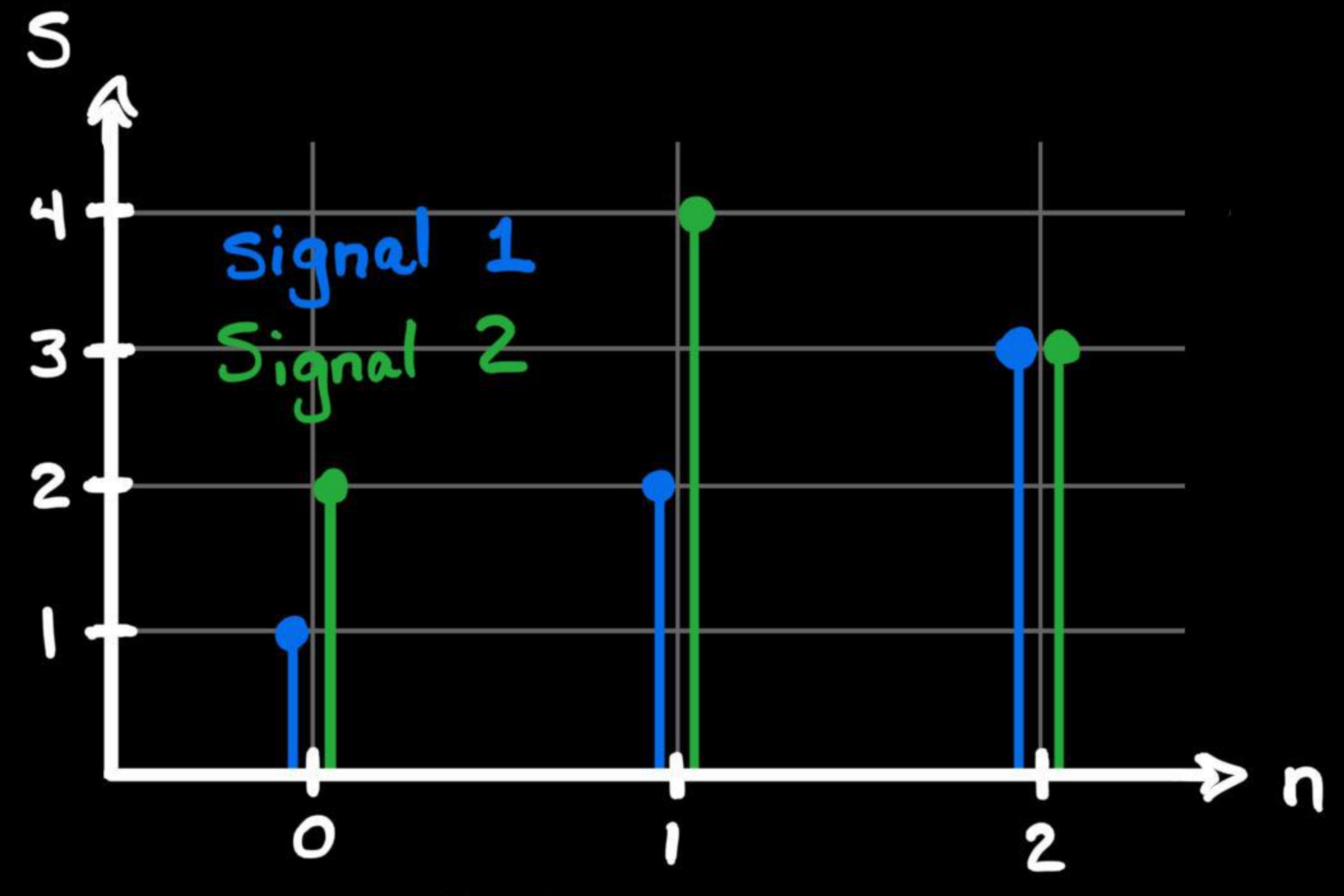
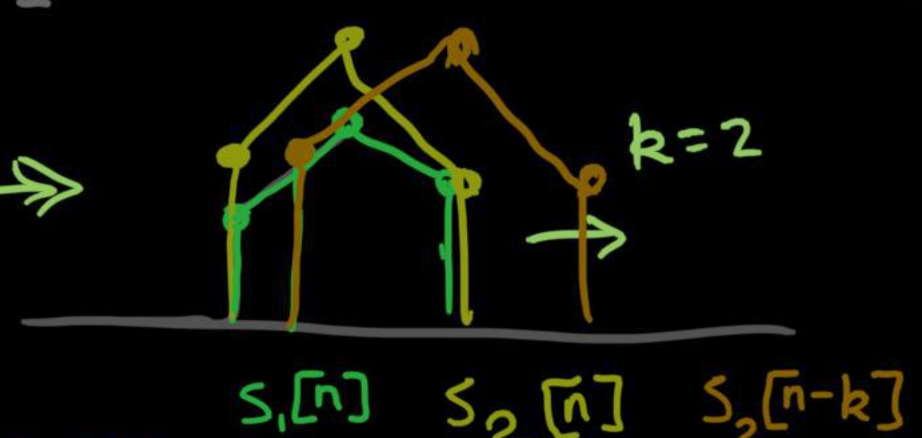
③ Correlation

$$\mathrm{Corr}_x(y)[k] = \sum_{i=-2}^{4} x[i] \cdot y[i-k]$$



* $S_1[n] = S_2[n] = 0$ if $n<0$ or $n>2$.

Think about 'k' as a sliding delay



a) Find $\mathrm{Corr}_{S_1}(S_2)[k]$

$S_2[0+2] = S_2[2]$

k=-2:

n	(-2	-1)	0	1	2	(3	4)	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n+2]$	2	4	3	0	0	0	0	
$\langle S_1, S_2[n+2]\rangle$	0 +	0 +	3 +	0 +	0 +	0 +	0	= 3

k=-1:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n+1]$	0	2	4	3	0	0	0	
$\langle S_1, S_2[n+1]\rangle$	0 +	0 +	4 +	6 +	0 +	0 +	0	= 10

k=0:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n]$	0	0	2	4	3	0	0	
$\langle S_1, S_2[n]\rangle$	0 +	0 +	2 +	8 +	9 +	0 +	0	= 19

k=+1:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n-1]$	0	0	0	2	4	3	0	
$\langle S_1, S_2[n-1] \rangle$	0 +	0 +	0 +	4 +	12 +	0 +	0	= 16

k=+2:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n-2]$	0	0	0	0	2	4	12	
$\langle S_1, S_2[n-2] \rangle$	0 +	0 +	0 +	0 +	6 +	0 +	0	= 6

$$Corr_{S_1}(S_2)[k] = \begin{bmatrix} 3 \\ 10 \\ 19 \\ 16 \\ 6 \end{bmatrix}$$

b) Find $Corr_{S_2}(S_1)[k]$

k=-2:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n+2]$	1	2	3	0	0	0	0	
$\langle S_2, S_1[n+2] \rangle$	0 +	0 +	6 +	0 +	0 +	0 +	0	= 6

k=-1:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n+1]$	0	1	2	3	0	0	0	
$\langle S_2, S_1[n+1] \rangle$	0 +	0 +	4 +	12 +	0 +	0 +	0	= 16

k=0:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n]$	0	0	1	2	3	0	0	
$\langle S_2, S_1[n] \rangle$	0 +	0 +	2 +	8 +	9 +	0 +	0	= 19

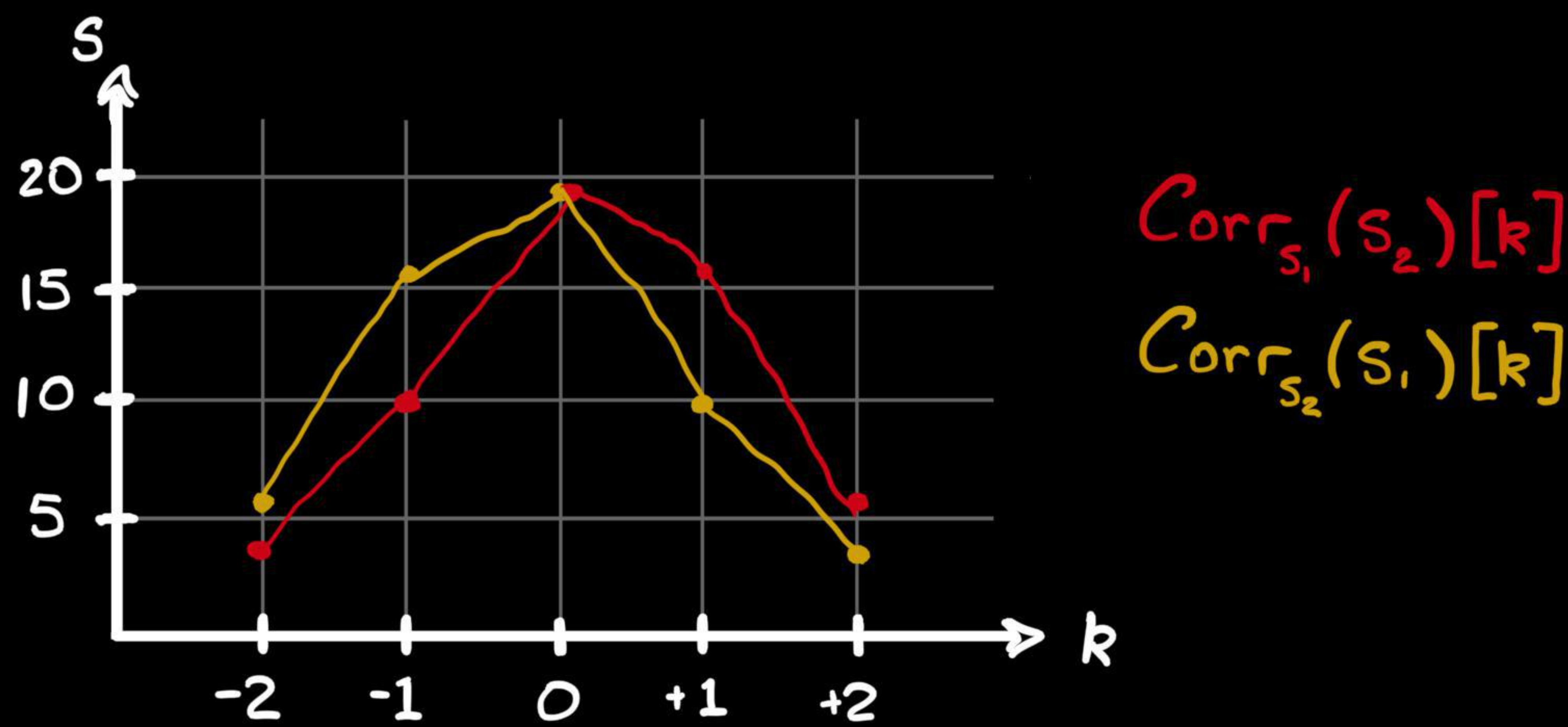
k = +1:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n-1]$	0	0	0	1	2	3	0	
$\langle S_2, S_1[n-1] \rangle$	0 +	0 +	0 +	4 +	6 +	0 +	0	= 10

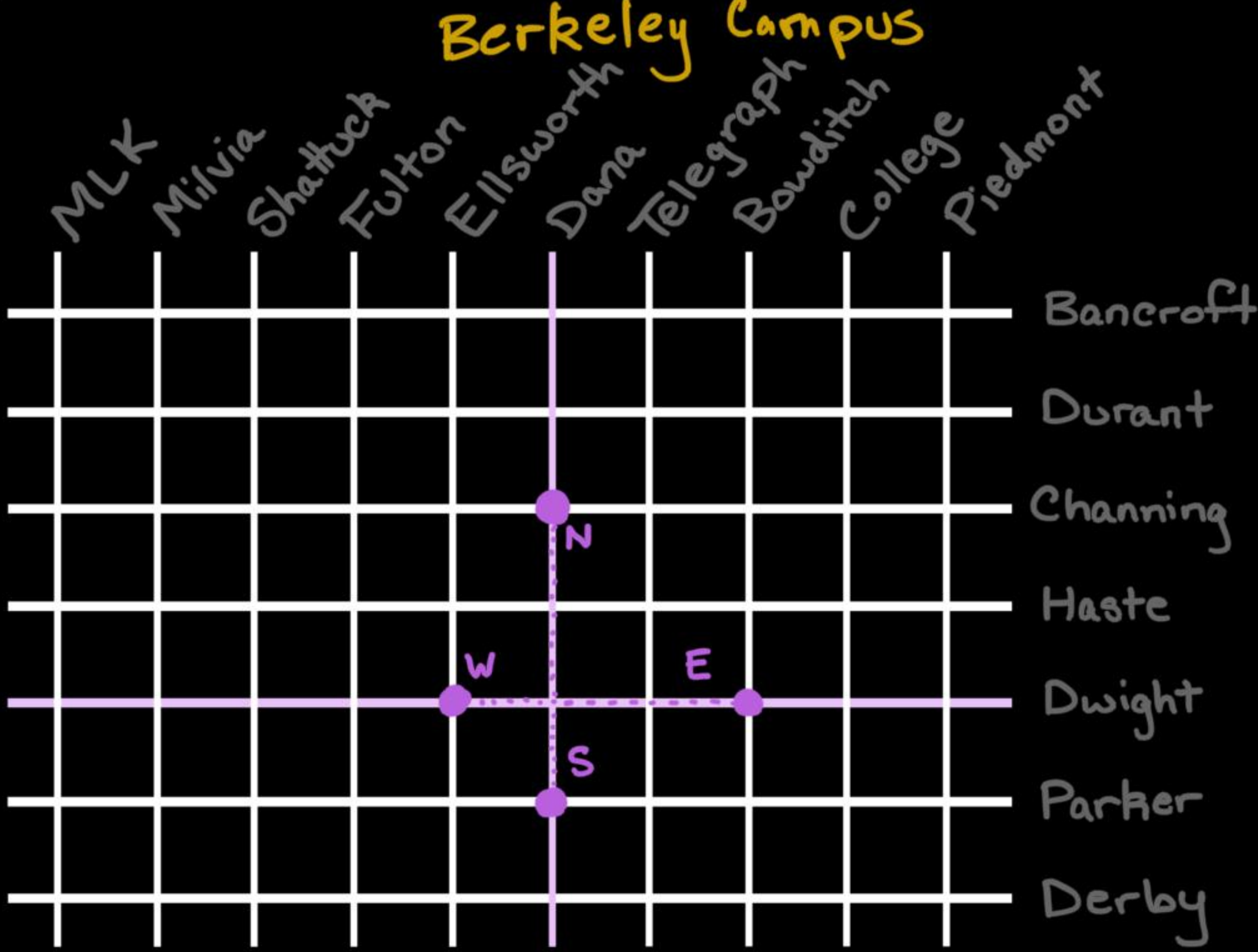
k = +2:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n-2]$	0	0	0	0	1	2	3	
$\langle S_2, S_1[n-2] \rangle$	0 +	0 +	0 +	0 +	3 +	0 +	0	= 3

$$Corr_{S_2}(S_1)[k] = \begin{bmatrix} 6 \\ 16 \\ 19 \\ 10 \\ 3 \end{bmatrix}$$

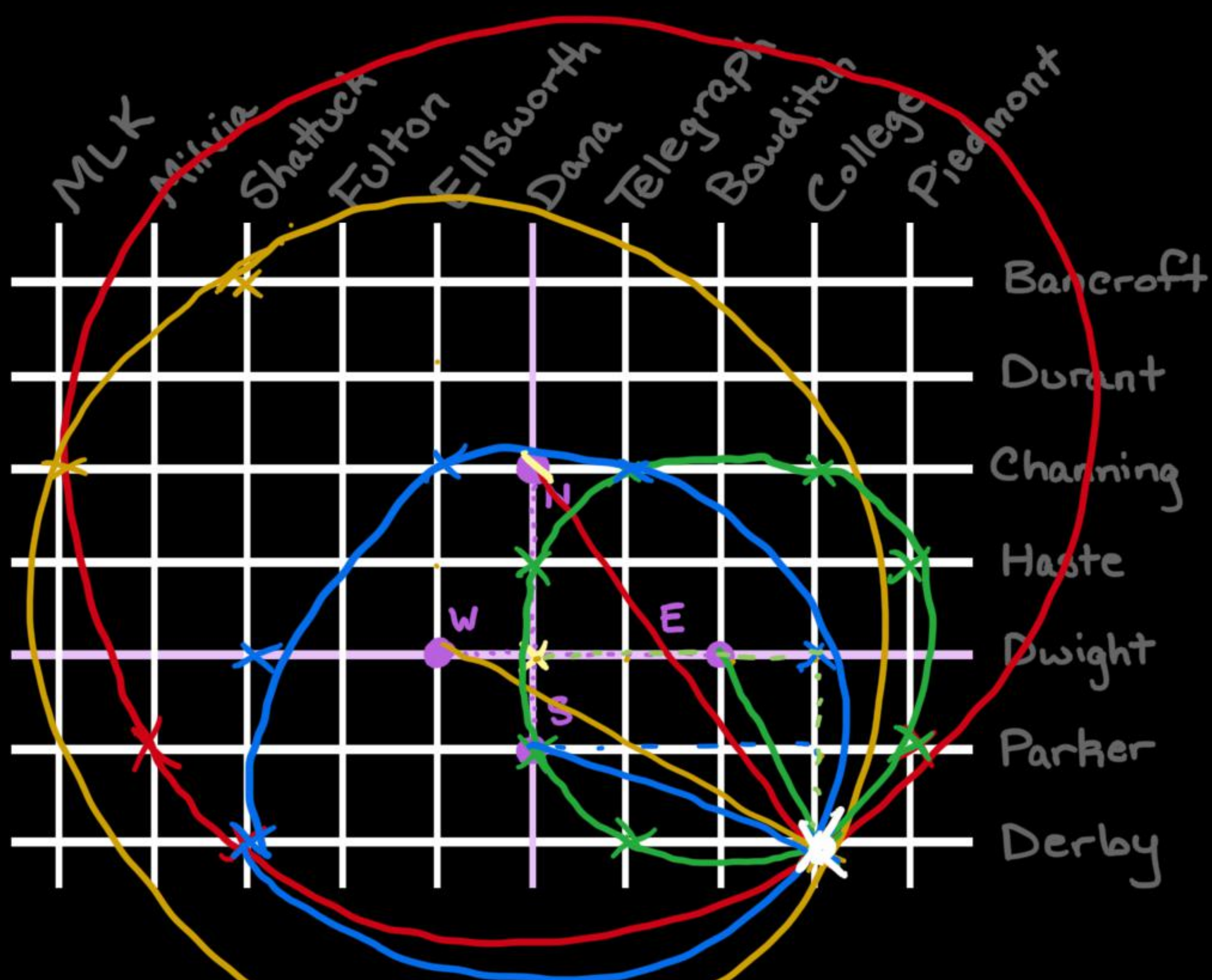


② Save Mr. Muffin!



Mr. Muffin is lost!
Luckily his Blue-Tooth ✱ collar pings 4 city sensors to provide an absolute distance between Mr. Muffin an each sensor. (Up to 5 Blocks)

a) Provided these data, can you identify where Mr. Muffin is?



(Distances, in city blocks)

Sensor Data:

$$\begin{array}{c} N \sim \\ W \sim \\ E \sim \\ S \sim \end{array} \begin{bmatrix} 5 \\ \sqrt{20} \\ \sqrt{5} \\ \sqrt{10} \end{bmatrix} \begin{array}{l} = \sqrt{3^2+4^2} \\ = \sqrt{2^2+4^2} \\ = \sqrt{1^2+2^2} \\ = \sqrt{1^2+3^2} \end{array}$$

$$\left[\ x=3 \quad y=-2\ \right]$$

$$D_N^2 = (x-0)^2 + (y-2)^2$$

b) Can you accomplish this by setting up a system of equations?

Sensor

N ~ (0, 2)

W ~ (-1, 0)

E ~ (+2, 0)

S ~ (0, -1)

$$D_N^2 = 25 = (x-0)^2 + (y-2)^2$$
$$= x^2 + y^2 - 4y + 4$$
$$\hookrightarrow 21 = (x^2+y^2) - 4y \qquad \text{"}EQN_N\text{"}$$

$$D_W^2 = 20 = (x+1)^2 + y^2$$
$$= x^2 + 2x + 1 + y^2$$
$$\hookrightarrow 19 = (x^2+y^2) + 2x$$

$$D_E^2 = 5 = (x-2)^2 + y^2$$
$$= x^2 - 4x + 4 + y^2$$
$$\hookrightarrow 1 = (x^2+y^2) - 4x$$

$EQN_N - EQN_E$: $20 = -4y + 4x$

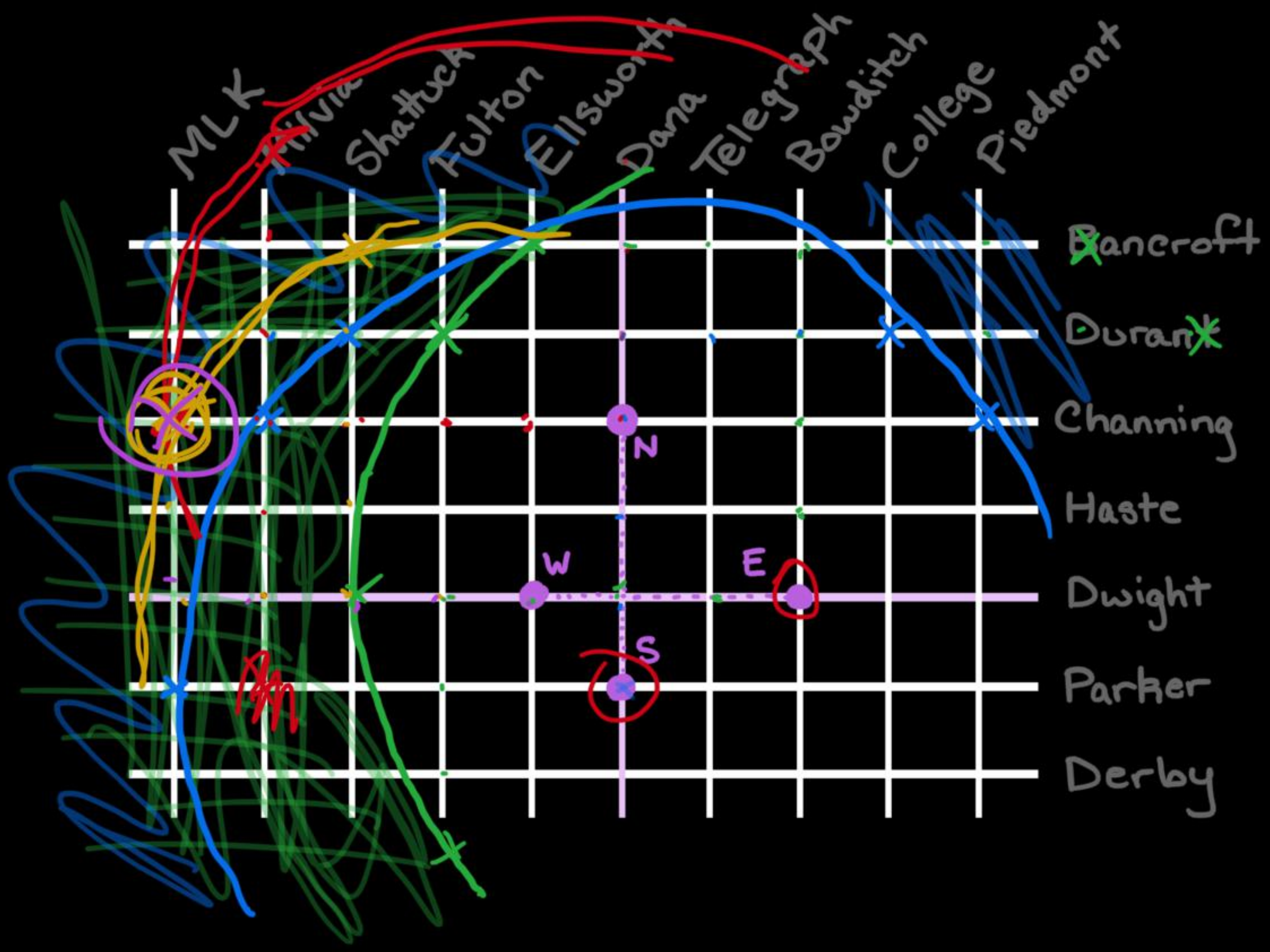
$EQN_W - EQN_E$: $18 = 2x + 4x = 6x$

$x = 3$

$$20 + 4y = 12$$
$$4y = -8$$
$$y = -2$$

✓

c) Provided new data in which E & S are now out of range, where is Mr. Muffin?



(Distances, in city blocks)

Sensor Data:

N ~ 5

W ~ $\sqrt{20}$

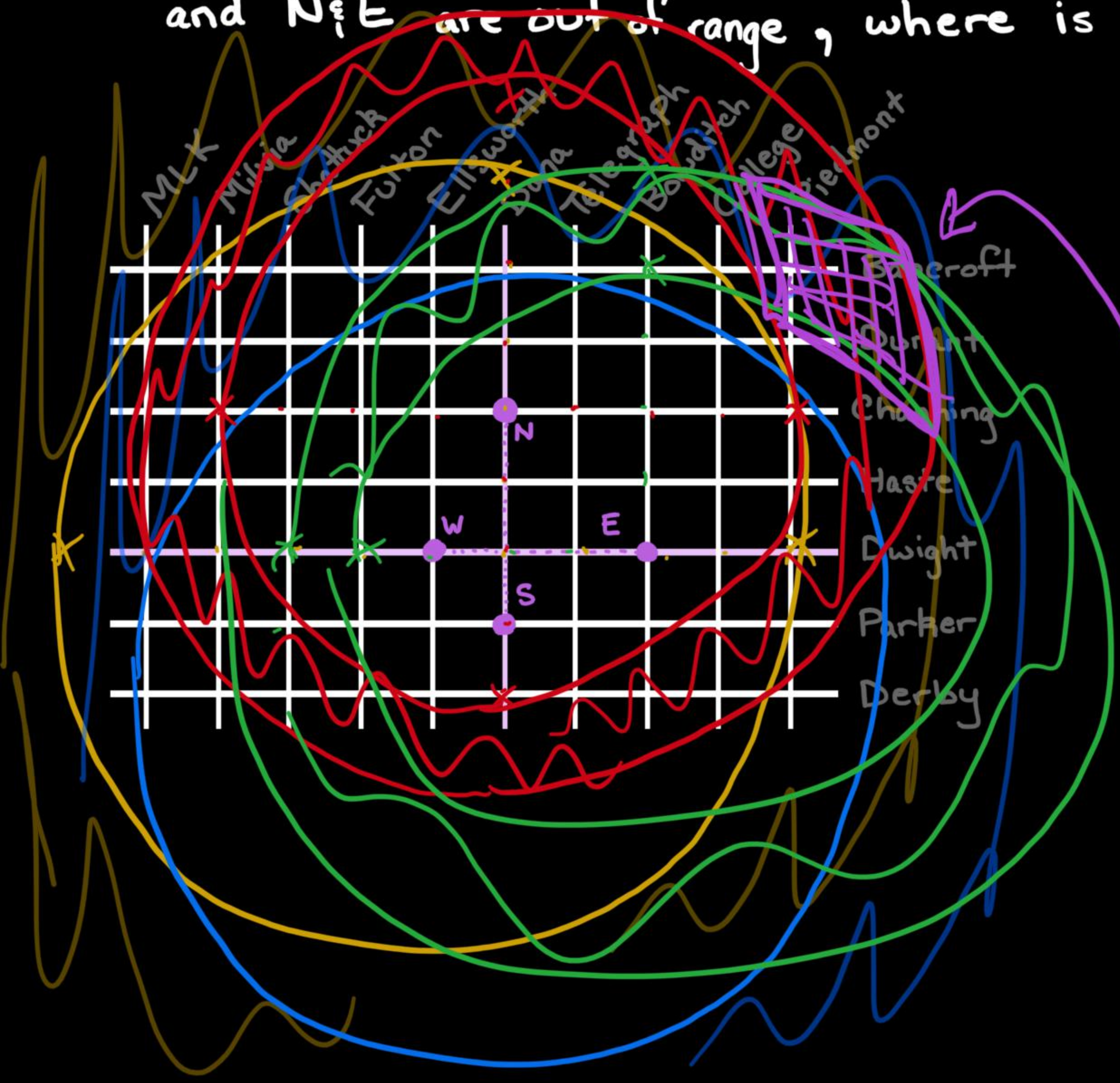
E ~ Out-of-range

S ~ Out-of-range

$$\begin{bmatrix} x = -5 \\ y = +2 \end{bmatrix}$$

✓

d) Provided uncertain date in which W & S are given with ±0.5 and N & E are out of range, where is Mr. Muffin?



(Distances, in city blocks)

Sensor Data:

N ~ 4.5 ± 0.5

W ~ Out-of-range

E ~ 4.5 ± 0.5

S ~ Out-of-range

Must be in this region, but exact block is unknown