Discussion 11A Notes

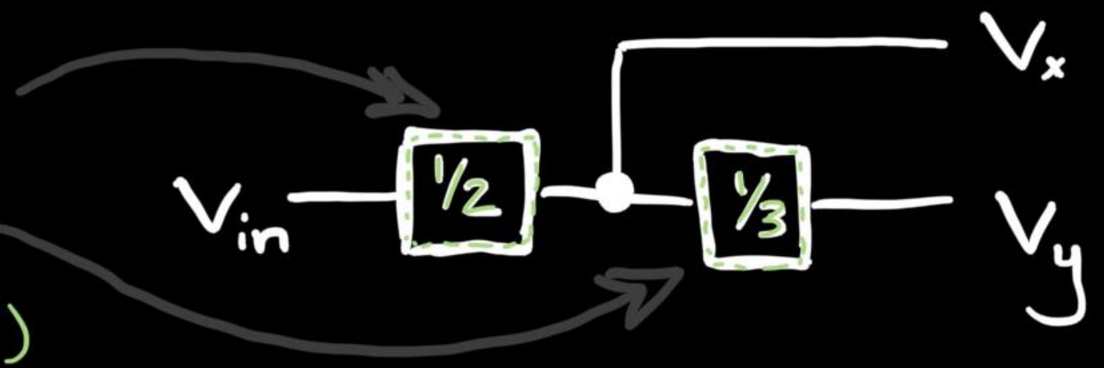
① Modular Circuit Buffer

"How to combine circuits"

Can we build a circuit that computes the following arithmetic? $V_x = \frac{1}{2} V_{in}$ $V_y = \frac{1}{3} V_x$

Circuits we'd like to implement

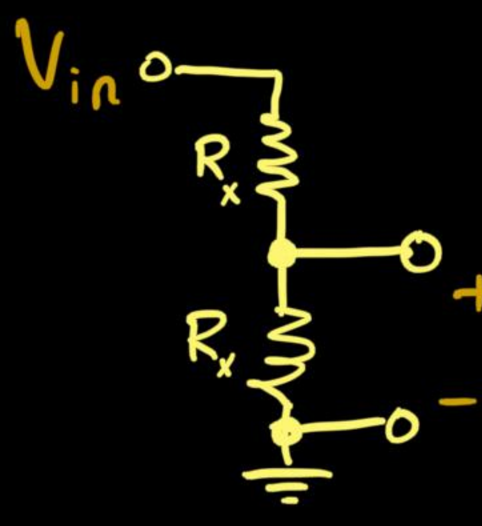
(these are the "modules")



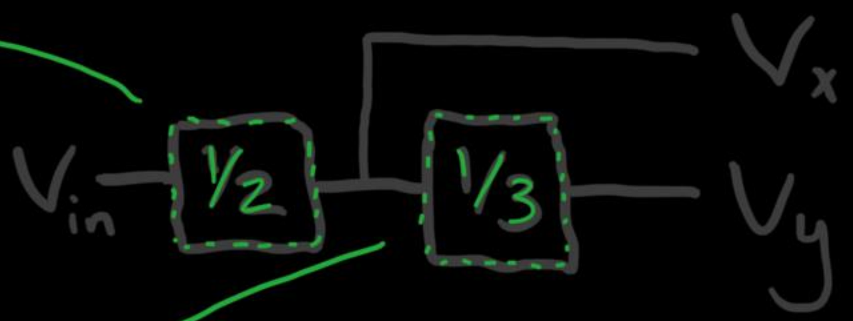
a) Draw a voltage divider for each operation:

$\boxed{1/2}$:

- → Takes in V_{in}
- → Returns V_x

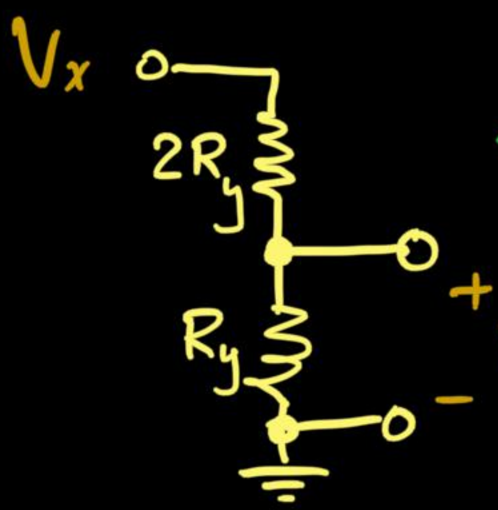


$$V_x = \left(\frac{R_x}{R_x + R_x}\right) V_{in} = \frac{1}{2} V_{in} \checkmark$$



$\boxed{1/3}$:

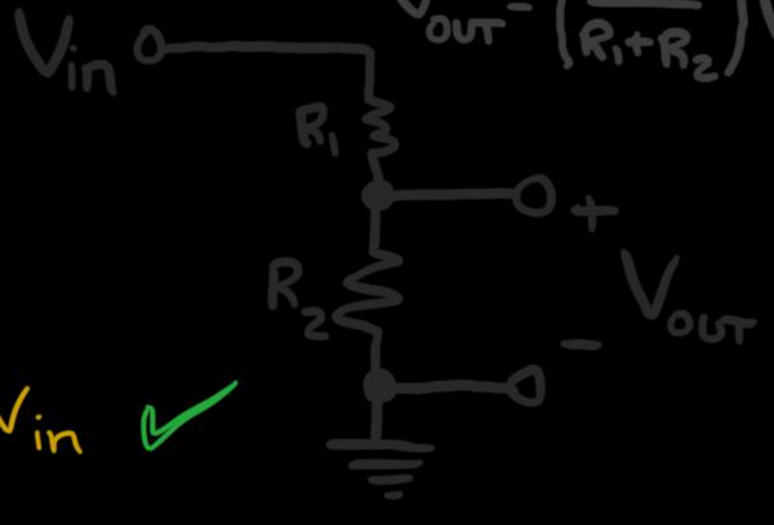
- → Takes in V_x
- → Returns V_y



$$V_y = \left(\frac{R_y}{R_y + 2R_y}\right) V_{in} = \frac{1}{3} V_{in} \checkmark$$

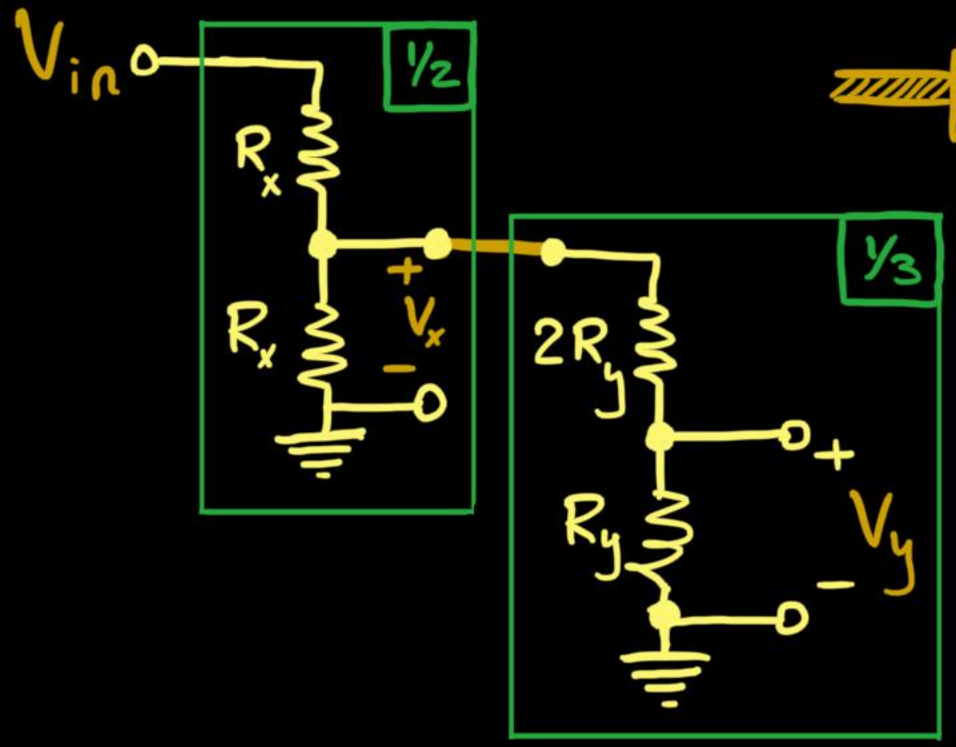
Recall:

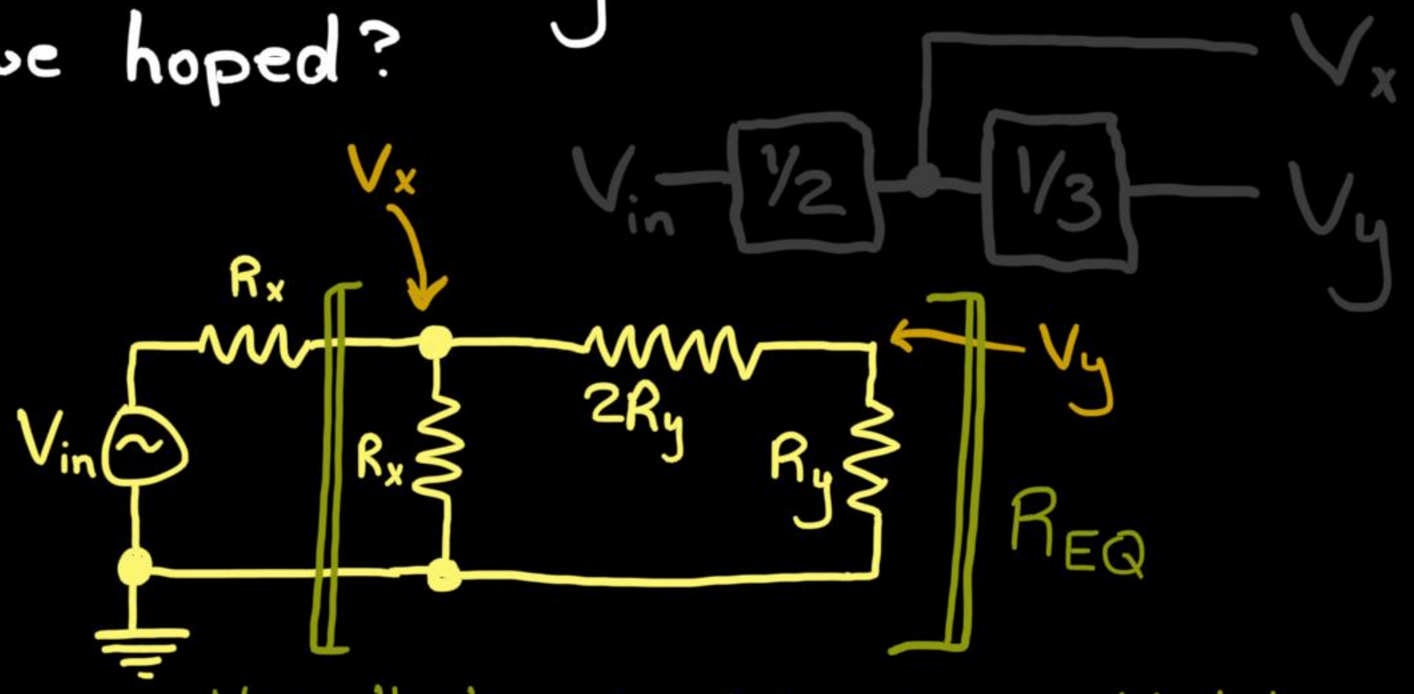
$$V_{OUT} = \left(\frac{R_2}{R_1 + R_2}\right) V_{in}$$



Note: While the ratio of resistor values within $\boxed{1/2}$ and $\boxed{1/3}$ circuits are separately fixed ($R_1 = R_2$ and $R_1 = 2R_2$ respectively), there is no relation of these values between circuits. Thus they've been left as R_x and R_y in general.

b) Link the two circuits as initially stated. Does it behave as we hoped?





Now that a load has been added to the [1/2] module, its behavior is altered by an alternate route for current!

$$R_{EQ} = \left(\frac{1}{R_x} + \frac{1}{2R_y + R_y}\right)^{-1} = \frac{3R_yR_x}{3R_y + R_x}$$

$$V_x = \left(\frac{R_{EQ}}{R_x + R_{EQ}}\right)V_{in} = \left(\frac{3R_xR_y/(R_x+3R_y)}{R_x + 3R_xR_y/(R_x+3R_y)}\right)V_{in}$$

$$= \left(\frac{3R_xR_y}{\underbrace{R_x^2 + 3R_xR_y}_{R_x(R_x+3R_y)} \underbrace{+\ 3R_xR_y}_{=6R_xR_y}}\right)V_{in}$$

$$= \left(\frac{1}{\frac{R_x^2}{3R_xR_y} + 2}\right)V_{in} = \left(\frac{1}{2 + \frac{R_x}{3R_y}}\right)V_{in}$$

$$V_x = \left(\frac{1}{2 + \frac{R_x}{3R_y}}\right)V_{in} \neq \frac{1}{2}V_{in} \quad ☹$$

$$V_y = \frac{1}{3}\left(\frac{1}{2 + \frac{R_x}{3R_y}}\right)V_{in} \neq \frac{1}{6}V_{in} \quad 😕$$

Since the latter [1/3] still has no load, $V_y = \frac{1}{3}V_x$.

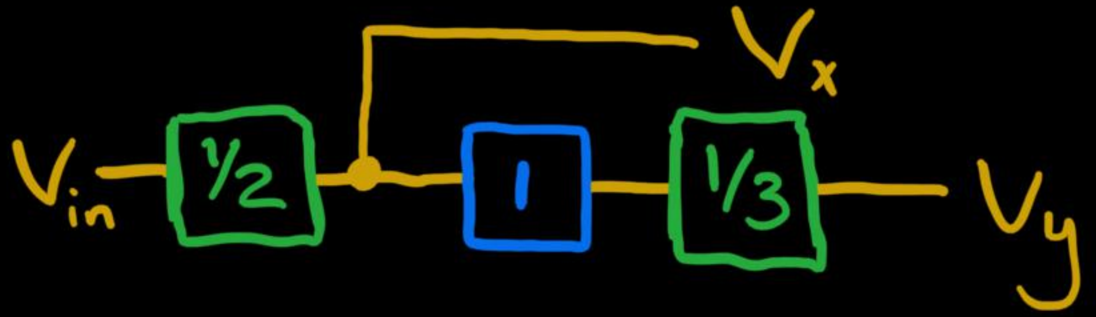
Oh no! I guess just slapping 2 voltage dividers together...

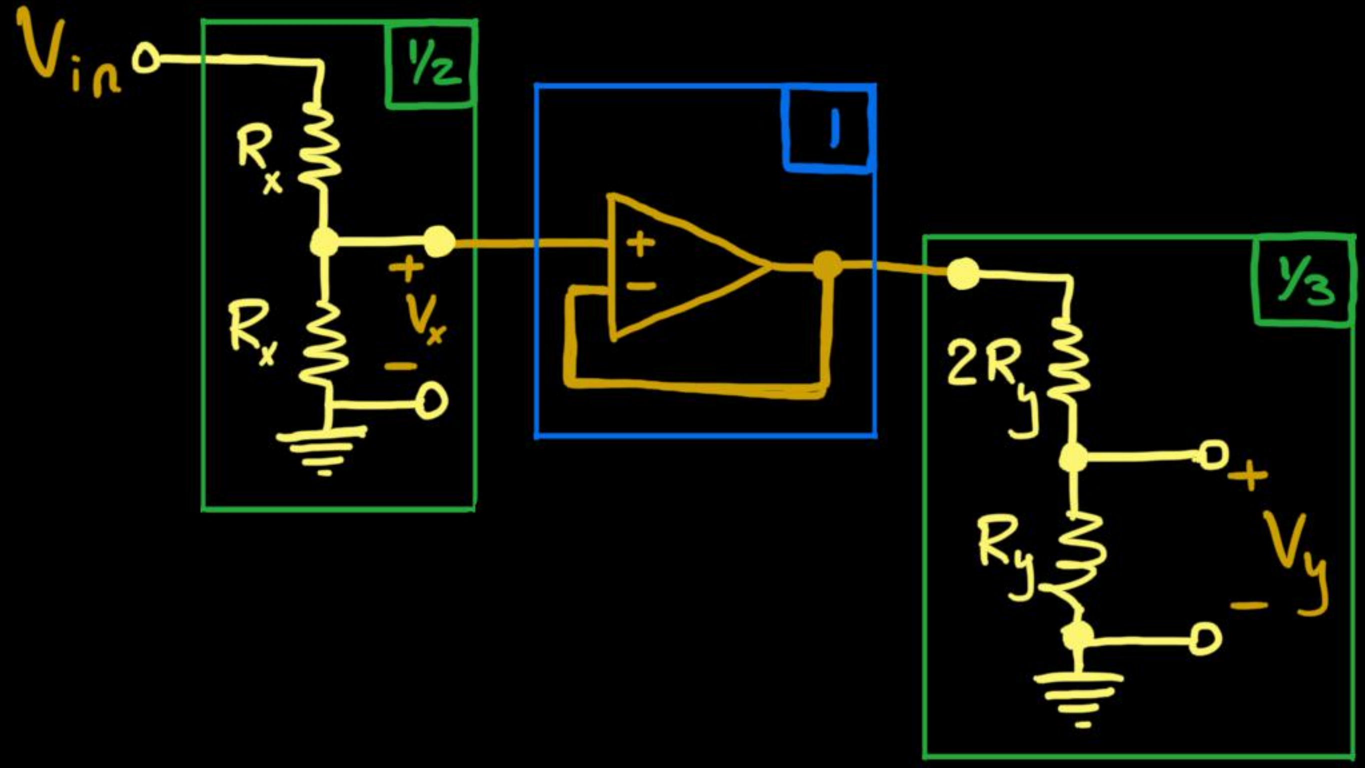
Notice though that for $R_y \gg R_x$ we find $V_x \approx \frac{1}{2}V_{in}$ and $V_y \approx \frac{1}{6}V_{in}$, but we want to be picky and have the circuits work exactly like in their isolated modules regardless of R_x, R_y. We need op-amps...

This is because effectively no current goes into the [1/3] circuit and it still "looks open" to the [1/2] circuit.

c) Try including an op-amp (in negative feedback) within the circuit to circumvent the loading issue!

Try inserting a unity gain op-amp circuit between them, so the output of $\boxed{1/2}$ feeds to an op-amp input terminal:

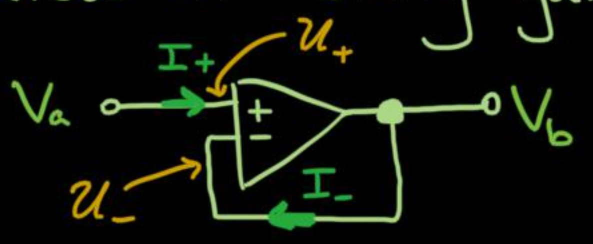




Since the inputs to an op-amp act like open circuits, the $\boxed{1/2}$ preserves its behavior! ✓

Quick aside...

Review of unity gain op-amp circuit



Golden rule: $I_+ = I_- = 0$

Gain: $V_b = A(u_+ - u_-)$

where A is HUGE ($A \approx 10^6$)

Now $u_+ = V_a$ and $u_- = V_b$ (since they're the same node), so we find:

$$V_b = A(V_a - V_b) \Rightarrow (1+A)V_b = A V_a$$

$$V_b = \left(\frac{1}{1+(1/A)}\right) V_a \approx V_a$$ ✓

② Modular Op-Amp Circuits

Perform the following operations using op-amps:

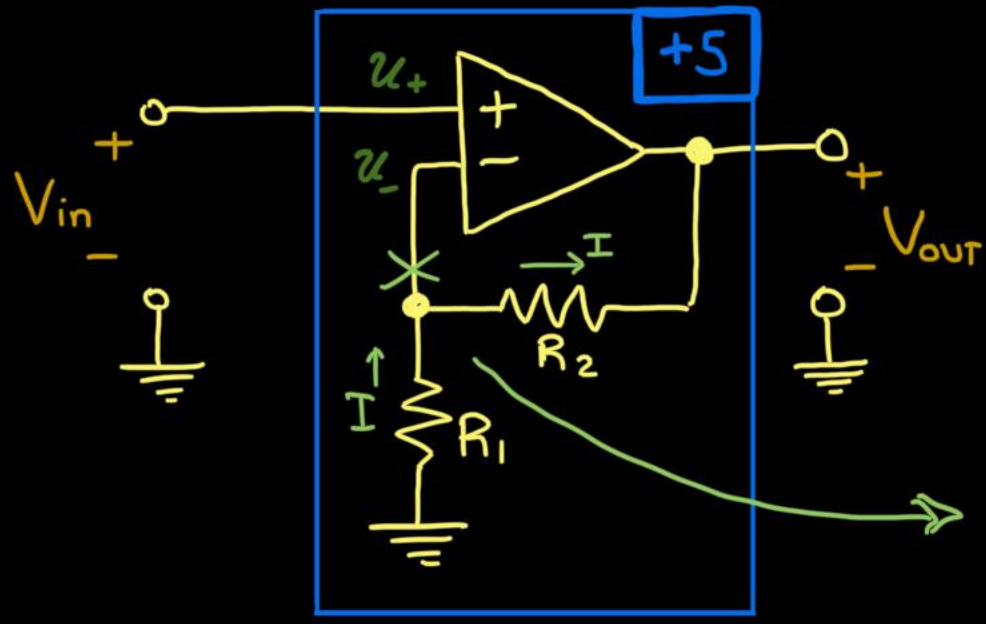
(a) $V_{OUT} = +5\,V_{in}$

(b) $V_{OUT} = -2\,V_{in}$

(c) $V_{OUT} = V_1 + V_2$

Can these circuits be combined while maintaing their function?

(a) We need a non-inverting amplifier:



Since $u_- = u_+ = V_{in}$, we know $I = \frac{V_{in} - 0}{R_1}$

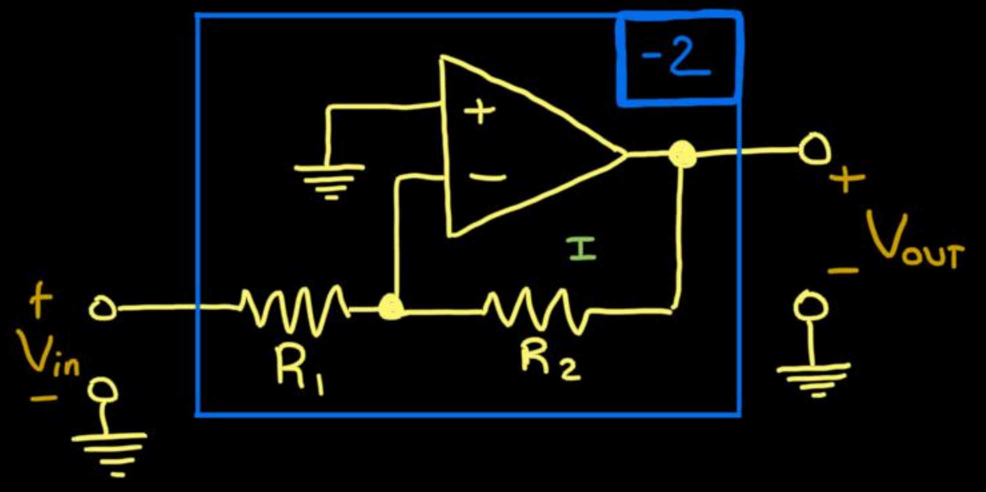
and so $V_{OUT} = V_{in} + IR_2$

$$= V_{in}\left(1 + \frac{R_2}{R_1}\right)$$

Now we need $(1 + R_2/R_1) = 5 \Rightarrow \boxed{R_2 = 4R_1}$

Given that 'Vin' leads into an op-amp input terminal (no current), we can safely connect this circuit to others without issue :)

(b) We need an inverting amplifier:



Since $\mathcal{U}_- = \mathcal{U}_+ = 0$, we know $I = \frac{V_{in} - 0}{R_1}$

and so $V_{OUT} = V_{in} + IR_2$

$$= V_{in}\left(1 + \frac{R_2}{R_1}\right)$$

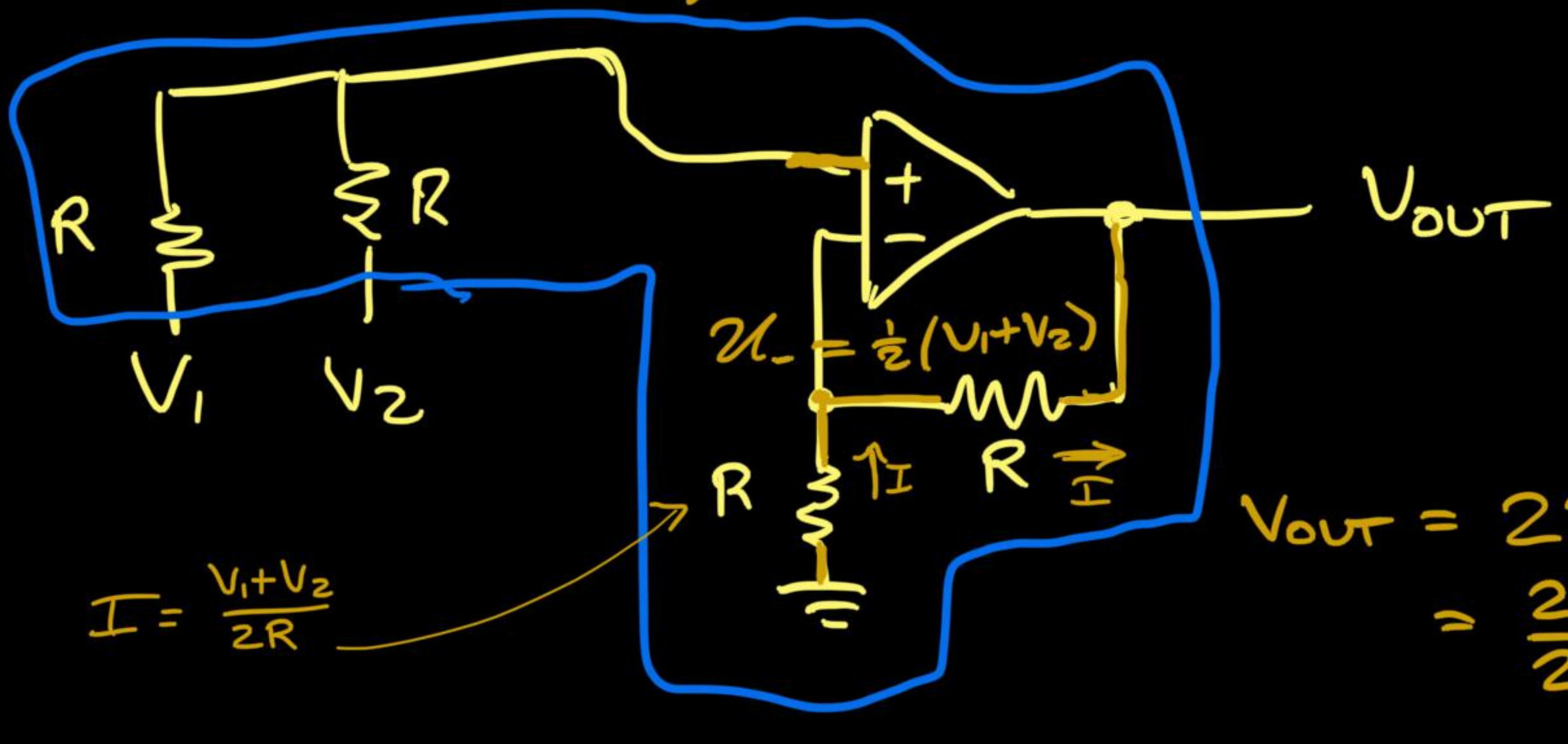
Now we need $(1 + R_2/R_1) = 5 \Rightarrow \boxed{R_2 = 4R_1}$

Given that 'V_{in}' does have a current connection to V_{OUT}, we would not be able to attach a voltage divider before this circuit without messing up that divider. However, the gain $\boxed{-2}$ works regardless!

we'd need a buffer :)

(c) $V_{OUT} = V_1 + V_2$

$\mathcal{U}_+ = \frac{1}{2}(V_1 + V_2)$



$I = \frac{V_1 + V_2}{2R}$

$$V_{OUT} = 2\mathcal{U}_-$$
$$= \frac{2}{2}(V_1 + V_2)$$
$$= V_1 + V_2$$

:)