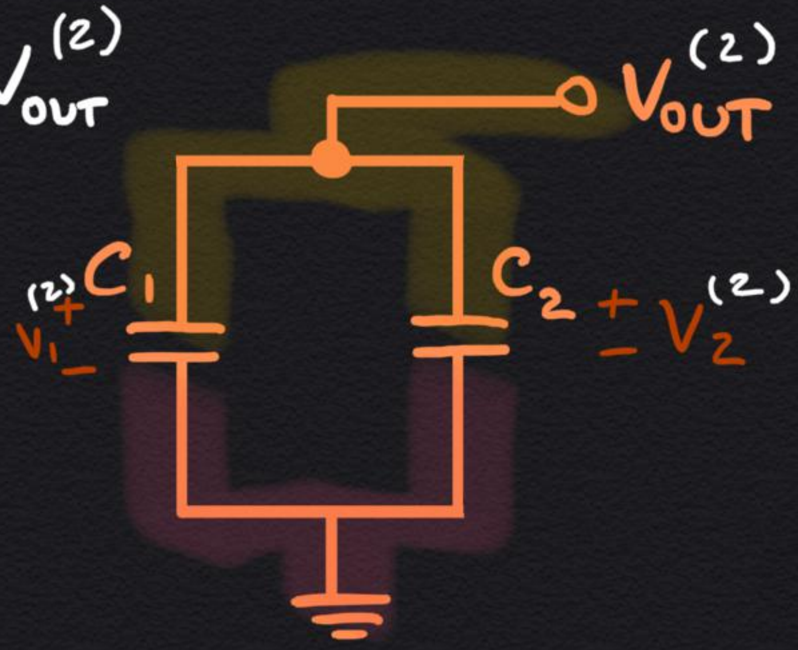
State 1





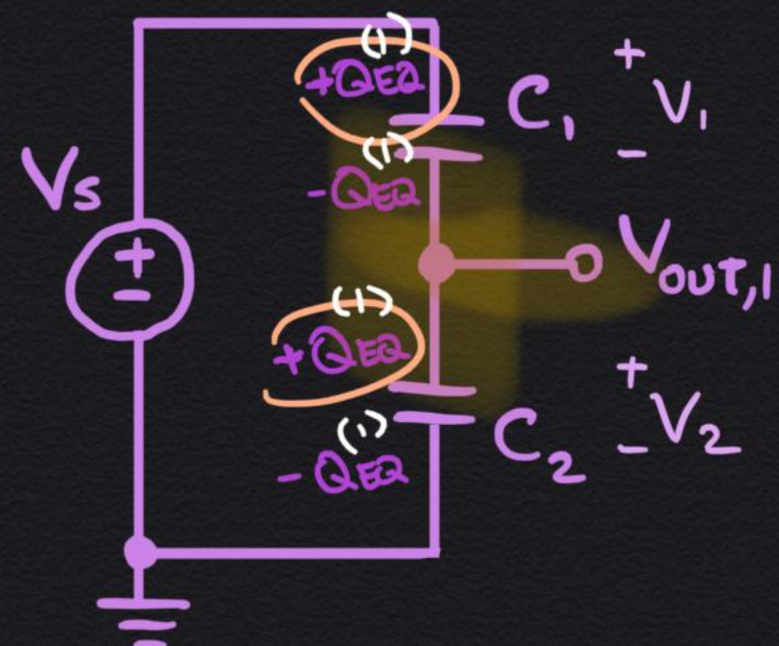
State 2

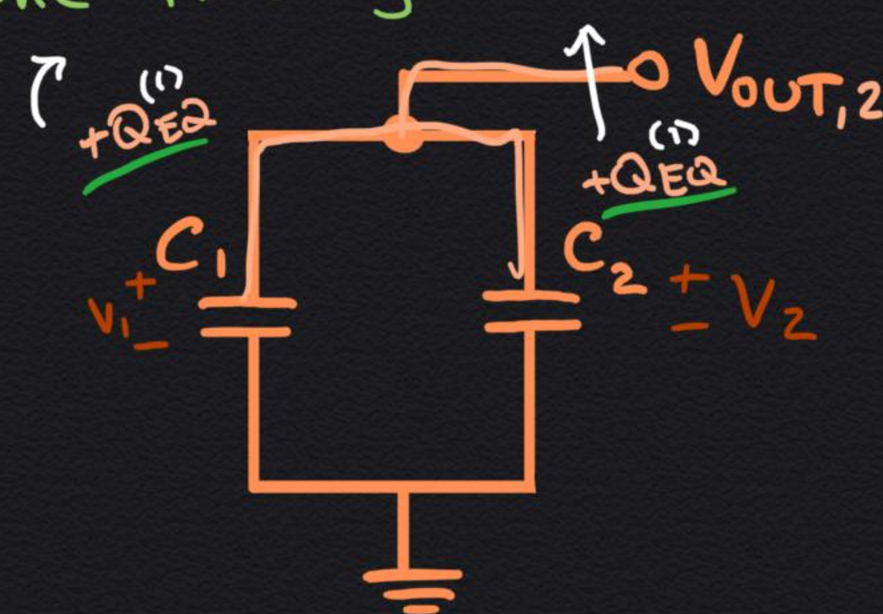




c] Find V_{OUT} in ϕ_2 as a function of V_s, C_1, and C_2:

The floating node has $Q_{EQ}^{(1)} + Q_{EQ}^{(2)}$ in total.





$$Q_{EQ}^{(2)} = 2Q_{EQ} = V_s \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{EQ}^{(1)} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{C_2}{C_1C_2} + \frac{C_1}{C_1C_2}\right)^{-1} = \left(\frac{C_1+C_2}{C_1C_2}\right)^{-1} = \frac{C_1C_2}{C_1+C_2}$$

$$Q_{EQ}^{(1)} = C_{EQ} V_s = \frac{C_1C_2}{C_1+C_2} V_s$$

$$V_{OUT}^{(2)} = \frac{Q_1^{(2)}}{C_1} = \frac{Q_2^{(2)}}{C_2}$$

$$C_{EQ}^{(2)} = C_1 + C_2$$

$$V_{OUT}^{(2)} = \frac{Q_{EQ}^{(2)}}{C_{EQ}^{(2)}} = \frac{2Q_{EQ}^{(1)}}{C_1+C_2}$$

$$\boxed{V_{OUT}^{(2)} = 2V_s \frac{C_1C_2}{(C_1+C_2)^2}}$$

✓

d] How are the charges distributed when $C_1 >> C_2$?

since $C_1 >> C_2$

$$Q_2^{(2)} = \left[\cancel{\frac{C_2}{C_1}}\right]^{\approx 0} Q_1^{(2)} \qquad Q_2^{(2)} \approx 0 \ ✓$$

All charge goes to the much bigger capacitor.

We can also solve this formally using the charge sum:

$$Q_1^{(2)} + Q_2^{(2)} = 2V_s\left(\frac{C_1C_2}{C_1+C_2}\right)$$

$$2V_s\left(\frac{C_1C_2}{C_1+C_2}\right) - Q_1^{(2)} \equiv Q_2^{(2)} \equiv \left(\frac{C_2}{C_1}\right)Q_1^{(1)}$$

$$2V_s\left(\frac{C_1C_2}{C_1+C_2}\right) = Q_1^{(1)}\left(\cancel{\frac{C_2}{C_1}+1}\right) \quad \frac{C_1+C_2}{C_1}$$

$$2V_s\left(\frac{C_1^2C_2}{(C_1+C_2)^2}\right) = Q_1^{(1)}$$

$$Q_1^{(2)} = 2V_sC_2\,\frac{1}{(1+C_2/C_1)^2} \approx 2V_sC_2$$

$$Q_2^{(2)} = 2V_sC_2\,\frac{C_2/C_1}{(1+C_2/C_1)^2} \approx 0 \ ✓$$